

Carrier Frequency Offset Recovery for CDMA Array-Receivers in Selective Rayleigh-Fading Channels*

Sofène AFFES¹, Jian ZHANG^{1,2}, and Paul MERMELSTEIN¹

¹ INRS-Télécommunications, Université du Québec

Place Bonaventure, 900, de la Gauchetière Ouest, Niveau C

Case Postale 644, Montréal, Québec, H5A 1C6, Canada

² NSI Communication Systems Inc.

6900 Trans Canada, Pointe-Claire, Quebec, H9R 1C2, Canada

Abstract— We propose a carrier frequency offset recovery (CFOR) module for CDMA to operate with STAR, the spatio-temporal array-receiver. The new CFOR module implements simple linear regression (LR), similar in approach to that implemented for time-delay synchronization in STAR. Simulations in selective Rayleigh-fading channels at various mobile speeds show that an increasing carrier frequency offset (CFO) severely degrades the capacity of a CDMA system, the relative loss being more significant at low mobility and transmission rate. CFOR with STAR reduces the effect of CFO and compensates almost completely the capacity loss. Relative capacity gains due to CFOR increase with the CFO. For nomadic voice and data-rates, the capacity gain is in the range of 270 and 150 %, respectively, at a CFO of about 1 ppm.

I. INTRODUCTION

Rapid and accurate synchronization in time and frequency are major requirements to address in future high-speed wireless access systems. Third generation standard proposals impose tight bounds on synchronization accuracy on both uplink [1] and downlink [2].

In previous work, we proposed a CDMA array-receiver, STAR, the spatio-temporal array-receiver [3]. STAR performs very accurate and fast channel acquisition and timing and hence achieves high spectrum efficiencies at low complexity [4]. So far, our focus has been on time synchronization for CDMA in selective Rayleigh-fading channels. In this contribution, we address the issue of frequency synchronization and incorporate an efficient CFOR technique in STAR [3],[4].

Recent research efforts on CFO estimation have been studied in the context of OFDM wireless access technology [5]-[7]. Additionally, the proposed CFO estimation techniques are all implemented in a single receive-antenna structure [5]-[8]. Most of them derive from a maximum likelihood (ML) approach. In contrast, we propose a CFOR module for CDMA that efficiently combines with an antenna-array receiver structure, namely STAR [3],[4]. The new CFOR module implements simple linear regression (LR) similar in approach to that implemented for time-delay synchronization in STAR [3].

* Work supported by the Bell/Nortel/NSERC Industrial Research Chair in Personal Communications and by the NSERC Research Grants Program.

For the sake of clarity and simplicity, we consider a simplified version of STAR with perfect synchronization [9] although the CFOR and timing solutions of STAR can be integrated easily in a joint structure. Without loss of generality, we consider the blind version of STAR (*i.e.*, without a pilot). Extension to pilot-aided versions of STAR [9] is *ad hoc*.

Simulations in selective Rayleigh-fading channels at various Doppler shifts (or mobile speeds) show that an increasing CFO severely degrades the capacity of a CDMA system, the relative loss being more significant at low mobility and transmission rate. CFOR with STAR reduces the effect of CFO and compensates almost completely the capacity loss. For nomadic voice and data-rate transmissions, the capacity gain is in the range of 270 and 150 %, respectively, at a CFO of about 1 ppm.

II. FORMULATION AND BACKGROUND

A. Data Model

We denote by M the number of the uplink receiving antennas at the base-station and consider a multipath Rayleigh fading environment with number of paths P . After channel coding and interleaving of the information data at the transmitter, the interleaved coded bits are DBPSK-modulated at the rate $1/T$ where T is the symbol duration and denoted as b_n where n is the symbol index (see more details about the transceiver in [10]). We assume a CFO Δf between the transmitter and the receiver and update the past data model [9] to reflect the presence of such a CFO.

