# Does Direction-of-Arrival Estimation Help Channel Identification in Multi-Antenna CDMA Receivers?\*

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Abstract— We investigated the benefits of explicit direction of arrival (DOA) estimation in the channel identification process of the STAR multi-antenna receiver for CDMA networks. We find that DOA extraction is beneficial only under more adverse conditions, *i.e.*, higher levels of interference and mobility. Receivers such as STAR already provide accurate channel estimates and SNR gains due to DOA exploitation are modest and become significant only with vary large antenna arrays.

## I. FORMULATION AND BACKGROUND

#### A. Data Model and Assumptions

We denote by M the number of uplink receiving antennas at the base-station (extension to downlink is *ad hoc*) and consider a multipath Rayleigh fading environment with number of paths P. For air-interface transmission, data symbols  $\underline{b}_n$  are BPSK-modulated then differentially encoded as  $b_n = \underline{b}_n b_{n-1}$ .

After despreading the data at the receiver, we form for each path p = 1, ..., P the corresponding  $M \times 1$  despread vector:

$$Z_{p,n} = G_{p,n} \varepsilon_{p,n} \psi_n b_n + N_{p,n} , \qquad (1)$$

where  $\psi_n^2$  is the total received power and  $\varepsilon_{p,n}^2$  is the normalized power fraction of the total power received over the *p*-th path (*i.e.*,  $\sum_{p=1}^{P} \varepsilon_{p,n}^2 = 1$ ). The  $M \times 1$  vector  $G_{p,n}$ , with norm  $\sqrt{M}$ , denotes the channel vector from the transmitter to the multi-antenna receiver over the *p*-th multipath. For more efficient joint space-time processing [1], we align the  $M \times 1$ -vectors  $G_{p,n}$ , *P* in number, to generate the following  $MP \times 1$  data observation vector [2]:

$$\underline{Z}_n = \begin{bmatrix} Z_{1,n}^T, \dots, Z_{P,n}^T \end{bmatrix}^T = \underline{H}_n s_n + \underline{N}_n , \qquad (2)$$

where  $s_n = \psi_n b_n$  denotes the signal component.  $\underline{\mathbf{H}}_n = \left[\varepsilon_{1,n}G_{1,n}^T, \ldots, \varepsilon_{P,n}G_{P,n}^T\right]^T$  is the  $MP \times 1$  spatio-temporal channel vector with norm  $\sqrt{M}$ .  $\underline{\mathbf{N}}_n = \left[N_{1,n}^T, \ldots, N_{P,n}^T\right]^T$  is a space-time uncorrelated Gaussian interference vector with mean zero and variance  $\sigma_N^2$  after despreading of the data channel. The resulting input SNR after despreading is  $SNR_{\rm in} = \psi^2/\sigma_N^2$  per antenna element.

## B. Overview of STAR

Exploiting the channel estimate  $\hat{\mathbf{H}}_n \simeq a \mathbf{H}_n$  derived below with a sign ambiguity  $a = \pm 1$ , STAR (spatio-temporal

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array-receiver) [1] first extracts the data signal component by spatio-temporal MRC [2]:

$$\hat{s}_n = \operatorname{Re}\left\{\hat{\mathbf{H}}_n^H \underline{\mathbf{Z}}_n / M\right\} . \tag{3}$$

and estimates the DBSPK data sequence  $b_n$  with a sign ambiguity as  $\hat{b}_n = \text{Sign} \{\hat{s}_n\} \simeq ab_n$ . Differential decoding of  $\hat{b}_n$  resolves the sign ambiguity a in the BPSK symbol estimates  $\underline{\hat{b}}_n = \hat{b}_n \hat{b}_{n-1} = \text{Sign} \{\hat{s}_n \hat{s}_{n-1}\}.$ 

In a second step, STAR feeds back the estimate of the data signal component  $\hat{s}_n$  (or  $\hat{\psi}_n \hat{b}_n$ ) in a decision feedback identification (DFI) scheme to update the channel estimate as follows (for details see [1],[2]):

$$\hat{\mathbf{H}}_{n+1} = \hat{\mathbf{H}}_n + \mu \left( \underline{\mathbf{Z}}_n - \hat{\mathbf{H}}_n \hat{s}_n \right) \hat{s}_n , \qquad (4)$$

where  $\underline{H}_n$  is the adaptive channel estimate and  $\mu$  the adaptation step-size. This simple DFI scheme [1] of Eqs. (3) and (4) identifies the channel within a constant sign ambiguity  $a = \pm 1$  thereby giving  $\underline{\hat{H}}_n \simeq a \underline{H}_n$ .

