# Spatio-Temporal Array-Receiver for Multipath Tracking in Cellular CDMA

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*Abstract*— ***Efficient multipath tracking is a crucial requirement for high performance in asynchronous cellular CDMA. Despite the recent increased focus on use of array receivers at the base stations, the problem of time-delay estimation for individual paths has not been specifically addressed. In this contribution we propose simple and fast multipath tracking procedures in a nonstationary CDMA environment, using the Spatio-Temporal Array-Receiver (STAR). Both the time-delays and their number are tracked in time in a partially blind scheme with a low order of computational complexity. We show by simulations the tracking capability of STAR at a high interference level.***

## I. Introduction

Time synchronization and tracking are important issues in cellular CDMA. The performance of the RAKE receiver particularly relies on correct timing [1]. So far, the acquisition process has involved heavy hypotheses testing of likely rough values of the multipath time-delays. Usually, their number is arbitrarily fixed. Once this approximate synchronization has been achieved, the estimates are refined to the required accuracy with a closed loop tracking technique [1]. Although recently array processing was advantageously introduced in cellular CDMA to improve its capacity, the crucial problem of timing was not specifically addressed. A 2D-RAKE receiver was proposed in [2] assuming perfect timing, while another similar structure was developed in [3] assuming approximate synchronization. On the other hand, blind channel identification or equalization schemes were proposed in [4],[5] without any need for multipath tracking (*i.e.*, number of paths and their time-delays). However, we showed in [6] that the combination of multipath tracking and blind identification reduces channel identification errors by a factor of the processing gain over the number of paths, and therefore improves the capacity of the CDMA system. Along this line, we proposed a Spatio-Temporal Array-Receiver (STAR) which outperforms previous methods [1]-[5]. Actually its time acquisition is very accurate [6] but too expensive to be used continuously for the tracking of nonstationarities. We herein develop for STAR simple and efficient procedures for multipath tracking.

We recently proposed a novel block formulation of the post-correlated data in the time domain [6]. This new formulation, presented in section II, amounts to an original data model of an instantaneous mixture in a one-dimensional spatio-temporal signal subspace. To the resulting "narrowband" model (*i.e.* iteration at the bit rate), we applied an adaptive subspace-tracking procedure which achieves a blind identification and equalization of the spatio-temporal propagation channel. This basic structure of STAR is described in section III. We also proposed a new localization procedure for estimating the number of paths and their time delays from the identified channel. This step which improves identification is just used here to start the time acquisition of STAR or to restart it whenever the tracking is interrupted.

We herein develop alternative new procedures for multipath tracking. From the narrowband model and the basic structure of STAR [6], we first make an analogy with DOA (direction of arrival) tracking and propose a time-delays tracking technique based on the algorithm proposed in [7]. This original step, described in section IV, avoids a continuous and expensive time acquisition (*i.e.*, localization) by [6] and improves channel identification errors. In section V, we secondly develop a new procedure for the tracking of the number of paths using techniques proposed in [7] for DOA tracking. Finally, STAR offers an attractive and simple structure with a low order of computational complexity. Simulations in section VI confirm its effectiveness and its multipath tracking capability at a high interference level.

## II. New Formulation and Block Modeling of the Receiver

We consider a CDMA cellular system base-station equipped with a receiving antenna of $M$ sensors. The PSK bit sequence of the desired user is first differentially encoded at the rate $1/T$ where $T$ is the bit duration. The resulting DPSK sequence $b(t)$ is then spread by a periodic personal code $c(t)$ at the rate $1/T_c$ where $T_c$ is the chip pulse duration. For the sake of simplicity and without loss of generality, the period of $c(t)$ is assumed to be equal to the bit duration $T$ although the results can be adequately extended to multiples of $T$. The processing gain is defined by $L = T/T_c$. We also assume a multipath environment where $P$ is the number of paths.

At time $t$, the observation vector received by the antenna array can be written as follows:

$$X(t)=\psi(t)\sum_{p=1}^{P}G_p(t)\varepsilon_p(t)b(t-\tau_p(t))c(t-\tau_p(t))+\mathcal{I}(t) \ , \quad (1)$$

where $\tau_p(t) \in [0,T[$ for $p = 1,\cdots,P$, are the propagation time delays along the $P$ paths, $G_p(t)$ are the propagation vectors with an equal norm to be fixed later, $\varepsilon_p^2(t)$ are the relative powers received from the desired user (*i.e.*, $\sum_{p=1}^{P}\varepsilon_p^2(t)=1$) along the different paths and $\psi^2(t)$ is the total received power. We assume their variations in time to be sufficiently slow relative to the bit duration $T$ to permit an assumption of constant path energies over the bit duration. The noise term $\mathcal{I}(t)$ includes the thermal noise received at the antenna elements and the total interference of the other users from either inside or outside the cell of interest.