After we despread the data channel at the receiver, we form from the $M \times P$ diversity branches the $MP \times 1$ data observation vector as [9]:

$$\dot{Z}_n = \{e^{j2\pi n \Delta f T} \underline{H}_n\} (\psi_n b_n) + \dot{N}_n = \dot{\underline{H}}_n s_n + \dot{N}_n, \quad (1)$$

where $s_n = \psi_n b_n$ is the data signal component and ψ_n^2 is the total received power. $\underline{H}_n = [\underline{H}_{1,n}, \dots, \underline{H}_{MP,n}]^T$ is the $MP \times 1$ spatio-temporal Rayleigh fading channel vector normalized to \sqrt{M} . We identify its i -th coefficient $\underline{H}_{i,n} = r_{i,n} e^{j\phi_{i,n}}$ by its magnitude $r_{i,n}$ and its phase $\phi_{i,n}$ and assume that it is slow and almost constant

over periods of K symbols (*i.e.*, $r_{i,nK+k} \simeq r_{i,nK}$ and $\phi_{i,nK+k} \simeq \phi_{i,nK}$ for $k = 1, \dots, K$). $\hat{\mathbf{H}}_n = e^{j2\pi n \Delta f T} \mathbf{H}_n$ is the channel vector including the CFO. Hence its i -th coefficient $\hat{H}_{i,n} = r_{i,n} e^{j\phi_{i,n}} = r_{i,n} e^{j2\pi n \Delta f T + j\phi_{i,n}}$ has a magnitude $r_{i,n}$ and a phase $\hat{\phi}_{i,n} = 2\pi n \Delta f T + \phi_{i,n}$. \mathbf{N}_n is a spatially-uncorrelated Gaussian interference vector with mean zero and variance σ_N^2 after despreading of the data channel. The resulting input SNR after despreading is $SNR_{in} = \psi^2 / \sigma_N^2$ per antenna element.

B. Overview of STAR

Using the channel estimate $\hat{\mathbf{H}}_n$ at iteration n , blind STAR first extracts the data signal component by spatio-temporal MRC [9]:

$$\hat{s}_n = \text{Re} \left\{ \frac{\hat{\mathbf{H}}_n^H \hat{\mathbf{Z}}_n}{M} \right\}. \quad (2)$$

This soft-decision output is passed on to a Viterbi decoder after differential decoding and deinterleaving [4],[10]. The data sequence b_n is then estimated by hard decision as:

$$\hat{b}_n = \text{Sign} \{ \hat{s}_n \}. \quad (3)$$

In a second step, blind STAR feeds back \hat{s}_n (or $\hat{\psi}_n \hat{b}_n$) in a decision feedback identification (DFI) scheme to update the channel estimate as follows (for details see [3],[9]):

$$\hat{\mathbf{H}}_{n+1} = \hat{\mathbf{H}}_n + \mu (\hat{\mathbf{Z}}_n - \hat{\mathbf{H}}_n \hat{s}_n) \hat{s}_n, \quad (4)$$

where $\hat{\mathbf{H}}_n$ is the adaptive channel estimate and μ the adaptation step-size. Details about other processing steps such as power estimation can be found in [9],[10].

Note that a CFO adds to the Doppler in increasing the time-variations of $\hat{\mathbf{H}}_n$, and hence increases estimation errors in $\hat{\mathbf{H}}_n$ thereby degrading the system's performance [9]. Next we introduce CFOR in STAR to reduce these time-variations, improve channel estimation and avoid the performance loss due to CFO.

III. CFOR IN STAR

We derive LR-based CFOR solutions for both frequency offset acquisition and tracking in STAR, using open- and closed-loop structures, respectively.