## II. STAR WITH DOA TRACKING

#### A. Data Model with DOA

So far we have made no assumptions on the propagation model. Here we assume that a DOA characterizes the multipath channel vector as follows:

$$G_{p,n} = r_{p,n} \mathcal{F}(\theta_{p,n}) = r_{p,n} \left[ \dots, e^{-j \frac{2\pi \sin(\theta_{p,n})}{\lambda} x_m, \dots} \right]^T, \quad (5)$$

where  $\theta_{p,n}$  is the DOA of the *p*-th path,  $\lambda$  is the wave length, and  $x_m$ ,  $m = 1, \ldots, M$  are the sensor positions of a linear antenna (extension to a two-dimensional antenna is possible [4]), and  $r_{p,n}$  is a phase shift due to Rayleigh fading<sup>1</sup>. The propagation vector, a parametric function  $\mathcal{F}$ of the DOA, is said to belong to an array manifold. In the following upgraded version of STAR, we attempt to improve channel identification by fitting the structure of the channel estimate in its array manifold.

## B. Structure Fitting by DOA Tracking in STAR

Let us denote the channel estimate prior to structure fitting as  $\tilde{\mathbf{H}}_n = \begin{bmatrix} \tilde{H}_{1,n}^T, \dots, \tilde{H}_{P,n}^T \end{bmatrix}^T$ . Its *p*-th  $M \times 1$  vector segment can be written as:

$$\tilde{H}_{p,n} = a \varepsilon_{p,n} G_{p,n} + E_{p,n} = \epsilon_{p,n} \mathcal{F}(\theta_{p,n}) + E_{p,n} , \qquad (6)$$

<sup>1</sup>For a given multipath, say p, Rayleigh fades across antennas are fully correlated and all equal to  $\psi_n \varepsilon_{p,n} r_{p,n}$  within a DOA phase shift.

where  $\epsilon_{p,n} = a\epsilon_{p,n}r_{p,n}$  and  $E_{p,n}$  denotes the  $M \times 1$  vector of identification errors over the *p*-th multipath. Providing means to extract both  $\hat{\theta}_{p,n}$  and  $\hat{\epsilon}_{p,n}$  from  $\tilde{H}_{p,n}$  allows its reconstruction as follows:

$$\hat{H}_{p,n} = \hat{\epsilon}_{p,n} \hat{G}_{p,n} = \hat{\epsilon}_{p,n} \mathcal{F}(\hat{\theta}_{p,n}) \quad , \tag{7}$$

yielding  $\hat{\mathbf{H}}_n = \left[\hat{H}_{1,n}^T, \dots, \hat{H}_{P,n}^T\right]^T$ . This step is explicitly referred to as structure fitting. We implement it in a DOA tracking loop as follows.

Suppose that structure fitting was already applied over the channel estimate at iteration n, yielding  $\hat{\mathbf{H}}_n$  (see initialization of DOA tracking below). Then apply the DFI procedure of Eq. (4) and denote the updated channel estimate at iteration n+1 as  $\hat{\mathbf{H}}_{n+1}$ . Assuming slow variations of  $G_{p,n}$ compared to the symbol duration (*i.e.*,  $G_{p,n+1} \simeq G_{p,n}$ ), we extract  $\epsilon_{p,n+1}$  from  $\tilde{H}_{p,n+1}$  by matched beamforming as follows:

$$\hat{\epsilon}_{p,n+1} = \hat{G}_{p,n}^H \tilde{H}_{p,n+1} / M ,$$
 (8)

then update the multipath channel vector estimate by:

$$\tilde{G}_{p,n+1} = \hat{G}_{p,n} + \mu_p \left( \tilde{H}_{p,n+1} - \hat{G}_{p,n} \hat{\epsilon}_{p,n+1} \right) \hat{\epsilon}_{p,n+1}^* , \quad (9)$$

where  $\mu_p$  is an adaptation step-size. Note the similarity of the adaptation above with the DFI procedure of Eq. (4). This LMS-type adaptation was originally proposed in the ASSET (adaptive source subspace extraction and tracking) algorithm [4] for multi-source beamforming and DOA tracking. We adjusted it in [1] to multipath timedelay tracking. Here we apply it as originally designed, *i.e.*, for DOA tracking and hence estimate  $\hat{\theta}_{p,n+1}$  from  $\tilde{G}_{p,n+1}$ by simple update of  $\hat{\theta}_{p,n}$ . Details of this DOA tracking step can be found in [4]. Estimation of  $\hat{G}_{p,n+1}$  and  $\hat{\mathbb{H}}_{n+1}$  in Eq. (7) using  $\hat{\epsilon}_{p,n+1}$  in Eq. (8) and  $\hat{\theta}_{p,n+1}$  completes structure fitting at iteration n + 1.