We now define the post-correlated observation vector by:

$$Z(t') = \frac{1}{T}\int_0^T X(t+t')c(t)dt \ . \tag{2}$$

With respect to the new formulation proposed in [6], we process $Z(t')$ over successive frames of period $T$. We hence define over the time-interval $[0,T[$ the post-correlated observation vector of the frame number $n$ by $Z_n(t') = Z(t'+nT)$ for $t' \in [0,T[$. We then sample $Z_n(t')$ for $t' \in [0,T[$ at the chip rate $1/T_c$ and form at each frame the $M \times L$ block data matrix of the post-correlated observation $\mathbf{Z}_n = [Z_n(0), Z_n(T_c), \cdots, Z_n((L-1)T_c)]$. From [6], $\mathbf{Z}_n$ can be explicitly written as follows:

$$\begin{aligned}\mathbf{Z}_n &= b_n\psi_n\sum_{p=1}^{P}G_{p,n}\varepsilon_{p,n}D_{p,n}^T + \mathbf{N}_n \\ &= b_n\psi_n\mathbf{G}_n\Upsilon_n\mathbf{D}_n^T + \mathbf{N}_n \ ,\end{aligned} \tag{3}$$

where $b_n$, $\psi_n$, $G_{p,n}$ and $\varepsilon_{p,n}$ are respectively equal to $b(nT)$, $\psi(nT)$, $G_p(nT)$ and $\varepsilon_p(nT)$, where $\mathbf{N}_n$ is the noise matrix and where $\mathbf{G}_n = [G_{1,n},\cdots,G_{P,n}]$, $\Upsilon_n = \text{diag}\{[\varepsilon_{1,n},\cdots,\varepsilon_{P,n}]\}$ and $\mathbf{D}_n = [D_{1,n},\cdots,D_{P,n}]$. In the above equation, $\mathbf{G}_n$ is the propagation matrix, $\Upsilon_n$ is the diagonal matrix of power partition and $\mathbf{D}_n$ is the sampled impulse response matrix of the multipath time-delays $\tau_{1,n},\cdots,\tau_{P,n}$. In the frequency domain, the column-by-column FFT of $\mathbf{D}_n$ denoted by $\mathcal{D}_n$ is given by:

$$\mathcal{D}_n = [\mathcal{D}_{1,n},\cdots,\mathcal{D}_{P,n}] \ , \tag{4}$$

$$\text{for } p=1,\cdots,P \quad \mathcal{D}_{p,n} = \mathcal{F}(\tau_{p,n}) =$$

$$\left[1, e^{-j2\pi\tau_{p,n}\frac{1}{L}},\cdots,e^{-j2\pi\tau_{p,n}\frac{L-1}{L}}\right]^T \in \mathbf{\Gamma} \ , \tag{5}$$

and its columns belong to a manifold say $\mathbf{\Gamma}$. For the sake of simplicity, we will equivalently say next that $\mathbf{D}_n$ or $\mathcal{D}_n$ belongs to the manifold $\mathbf{\Gamma}$ without distinction. Later, we shall use this feature to implement the time-delays tracking step.

We rewrite Eq. (3) in a compact form for further processing:

$$\mathbf{Z}_n = \mathbf{H}_n s_n + \mathbf{N}_n \ , \tag{6}$$

where $\mathbf{H}_n = \mathbf{G}_n\Upsilon_n\mathbf{D}_n^T = \mathbf{J}_n\mathbf{D}_n^T$ is the spatio-temporal response matrix and $s_n = b_n\psi_n$ is the signal component. Notice here that while the columns of the time response matrix $\mathbf{D}_n$ are defined in the manifold $\mathbf{\Gamma}$, the spatial response matrix $\mathbf{J}_n = \mathbf{G}_n\Upsilon_n$ is unmodeled and defines the unknown part of $\mathbf{H}_n$. Finally, we transform the matrices $\mathbf{Z}_n$,$\mathbf{H}_n$ and $\mathbf{N}_n$ into $(M \times L)$-dimensional vectors by putting their columns in one column and obtain:

$$\underline{Z}_n = \underline{H}_n s_n + \underline{N}_n \ , \tag{7}$$

where $\underline{Z}_n$, $\underline{H}_n$ and $\underline{N}_n$ respectively denote the resulting vectors. To avoid the ambiguity due any multiplicative factor between $\underline{H}_n$ and $s_n$, we fix the norm of $\underline{H}_n$ to $\sqrt{M}$. As mentioned earlier below Eq. (1), the norm of propagation vectors $G_{p,n}$ are implicitly fixed to $\sqrt{K_n}$ for $p=1,\cdots,P$ in such a way that $\|\underline{H}_n\|^2 = M$. We will explicitly give its expression later.