A. CFO Acquisition in STAR

Exploiting the assumption that the Rayleigh component of the i -th channel coefficient $\hat{H}_{i,n}$ is almost constant over periods of K symbols, we can develop its phase $\hat{\phi}_{i,n}$ at iterations $nK + k$ for $k = 1, \dots, K$ as:

$$\hat{\phi}_{i,nK+k} \simeq (2\pi \Delta f T) \times k + (\phi_{i,n} + 2\pi n K \Delta f T), \quad (5)$$

where the CFO Δf appears as part of the slope of a linear function versus the symbol index k . Hence, to estimate Δf we simply resort to a length- K linear regression over the phases $\hat{\phi}_{i,n}$ of the i -th channel coefficient estimate $\hat{H}_{i,n} = \hat{r}_{i,n} e^{j\hat{\phi}_{i,n}}$ after convergence of Eq. (4).

For each diversity finger for $i = 1, \dots, MP$, we buffer $\hat{\phi}_{i,n}$ over successive blocks of K symbols in the following phase vector:

$$\hat{\Phi}_{i,nK} = [\hat{\phi}_{i,(n-1)K+1}, \dots, \hat{\phi}_{i,(n-1)K+k}, \dots, \hat{\phi}_{i,nK}]^T, \quad (6)$$

then estimate Δf at the symbol iteration nK as the slope of a linear regression as follows for $i = 1, \dots, MP$:

$$\hat{\Delta f}_{i,nK} = \frac{\|R_0\|^2 (R_1^T \hat{\Phi}_{i,nK}) - (R_1^T R_0) (R_0^T \hat{\Phi}_{i,nK})}{2\pi T \left\{ \|R_0\|^2 \|R_1\|^2 - (R_1^T R_0)^2 \right\}}, \quad (7)$$

where $R_0 = [1, \dots, 1, \dots, 1]^T$ and $R_1 = [1, \dots, k, \dots, K]^T$ are K -dimensional vectors. There are MP such estimates of Δf available, one for each diversity branch. To minimize estimation errors, we define the estimate of Δf as the following weighted summation:

$$\hat{\Delta f}_{nK} = \frac{1}{M} \sum_{i=1}^{MP} \left(\frac{\sum_{k=1}^K \hat{r}_{i,(n-1)K+k}^2}{K} \right) \hat{\Delta f}_{i,nK}. \quad (8)$$

We weight $\hat{\Delta f}_{i,nK}$ for $i = 1, \dots, MP$, by the power of the i -th diversity finger averaged over the length- K regression block, then normalize the summation by M , the sum of the power averages (since \mathbf{H}_n is normalized to \sqrt{M}).

Note that if different CFOs over antennas arise from decoupled baseband demodulations, then their values can be estimated by M separate weighted summations over paths in Eq. (8).

The initial step above implements the CFO acquisition in an open-loop structure (*i.e.*, no feedback of $\hat{\Delta f}$) within STAR. Next, we introduce the CFO tracking step in a closed-loop structure.

B. CFO Tracking in STAR

Once an estimate $\hat{\Delta f}$ is available¹ after initial acquisition, say at iteration n_0 , we feed it back to the input of the receiver by multiplying the despread vector $\hat{\mathbf{Z}}_n$ of Eq. (1) with the conjugated complex carrier of the CFO estimate, giving for $n > n_0$:

$$\begin{aligned} \mathbf{Z}_n &= e^{-j2\pi \hat{\Delta f} (n-n_0)T} (\hat{\mathbf{H}}_n \mathbf{s}_n + \mathbf{N}_n) \\ &= e^{j2\pi (\Delta f - \hat{\Delta f}) n T} \left(e^{j2\pi \hat{\Delta f} n_0 T} \mathbf{H}_n \right) \mathbf{s}_n + \mathbf{N}_n \\ &= (e^{j2\pi \delta f n T} \mathbf{H}'_n) \mathbf{s}_n + \mathbf{N}_n = \hat{\mathbf{H}}_n \mathbf{s}_n + \mathbf{N}_n, \end{aligned} \quad (9)$$

¹For now, we skip the symbol index of the CFO estimate for simplicity.