## C. Disabling/Activation of DOA Tracking in STAR

Below a detection threshold  $\delta_{\text{TH}}^2$  of the multipath energy (see [3]), DOA tracking is no longer reliable. Structure fitting must be disabled, say for the *p*-th path at symbol iteration  $n_d$ , if the estimate of the *p*-th power fraction, given by  $|\hat{\epsilon}_{p,n}|^2 = \hat{\epsilon}_{p,n}^2$  remains continuously below  $\delta_{\text{TH}}^2$ over  $n_v$  symbol durations (*i.e.*,  $n \in \{n_d - n_v + 1, \dots, n_d\}$ ). For  $n > n_d$ , we skip Eqs. (8) and (9) for the *p*-th path and set  $\hat{H}_{p,n} = \tilde{H}_{p,n}$  in Eq. (7) until the multipath energy of the *p*-th path exceeds the detection threshold again.

Indeed, if DOA tracking was inactive say for the *p*-th path, we activate it at symbol iteration  $n_d$  if the estimate of the *p*-th power fraction, given by  $\|\tilde{H}_{p,n}\|^2/M$  continuously exceeds  $\delta_{\text{TH}}^2$  over  $n_a$  symbol durations (*i.e.*,  $n \in \{n_d - n_a + 1, \ldots, n_d\}$ ). We initialize/reactivate the DOA tracking described above for each of the *P* paths by:

$$\hat{\theta}_{p,n_d} = \operatorname*{argmin}_{\theta \in \Theta} \left\{ |\mathcal{F}(\theta)^H \tilde{H}_{p,n_d}|^2 \right\} , \qquad (10)$$

where  $\Theta$  denotes a search set of tentative DOA values (in the range of M in number) over which projected energy from  $\tilde{H}_{p,n_d}$  is maximized.



Fig. 1. SNR gains in dB vs. the number of antennas for DOA tracking in STAR (carrier 1.9 GHz, data rate 288 kbps, P = 3 equal-power paths, average 10% power control errors at a PC rate of 1600 Hz with  $\pm 0.25$  dB increment and 0.625 ms Tx delay).

## III. SIMULATIONS RESULTS AND CONCLUSIONS

In Fig. 1 we plot the SNR gain at a given BER over  $\underline{b}_n$  (before channel decoding) due to enhancement of channel identification by DOA tracking in STAR. Results suggest the following:

• The relative SNR gains of DOA-based structure fitting increase with degrading conditions for channel identification. A larger number of antennas enables operation at higher noise levels and/or with faster channel variations, providing higher gains but increased channel-identification errors.

• The relative SNR gains of DOA-based structure fitting are modest with small antenna arrays (fractions of a dB) and even less at higher BER thresholds resulting from better channel coding. A large array of 32 elements enables more than 2 dB gain at vehicular speed and 5% BER. However, this size antenna array appears impractical today.

• Structure fitting by Eqs. (7) and (8) amounts to a projection that reduces identification errors by no more than factor M [1]. If the channel identification is already quite accurate, such as the DFI technique used in STAR, enhancement of channel identification by DOA-based structure fitting becomes negligible when the antenna array is small and/or when channel identification conditions are already favorable.

#### References

- S. Affes, P. Mermelstein, "A new receiver structure for asynchronous CDMA: STAR - the spatio-temporal array-receiver", *IEEE Journal on Selected Areas in Communications*, vol. 16, no. 8, pp. 1411-1422, October 1998.
- [2] S. Affes and P. Mermelstein, "Comparison of pilot-assisted and blind CDMA array-receivers adaptive to Rayleigh fading rates", *Proc. of IEEE PIMRC'99*, 1999, vol. 3, pp. 1186-1192.
- [3] K. Cheikhrouhou, S. Affes, and P. Mermelstein, "Impact of synchronization on performance of enhanced array-receivers in wideband CDMA networks", *Journal on Selected Areas in Communications*, vol. 19, no. 12, pp. 2462-2476, December 2001.
- [4] S. Affes, S. Gazor, and Y. Grenier, "An algorithm for multisource beamforming and multitarget tracking", *IEEE Transactions on Signal Processing*, vol. 44, no. 6, pp. 1512-1522, June 1996.