Contrary to previous methods [1]-[5], STAR formulates from the block despread data an attractive instantaneous mixture model of a narrowband source with a one-dimensional spatio-temporal channel and processes it at the bit rate. With this new formulation and block modeling of the receiver, we next describe the basic structure of STAR developed in [6]. Then we explain how to apply the beamforming and subspace-tracking method developed in [7] for multipath tracking.

## III. Spatio-Temporal Identification and Equalization

We assume that the time-delays $\hat{\tau}_{1,n},\cdots,\hat{\tau}_{\hat{P},n}$ (*i.e.*, $\hat{\mathbf{D}}_n$) and their number $\hat{P}$ have been once estimated along with $\hat{\mathbf{G}}_n$, $\hat{\Upsilon}_n$ and $\hat{\mathbf{H}}_n = \hat{\mathbf{G}}_n\hat{\Upsilon}_n\hat{\mathbf{D}}_n^T$, as explained in [6]. In the next sections, we will explain how to adaptively update these quantities at each block iteration. We will also show how to avoid the localization step of [6] in the following. From the resulting estimate of $\underline{H}_n$ say $\hat{\underline{H}}_n$, we can extract the signal component $s_n$ by any distortionless beamformer $W_n$ (*i.e.*, $W_n^H\hat{\underline{H}}_n = 1$). As in [6], we assume that $\underline{N}_n$ is an uncorrelated white noise vector and apply a Delay-Sum (DS) beamformer (*i.e.*, $W_n = \hat{\underline{H}}_n/M$) for source extraction as follows:

$$\hat{s}_n = \text{Real}\left\{W_n^H\underline{Z}_n\right\} = \text{Real}\left\{\frac{\hat{\underline{H}}_n^H\underline{Z}_n}{M}\right\} \ , \tag{8}$$

where the Real{.} function is used to extract the real sequence $s_n = \psi_n b_n$. The total power received from the desired user is estimated as follows:

$$\hat{\psi}_n^2 = (1-\alpha)\ \hat{\psi}_{n-1}^2 + \alpha\ |\hat{s}_n|^2 \ , \tag{9}$$

where $\alpha \ll 1$ is a smoothing factor of the power estimate. This estimate is used for power control. As in [6], the bit sequence $b_n$ is estimated by the following decision equation:

$$\hat{b}_n = \text{Sign}\left\{\hat{s}_n\right\} \ . \tag{10}$$

The Sign{.} function is introduced to quantize the received binary DPSK sequence. The decision equation (10) can be easily adapted to other constellations.

The propagation vector $\underline{H}_n$ is finally tracked by the following subspace-based tracking procedure [7]:

$$\hat{\underline{H}}_{n+1} = \hat{\underline{H}}_n + \mu_n(\underline{Z}_n - \hat{\underline{H}}_n\hat{s}_n)\hat{s}_n^* \ , \tag{11}$$

where $\mu_n$ is an adaptation step-size, possibly normalized, and where $*$ denotes complex conjugation. As explained in [6], this equation converges to $\underline{\mathbf{H}}_n$ with norm $\sqrt{M}$ within a sign ambiguity (*i.e.*, $\hat{\underline{\mathbf{H}}}_n \simeq \pm\underline{\mathbf{H}}_n$ when $n \to \infty$). This ambiguity has no consequences other than changing the signs of $\hat{s}_n$ and $\hat{b}_n$ in (8) and (10) respectively, since $b_n$ is a DPSK sequence. For the sake of simplicity, we assume in the following that $\hat{\underline{\mathbf{H}}}_n$ converges to $\underline{\mathbf{H}}_n$ without loss of generality.

Notice that the unconstrained estimate of $\underline{\mathbf{H}}_{n+1}$ is denoted at present by $\tilde{\underline{\mathbf{H}}}_{n+1}$ in (11). Indeed, it is not readily defined that the resulting estimate of $\mathbf{H}_{n+1}$ say $\tilde{\mathbf{H}}_{n+1}$ is precisely of the form $\hat{\mathbf{J}}_{n+1}\hat{\mathbf{D}}_{n+1}^T$. From this estimate, we should extract the time response matrix $\hat{\mathbf{D}}_{n+1}$ in the manifold $\mathbf{\Gamma}$ and identify the blind part of propagation defined by the unmodeled spatial response matrix $\hat{\mathbf{J}}_{n+1}$. Then we should reconstruct the constrained estimate of the spatio-temporal response matrix by $\hat{\mathbf{H}}_{n+1} = \hat{\mathbf{J}}_{n+1}\hat{\mathbf{D}}_{n+1}^T$. This "space/time" separation step which amounts to partially fitting the structure of $\hat{\mathbf{H}}_{n+1}$ with respect to the manifold $\mathbf{\Gamma}$ reduces identification errors of $\mathbf{H}_{n+1}$ by a factor of $L/P$, as shown in [6]. However, the adaptive implementation we propose in the following section avoids the localization step used therein.