where $\delta f = \Delta f - \widehat{\Delta f}$ is the CFO estimation error, $\underline{H}'_n = e^{j2\pi\widehat{\Delta f}n_0T}\underline{H}_n$ is the Rayleigh channel vector within a constant phase rotation (*i.e.*, its time variations are identical to those of \underline{H}_n), $\underline{\tilde{H}}_n = e^{j2\pi\delta f nT}\underline{H}_n$ is the channel vector including the CFO estimation residual, and $\underline{N}_n = e^{-j2\pi\Delta f nT}\underline{N}_n$ is a spatially-uncorrelated Gaussian interference vector with mean zero and variance σ_N^2 .

Note that the CFO compensation in Eq. (9) reduces the time-variations in \underline{H}_n due to Δf to much weaker fluctuations in $\underline{\tilde{H}}_n$ due to the residual $\delta f \ll \Delta f$. Tracking of $\underline{\tilde{H}}_n$ instead of \underline{H}_n in Eq. (4) should result in much weaker identification errors and should enable further reductions of the CFO estimation error δf (*e.g.*, due to time-variations of the CFO).

Hence we modify Eqs. (2) and (4) by the following equations, respectively, for $n > n_0$:

$$\hat{s}_n = \text{Re} \left\{ \frac{\hat{\underline{H}}_n^H \underline{Z}_n}{M} \right\}, \quad (10)$$

$$\hat{\underline{H}}_{n+1} = \hat{\underline{H}}_n + \mu \left(\underline{Z}_n - \hat{\underline{H}}_n \hat{s}_n \right), \quad (11)$$

where $\hat{\underline{H}}_{n_0} = \underline{H}_{n_0}$. The i -th coefficient of $\hat{\underline{H}}_n$ is identified as $\hat{H}_{i,n} = \hat{r}_{i,n} e^{j\hat{\phi}_{i,n}}$.

Similarly to CFO acquisition, we estimate δf by linear regressions over successive blocks of length K by replacing Eqs. (6), (7) and (8) by the following equations, respectively:

$$\hat{\underline{\Phi}}_{i,nK} = \left[\hat{\phi}_{i,(n-1)K+1}, \dots, \hat{\phi}_{i,(n-1)K+k}, \dots, \hat{\phi}_{i,nK} \right]^T, \quad (12)$$

$$\widehat{\delta f}_{i,nK} = \frac{\|R_0\|^2 \left(R_1^T \hat{\underline{\Phi}}_{i,nK} \right) - \left(R_1^T R_0 \right) \left(R_0^T \hat{\underline{\Phi}}_{i,nK} \right)}{2\pi T \left\{ \|R_0\|^2 \|R_1\|^2 - \left(R_1^T R_0 \right)^2 \right\}}, \quad (13)$$

$$\widehat{\delta f}_{nK} = \frac{1}{M} \sum_{i=1}^{MP} \left(\frac{\sum_{k=1}^K \hat{r}_{i,(n-1)K+k}^2}{K} \right) \widehat{\delta f}_{i,nK}. \quad (14)$$

From the above residual estimate, we update the CFO estimate over successive blocks of K symbols as follows:

$$\widehat{\Delta f}_{nK} = \widehat{\Delta f}_{(n-1)K} + \widehat{\delta f}_{nK}, \quad (15)$$

and take these updates into account when compensating for the CFO in Eq. (9).

To underline the differences between the time-variations during acquisition and tracking, we introduced two different notations for the channel vector \underline{H}_n and $\underline{\tilde{H}}_n$, respectively. For the sake of simplicity, we also introduced two different notations for the channel estimate $\hat{\underline{H}}_n$ and $\hat{\underline{\tilde{H}}}_n$, respectively. Note however that such distinction is not necessary in implementation where Eqs. (10) to (14) actually hold both in acquisition and tracking.

IV. PERFORMANCE EVALUATION

A. Simulation Setup

We consider the uplink of a wideband CDMA system with $M = 2$ receive antennas and 5 MHz bandwidth operating at a carrier frequency of 1.9 GHz. We assume a selective Rayleigh fading channel characterized by $P = 3$ equal-power multipaths. We implement a closed power control loop at 1600 Hz, corresponding to an update period of 1.25 ms. To make this power control loop even more realistic, we degrade the power control link by a transmission delay of 1.25 ms and a BER of 10%. The power control increments are fixed at ± 0.25 dB.

We also consider voice and data-rate transmissions of 9.6 and 144 Kbps with qualities of service (QoS) of 10^{-3} and 10^{-5} BER after Viterbi decoding, respectively (see more details about the codec in [10]). We use the capacity computation tool described in [11] to evaluate STAR with/without CFOR. This tool populates a multi-cellular system with spatially uniformly distributed mobiles up to the capacity of the cell and ensures that the received SNR meets the required value at the required QoS. The transmission activity factor [11],[4] is 45% and 100%, respectively. Finally, we fix the regression length $K = 64$.

B. Simulation Results

Simulations are run for two mobile speeds of 5 and 100 Kmph, corresponding to Doppler shifts of 8.8 and 176 Hz with our setup. Required SNR is computed then translated into capacity in numbers of users per cell for a CFO range from 0 up to 7 KHz, *i.e.*, up to about 3.7 ppm at 1.9 GHz (74 times the 0.05 ppm maximum error tolerated on the uplink in 3G standard proposals [1]).

Figure 1 shows the capacity performance without CFOR. It suggests that even for low CFOs below 500 Hz (*i.e.*, about 0.25 ppm), capacity degradation due to CFO is severe for the voice rate (see Fig. 1-a), from about 45 % loss at low Doppler to 25 % loss at high Doppler. In the data-rate case (see Fig. 1-b), losses are in the range of 20 and 65 %, respectively. Increased Doppler readily decreases performance (see curves in the vicinity of 0 Hz CFO). However, relatively larger CFOs become the dominant factor in performance losses (see curves superimposed at high CFO). At large CFOs, degradation for data-rate transmissions is substantial from 60 to 65 % around 3 KHz, and much more severe for voice-rate transmissions where no capacity is achievable beyond 2.5 to 3 KHz. In either case, CFOR is definitely required.

Figure 2 shows the capacity performance with CFOR. For voice-rate transmissions, Fig. 2-a indicates that the proposed CFOR technique is very efficient in the CFO range of 2 to 2.5 KHz (*i.e.*, 1 ppm) in that it restores capacities at different Dopplers to their respective initial ranges without CFO (*i.e.*, at 0 Hz CFO). Beyond that range, it allows nonzero capacity despite the CFO