The basic STAR receiver given by Eq. (8) to (11), as developed in [6], offers many advantages when compared to previous methods [1]-[5]. Indeed, STAR has a very simple structure of narrowband processors and requires only an order of $O(ML)$ operations per bit or equivalently $O(M)$ operations per chip. Besides, STAR shows fast convergence and adaptation to nonstationarities, optimally reduces interference and offers a potential capacity to accommodate a larger number of users per cell. We next explain its new multipath tracking procedures.

## IV. The Tracking of the Multipath Delays

Based on the new formulation of section II, we can model the unconstrained estimate of the spatio-temporal response matrix $\tilde{\mathbf{H}}_{n+1}$ as a "space/time" observation matrix of an instantaneous mixture between the space and time response matrices $\mathbf{J}_{n+1}$ and $\mathbf{D}_{n+1}$. The model is given by:

$$\tilde{\mathbf{H}}_{n+1}^T = \mathbf{D}_{n+1}\mathbf{J}_{n+1}^T + \mathbf{E}_{n+1}^T , \quad (12)$$

where $\mathbf{E}_{n+1}^T$, the estimation error matrix, is considered as an additive noise matrix. From this equation, the estimation of $\mathbf{D}_{n+1}$ and $\mathbf{J}_{n+1}$ which amounts to a "space/time"-like separation can be achieved in a way similar to source separation.

By analogy to source extraction and DOA (direction of arrival) localization and tracking, the column vectors of $\tilde{\mathbf{H}}_{n+1}^T$ constitute observation vectors in $M$ parallel spaces. With respect to this view, the column vectors of $\mathbf{J}_{n+1}^T$ can be seen as signal vectors of $P$ sources different from one space to another. However, all the sources propagate in the $M$ different spaces along the same trajectories defined by the common propagation matrix $\mathbf{D}_{n+1}$. The manifold $\mathbf{\Gamma}$ contains $\mathbf{D}_{n+1}$ (actually $\mathcal{D}_{n+1}$, see (5)) and can be seen as an "array" manifold where the time delays $\tau_{p,n}$ stand for the DOAs. In this case, any DOA localization or tracking technique can be used to estimate the time-delays in each space.

Now consider the application of the above formulation to DOA localization and tracking in cellular CDMA. Based on the above analogy, we developed in [6] a formulation of the initial time-delay acquisition as a localization operation. Here, we apply the algorithm proposed in [7] to time-delay tracking. Using the estimate $\hat{\mathbf{D}}_n$ as an approximation of $\mathbf{D}_{n+1}$, we compute the "signal-like" matrix $\hat{\mathbf{J}}_{n+1}^T$ as the output of a multi-dimensional DS (delay-sum) beamformer [7]:

$$\begin{aligned}\hat{\mathbf{J}}_{n+1}^T &= \left[\hat{J}_{1,n+1}, \cdots, \hat{J}_{\hat{P},n+1}\right]^T \\ &= \left(\hat{\mathbf{D}}_n^T\hat{\mathbf{D}}_n\right)^{-1}\hat{\mathbf{D}}_n^T\tilde{\mathbf{H}}_{n+1}^T , \end{aligned} \quad (13)$$

This beamformer is optimal if $\mathbf{E}_{n+1}^T$ in (12) can be considered as an uncorrelated white noise matrix. Otherwise, we may apply the optimal beamformer proposed in [8]. The required computational complexity order in (13) is of $O(LP^2 + P^3 + MPL + MP^2)$, but it can be reduced to $O(MPL + MP)$ with the iterative implementation of [8]. From (13), the estimation of $\hat{\mathbf{G}}_{n+1}$ and $\hat{\Upsilon}_{n+1}$ follows by:

$$\hat{\Upsilon}_{n+1} = \mathrm{diag}\left\{\left[\frac{\|\hat{J}_{1,n+1}\|}{\|\hat{\mathbf{J}}_{n+1}\|_F}, \cdots, \frac{\|\hat{J}_{\hat{P},n+1}\|}{\|\hat{\mathbf{J}}_{n+1}\|_F}\right]\right\} , \quad (14)$$

$$\hat{\mathbf{G}}_{n+1} = \|\hat{\mathbf{J}}_{n+1}\|_F \left[\frac{\hat{J}_{1,n+1}}{\|\hat{J}_{1,n+1}\|}, \cdots, \frac{\hat{J}_{\hat{P},n+1}}{\|\hat{J}_{\hat{P},n+1}\|}\right] , \quad (15)$$

where $\|.\|_F$ denotes the Frobinius norm of a matrix and where $\sqrt{K_n}$, the constant norm of propagation vectors mentioned earlier below Eq. (7), is equal to $\|\hat{\mathbf{J}}_n\|_F = \sqrt{\sum_{p=1}^{\hat{P}} \|\hat{J}_{p,n}\|^2}$ such that $\|\hat{\underline{\mathbf{H}}}_n\| = \sqrt{M}$.

We now update the time response matrix $\hat{\mathbf{D}}_n$ in a subspace-tracking equation similar to (11). Although one observation space would be sufficient (*i.e.*, $M = 1$, an application to downlink can be viewed in such a case), the combination of the estimates over the $M$ spaces (*i.e.*, exploiting antenna diversity) improves the tracking performance. Taking the average, the tracking equation can be stated in the following compact form:

$$\tilde{\mathbf{D}}_{n+1} = \hat{\mathbf{D}}_n + \frac{\eta_n}{M}\left(\tilde{\mathbf{H}}_{n+1}^T - \hat{\mathbf{D}}_n\hat{\mathbf{J}}_{n+1}^T\right)\hat{\mathbf{J}}_{n+1}^* , \quad (16)$$

where $\eta_n$ is an adaptation step-size, possibly normalized. It requires a computational complexity of the order $O(MPL + PL)$.

Notice here that if we replace $\tilde{\mathbf{H}}_{n+1}^T$ in Eq. (16) by $\mathbf{Z}_n^T$, we obtain the exact expression of the tracking equation of the multisource beamforming and multitarget tracking algorithm in [7], averaged over sensors. Here, we have the advantage of reducing the noise present in $\mathbf{Z}_n^T$ to simple identification errors in $\tilde{\mathbf{H}}_{n+1}^T$ (see Eq. (12)). Thus, Eq. (16) allows for multipath tracking at a higher interference level and STAR is able to work at a higher capacity.

The above analogy with [7] is the subject of another interesting observation. By virtue of the structure fitting step in the array manifold developed in [7],[8], the algorithm proposed therein is optimal in the presence of colored noise and correlated sources. In the same manner, we implement this feature in STAR as shown below, so that multipath tracking is still possible in the presence of colored noise and correlated multipath. In such a case, an adaptive beamformer could be used in (8) instead of the DS structure for optimal colored noise reduction. Therefore, STAR offers a potential robustness to colored noise and to multipath correlation. The performance in colored noise and with correlated multipath will be reported in a later publication.

We now define $\tilde{\mathcal{D}}_{n+1}$, the column-by-column FFT of $\tilde{\mathbf{D}}_{n+1}$, and constrain its column vectors $\tilde{\mathcal{D}}_{p,n+1}$ to lie in $\mathbf{\Gamma}$ for $p = 1, \cdots, \hat{P}$, as in [7]. To do so, we make linear regressions of the phase variations between the $L/2$ first components of $\tilde{\mathcal{D}}_{p,n+1}$ and $\hat{\mathcal{D}}_{p,n}$ over the $L/2$ frequency bins $0, 1, \cdots, L/2 - 1$. If we define these variations for $l = 1, \cdots, L/2$ and for $p = 1, \cdots, \hat{P}$ by:

$$\delta\hat{\varphi}_{p,l,n+1} = \mathrm{Im}\left\{\log\left(\tilde{\mathcal{D}}^*_{p,l,n+1}\hat{\mathcal{D}}_{p,l,n}\right)\right\} , \tag{17}$$

then we can have $\hat{\tau}_{p,n+1}$ as the slopes of the linear regressions and reconstruct $\hat{\mathcal{D}}_{p,n+1} \in \mathbf{\Gamma}$ as follows:

$$\begin{aligned}
\hat{\tau}_{p,n+1} &= \hat{\tau}_{p,n} + \frac{L}{2\pi} \times \qquad\qquad (18)\\
&\left\{ \frac{(\frac{L}{2}-1)\sum_{l=1}^{\frac{L}{2}}(l-1)\delta\hat{\varphi}_{p,l,n+1}}{(\frac{L}{2}-1)\sum_{l=1}^{\frac{L}{2}}(l-1)^2 - \left(\sum_{k=1}^{\frac{L}{2}}(k-1)\right)^2} \right.\\
&\left. - \frac{\sum_{k=1}^{\frac{L}{2}}(k-1)\sum_{l=1}^{\frac{L}{2}}\delta\hat{\varphi}_{p,l,n+1}}{(\frac{L}{2}-1)\sum_{l=1}^{\frac{L}{2}}(l-1)^2 - \left(\sum_{k=1}^{\frac{L}{2}}(k-1)\right)^2} \right\}\\
\hat{\mathcal{D}}_{p,n+1} &= \mathcal{F}(\hat{\tau}_{p,n+1})\\
&= \left[1, e^{-j2\pi\hat{\tau}_{p,n+1}\frac{1}{L}}, \cdots, e^{-j2\pi\hat{\tau}_{p,n+1}\frac{L-1}{L}}\right]^T , \qquad (19)
\end{aligned}$$

where $\mathrm{Im}\{.\}$ denotes the imaginary part of a complex number. The impulse response matrix $\hat{\mathbf{D}}_{n+1}$ is estimated in the time domain as the column-by-column inverse FFT of $\hat{\mathcal{D}}_{n+1} = \left[\hat{\mathcal{D}}_{1,n+1}, \cdots, \hat{\mathcal{D}}_{\hat{P},n+1}\right]$. This step requires an order of complexity of $O(PL + P\log L + P)$.