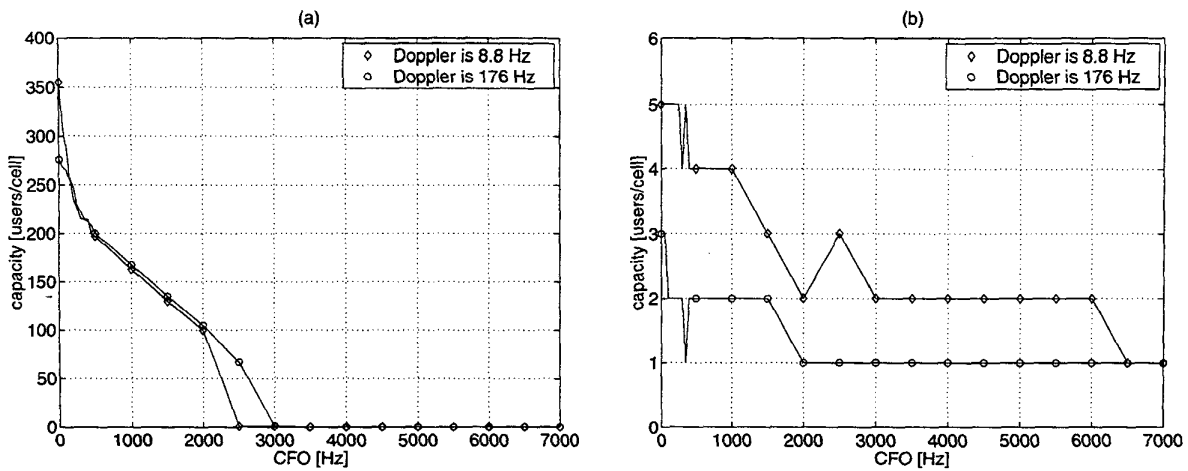


Fig. 1. Capacity of STAR without CFOR vs. the CFO in 5 Mz at different Doppler shifts, (a): for a voice-rate of 9.6 Kbps, (b): for a data-rate of 144 Kbps.

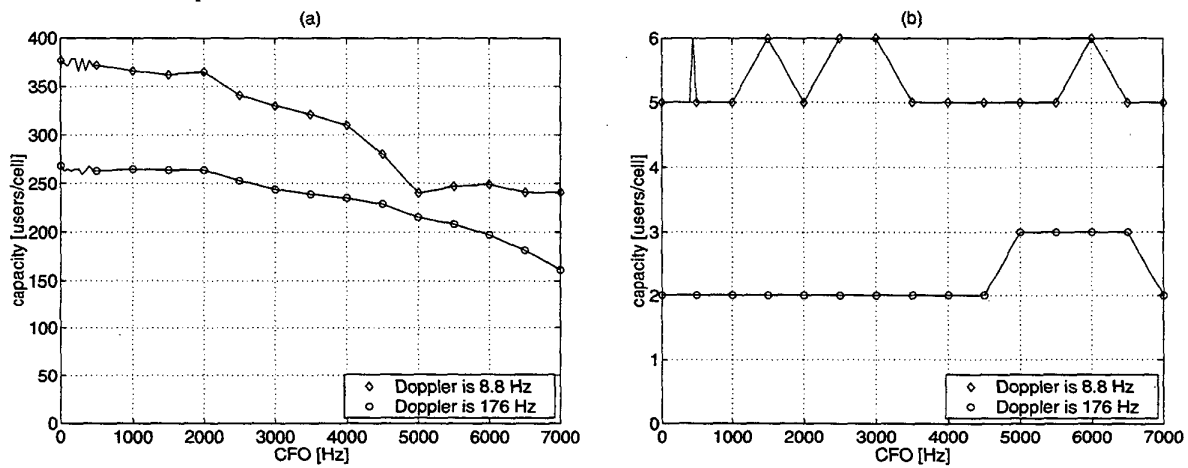


Fig. 2. Capacity of STAR with CFOR vs. the CFO in 5 Mz at different Doppler shifts, (a): for a voice-rate of 9.6 Kbps, (b): for a data-rate of 144 Kbps.