We finally reconstruct the spatio-temporal response matrix $\hat{\mathbf{H}}_{n+1}$ by:

$$\hat{\mathbf{H}}_{n+1} = \hat{\mathbf{J}}_{n+1}\hat{\mathbf{D}}_{n+1}^T = \hat{\mathbf{G}}_{n+1}\hat{\Upsilon}_{n+1}\hat{\mathbf{D}}_{n+1}^T , \tag{20}$$

The computational complexity order of this step is of $O(MPL + MP)$. The total order of computational complexity required for STAR, including the multipath tracking, is of $O(MPL + ML + PL + P\log L + MP + P)$.

## V. The Tracking of the Number of Paths

Once we established the adaptive procedure for the multipath delays tracking, we now focus on the tracking of their number $\hat{P}$ as $P$ may change in time due to vanishing or "new born" paths. The strategy we propose relies on the observation in time of energy-based detection criteria proposed in [7] for DOA tracking.

### A. The case of vanishing paths

The matrix $\hat{\Upsilon}_n = \mathrm{diag}\left\{\left[\hat{\varepsilon}_{1,n}, \cdots, \hat{\varepsilon}_{\hat{P},n}\right]\right\}$ of power partition is useful for the detection of vanishing paths. We decide that the $p^{th}$ path vanishes at the block iteration $n_d$ if for a number of block iterations, say $n_{\text{it}}$, we always observe its power $\hat{\varepsilon}^2_{p,n}$ to lie below a minimum threshold $\varepsilon^2_{\min}$:

$$\hat{\varepsilon}^2_{p,n} < \varepsilon^2_{\min} \quad \text{for } n \in \{n_d - n_{\text{it}} + 1, \cdots, n_d\} . \tag{21}$$

Instead of evaluating this condition over $n_{\text{it}}$ block iterations, we may smooth the elements of $\hat{\Upsilon}_n$ to introduce a forgetting factor. Both techniques can be combined, if necessary, to avoid false detections. When the above condition is satisfied, we eliminate the $p^{th}$ path and the corresponding estimates, decrement $\hat{P}$ by 1 (*i.e.*, $\hat{P} = \hat{P} - 1$) and update the parameters of the algorithm.

### B. The case of merging paths

Another case where the number of paths $\hat{P}$ should be decremented corresponds to the situation when two time delays get very close and their paths appear to merge into a single one for more than say $n_m$ block iterations. Although this case is unlikely to happen in practice, its absence guarantees a full column-rank condition of $\hat{\mathbf{D}}_n$ and provides a better stability of the algorithm. This situation can be stated at the block iteration $n_d$ say for the $p^{th}$ and $k^{th}$ merging paths by the following condition:

$$|\hat{\tau}_{p,n} - \hat{\tau}_{k,n}| < T_c \quad \text{for } n \in \{n_d - n_m + 1, \cdots, n_d\} . \tag{22}$$

In this case, we eliminate either path, decrement $\hat{P}$ by 1 (*i.e.*, $\hat{P} = \hat{P} - 1$) and update the parameters of the algorithm. Contrary to DOA tracking, notice that no data association is required when after less than $n_m$ iterations the two merged paths split again as after a simple crossing, since they both belong to the same source.

### C. The case of new appearing paths

We now study the case when a new path appears. Such an event would involve an identification error on the spatio-temporal response matrix $\mathbf{H}_n$ denoted by $\delta\mathbf{H}_n = \varepsilon_{P+1,n}G_{P+1,n}D^T_{P+1,n}$ whose energy can be compared to a maximum threshold for the detection. We can give an estimate say $\delta\hat{\mathbf{H}}_n$ of this error in two ways. We can define the noise estimate $\hat{\mathbf{N}}_n = \mathbf{Z}_n - \hat{\mathbf{H}}_n\hat{s}_n$ and directly identify the error in a LMS-type tracking equation given by:

$$\delta\hat{\underline{\mathrm{H}}}_{n+1} = \delta\hat{\underline{\mathrm{H}}}_n + \zeta_n(\hat{\underline{\mathrm{N}}}_n - \delta\hat{\underline{\mathrm{H}}}_n\hat{s}_n)\ \hat{s}^*_n , \tag{23}$$

where $\zeta_n$ is an adaptation step-size and where $\delta\hat{\underline{\mathrm{H}}}_n$ and $\hat{\underline{\mathrm{N}}}_n$ are the $M \times L$ vectors resulting from the aligned columns of $\hat{\mathbf{H}}_n$ and $\hat{\mathbf{N}}_n$ respectively. With respect to the sequence $\hat{s}_n$ already estimated in (8), this equation is based on an "almost exact" LMS implementation. We can also identify an unconstrained estimate of the spatio-temporal response matrix say $\breve{\mathbf{H}}_n$ in another "almost exact" LMS tracking equation given by:

$$\underline{\breve{\mathbf{H}}}_{n+1} = \underline{\breve{\mathbf{H}}}_n + \zeta_n(\underline{\mathbf{Z}}_n - \underline{\breve{\mathbf{H}}}_n \hat{s}_n)\ \hat{s}_n^* \ , \qquad (24)$$

where $\underline{\breve{\mathbf{H}}}_n$ is the $M \times L$ vector resulting from the aligned columns of $\breve{\mathbf{H}}_n$. This estimate, without the structure fitting constraint in the manifold $\mathbb{\Gamma}$, tracks all the spatio-temporal components of $\mathbf{H}_n$ including the unresolvable paths. In which case, $\delta\hat{\mathbf{H}}_n = \breve{\mathbf{H}}_n - \hat{\mathbf{H}}_n$ is an alternative estimator of the required error. Actually, other estimators can be possibly tested.

Now given an estimate $\delta\hat{\mathbf{H}}_n$ of the identification error, we consider that a new path appears at the block iteration $n_d$ if after a given number of block iterations $n_{\text{it}}$ we have always observed that the relative distortion over $\hat{\mathbf{H}}_n$ given by $\|\delta\hat{\mathbf{H}}_n\|_F^2/\|\hat{\mathbf{H}}_n\|_F^2$ exceeds a maximum threshold say $\delta^2_{\max}$:

$$\frac{\|\delta\hat{\mathbf{H}}_n\|_F^2}{M} > \delta^2_{\max} \quad \text{for } n \in \{n_d - n_{\text{it}} + 1, \cdots, n_d\}\ . \qquad (25)$$

Again a smoothing of the distortion instead of (25) or a combination of both can be used to avoid false detections. Whenever we detect an identification error, we can apply the localization procedure of [6] to reestimate $P$ and the time-delays $\hat{\tau}_{p,n}$ for $p = 1, \cdots, \hat{P}$. Finally, we adequately update the parameters of the algorithm.

## VI. Simulation Results

We consider for our simulations the case of a code length $L = 128$, a number of sensors $M = 4$ and a number of paths $P = 3$. The propagation vectors $G_{p,n}$ are generated at random to simulate nonstationary Rayleigh fading. For the sake of simplicity, we fix the total received power $\psi^2(t)$ to 1. This power is generally equally distributed over the 3 paths (*i.e.*, $\varepsilon^2_{p,n} = 1/3$). We assume, perhaps due to some obstacles, that the $1^{st}$ path vanishes at $n = 4000\ T$ and reappears at $n = 7000\ T$ as shown in Fig. 1. The multipath time-delays are varying along the trajectories of Fig. 2. We further assume the presence of strong interference with a total received power $\sigma_I^2 = 60$ (*i.e.*, $SINR_{in} = -17.8$ dB).

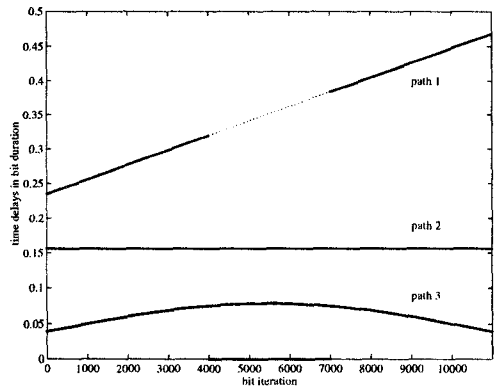

Fig. 2. True time-delays $\tau_{p,n}$.

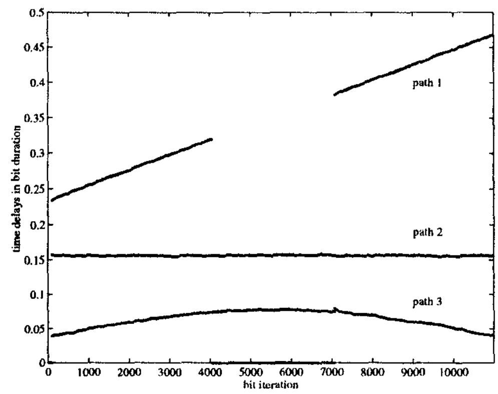

Fig. 3. Estimated time-delays $\hat{\tau}_{p,n}$.

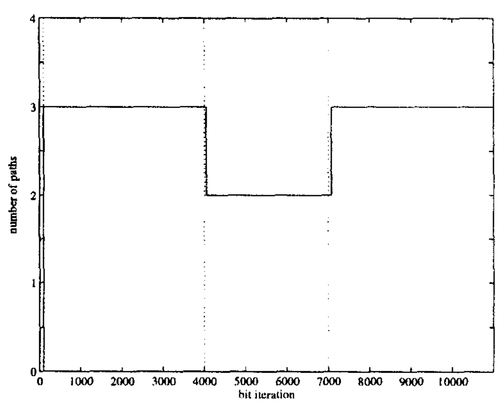

Fig. 4. Estimated number of paths $\hat{P}$ (solid).

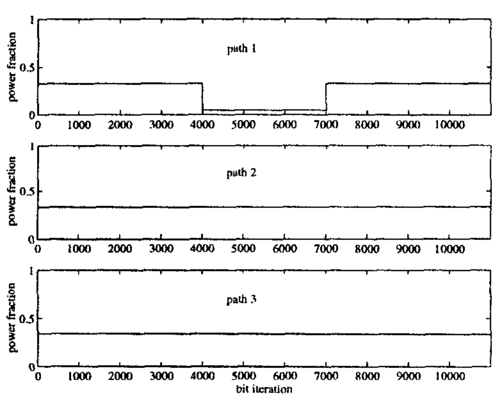

Fig. 1. True power fractions $\varepsilon^2_{p,n}$.

We initialize STAR and start time acquisition within $100\ T$ with the localization technique of [6]. Then we apply the proposed tracking procedures. In Fig. 3, we see that the time-delays are properly tracked despite the disappearance and appearance of the $1^{st}$ path. Indeed, we show in Fig. 4 that the number of paths is correctly estimated. The disappearance and appearance of the $1^{st}$ path are rapidly detected with estimation delays less than $100\ T$. In Fig. 5, we plot the average MSE of time-delay tracking and show that absolute errors are as small as $0.001\ T$. Channel identification errors are also reduced below -25 dB as shown in Fig. 6. Notice the small increase in MSE due to the negligible and unresolvable contribution of the $1^{st}$ path when it vanishes. Notice also the abrupt increase in MSE in Fig. 6 as well as in Fig. 5 as soon as the reappearing path is detected. At that iteration, the restart of the time-delay acquisition by the localization procedure of [6] instantaneously entails larger errors. However, their MSE is immediately reduced to its normal range by the time-delay tracking response. Despite the nonstationary multipath fading and the high interference, we measure over the total DPSK sequence $\hat{b}_n$, including the learning phase, a bit error rate less than $10^{-3}$.

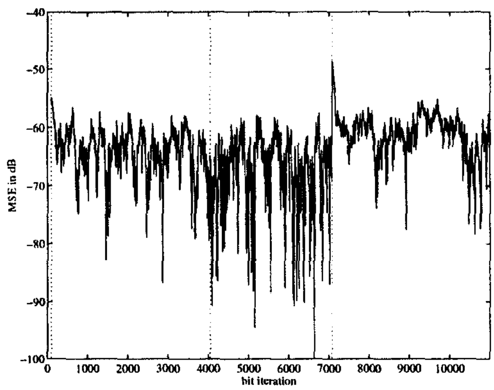


Fig. 5. Average MSE of time-delay tracking $\sum_{p=1}^{\hat{P}} |\tau_{p,n} - \hat{\tau}_{p,n}|^2/\hat{P}$.

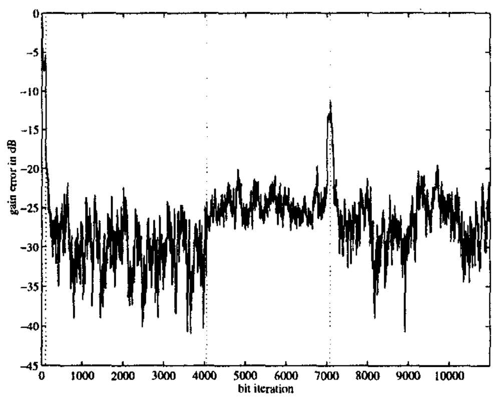


Fig. 6. Average gain error of channel identification $|1 \pm \hat{\underline{H}}_n^H \underline{H}_n/M|^2$.

## VII. Conclusions

We proposed a Spatio-Temporal Array-Receiver (STAR) for multipath tracking in cellular CDMA. Contrary to previous methods [1]-[5], we addressed the crucial problem of time-delay tracking. We first proposed a new formulation which models the post-correlated data as a one-dimensional instantaneous mixture. This allows us to design STAR with a simple structure of narrowband processors where timing can be treated by DOA tracking techniques. Then we developed efficient procedures for the tracking of the multipath delays and their number.

The proposed CDMA array-receiver requires a low order of computational complexity. It achieves a rapid time-acquisition of paths and accurately captures their number and their time-delays. It also displays good time-delay tracking behavior and promptly adapts to vanishing or new appearing paths. These properties are essential for high performance in asynchronous cellular CDMA. Therefore, STAR shows a potential capacity to accommodate a larger number of users in the presence of nonstationary multipath.

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