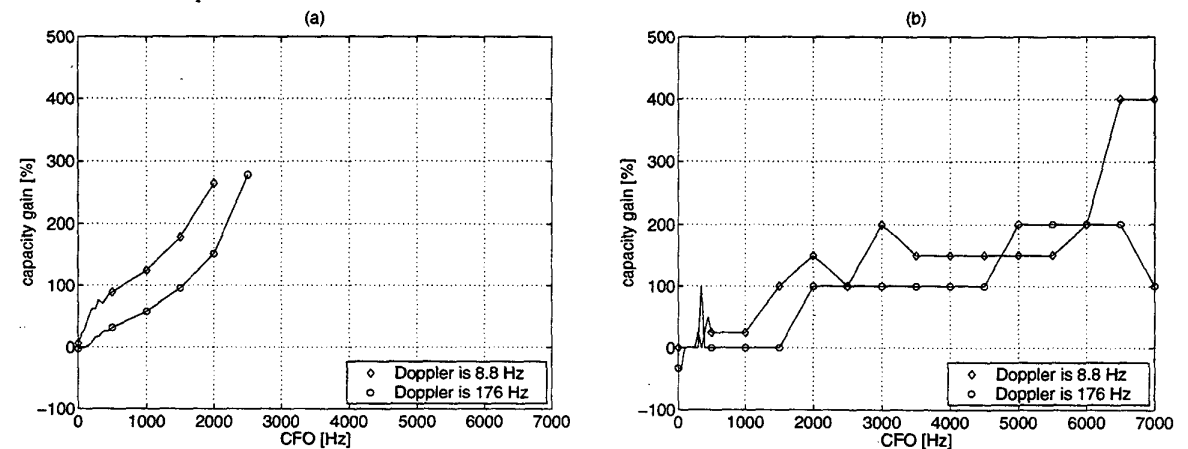


Fig. 3. Capacity gain of STAR due to CFOR vs. the CFO in 5 Mz at different Doppler shifts, (a): for a voice-rate of 9.6 Kbps, (b): for a data-rate of 144 Kbps.

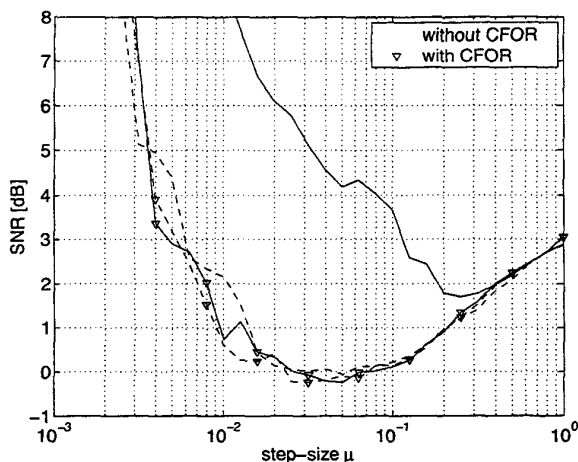


Fig. 4. Required SNR vs. the adaptation step-size for a voice-rate transmission of 9.6 Kbps with low speed at CFOs of 0 Hz (dashed) and 200 Hz (solid).

(see Fig. 1-a). For data-rate transmissions, the proposed CFOR technique restores capacities at different Dopplers to their respective initial values no matter what the actual CFO is in the tested 7 KHz range (see Fig. 2-b). Overall, the new CFOR module is very stable in tracking and compensating the average component of the whole Doppler spread resulting from both the CFO and the user mobility.

Figure 3 suggests that the relative capacity gains² due to CFOR are more significant at higher CFOs and lower Doppler shifts or transmission rates. In the voice-rate case at either speed, the capacity gain³ increases steadily with the CFO up to around 270 % at 2 to 2.5 KHz (*i.e.*, around 1 ppm). Beyond this CFO range the CFOR capacity gain is compared to zero capacity without CFOR resulting in very high relative capacity gain. In the data-rate case, the gain increases steadily along the entire 7KHz CFO-range up to about 400 and 200 % for low and high speeds, respectively. At about 2 to 2.5 KHz (*i.e.*, around 1 ppm), the gain is around 150 and 100 %, respectively.

Capacity results provided above (*a priori* SNR results as well) are actually optimized over the adaptation step-size μ for channel identification in STAR [9]. Required SNR results at a BER of 10^{-3} after channel decoding for a 9.6 Kbps link with low Doppler in Fig. 4 indicate that the CFOR module extends and stabilizes the optimal range for the adaptation step-size value μ . The reference curves in dashed line show that CFOR has no impact on the system performance in the absence of CFO (*i.e.*, 0 Hz CFO). On the other hand, the solid-line curves show that a CFO of 200 Hz shrinks the optimal range of μ when

²The negligible gains/losses measured in the vicinity of 0 Hz CFO should be attributed to stochastic errors in the simulations.

³At CFOs below the Doppler shift range, the capacity gains are negligible (within ± 5 % margin).

CFOR is not active. The CFOR module enlarges the optimal range of μ back to its initial span and therefore stabilizes it. Much less tuning effort is required in practice to keep performance optimal both in SNR and capacity.

V. CONCLUSIONS

In this contribution we proposed an LR-based CFOR module for CDMA to operate with STAR. The new module is designed to cope with the severe degradation in capacity incurred by the network due to CFO, the relative loss being more significant at higher CFO and lower mobility or transmission rate. Its incorporation into STAR reduces the effect of CFO so as to compensate almost completely the capacity loss that results otherwise. For nomadic voice and data-rates, the capacity gain is in the range of 270 and 150 %, respectively, at a CFO of about 2 to 2.5 KHz (*i.e.*, around 1 ppm at a carrier of 1.9 GHz).

Additionally, the new CFOR module stabilizes channel identification by enlarging the optimal range of the adaptation step-size back to its initial span. Much less tuning effort is required in practice to keep performance optimality both in SNR and capacity.

REFERENCES

- [1] 3rd Generation Partnership Project (3GPP), Technical Specification Group (TSG), Radio Access Network (RAN), Working Group (WG4), *UE Radio Transmission and Reception (FDD)*, TS 25.101, V3.4.1, 2000.
- [2] 3rd Generation Partnership Project (3GPP), Technical Specification Group (TSG), Radio Access Network (RAN), Working Group (WG4), *Base Station Conformance Testing (FDD)*, TS 25.141, V3.3.0, 2000.
- [3] S. Affes, P. Mermelstein, "A new receiver structure for asynchronous CDMA: STAR - the spatio-temporal array-receiver", *IEEE J. Select. Areas Comm.*, vol. 16, no. 8, pp. 1411-1422, October 1998.
- [4] K. Cheikhrouhou, S. Affes, and P. Mermelstein, "Impact of synchronization on performance of enhanced array-receivers in wideband CDMA networks", *IEEE J. Select. Areas Comm.*, vol. 19, no. 12, pp. 2462-2476, December 2001.
- [5] P. H. Moose, "A technique for orthogonal frequency division multiplexing frequency offset correction", *IEEE Trans. Comm.*, vol. 42, no. 10, pp. 2908-2914, October 1994.
- [6] J.-J. van de Beek, M. Sandell, and P. O. Börjesson, "ML estimation of time and frequency offset in OFDM systems", *IEEE Trans. Sig. Process.*, vol. 45, no. 7, pp. 1800-1804, July 1997.
- [7] T. Keller, P. Mandarini, and L. Hanzo, "Orthogonal frequency division multiplex synchronization techniques for frequency-selective fading channels", *IEEE J. Select. Areas Comm.*, vol. 19, no. 6, pp. 999-1008, June 2001.
- [8] H. Viswanathan and R. Krishnamoorthy, "A frequency offset estimation technique for frequency-selective fading channels", *IEEE Comm. Letters*, vol. 5, no. 4, pp. 166-168, April 2001.
- [9] S. Affes, A. Louzi, N. Kandil, and P. Mermelstein, "A high capacity CDMA array-receiver requiring reduced pilot power", *Proc. of IEEE GLOBECOM'2000*, San Francisco, USA, vol. 2, pp. 910-916, November 27-December 1, 2000.
- [10] H. Xu, S. Affes, and P. Mermelstein, "Enhanced array-receiver operation with turbo codes for increasing the capacity of wideband CDMA networks", *Proc. of IEEE VTC'2002*, Birmingham, USA, to appear, November 27-December 1, 2000.
- [11] A. Jalali and P. Mermelstein, "Effects of diversity, power control, and bandwidth on the capacity of microcellular CDMA systems", *IEEE J. Select. Areas Comm.*, vol. 12, no. 5, pp. 952-961, June 1994.