Maximum *a posteriori* detection of the primary synchronization channel of the UMTS/FDD mode

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Abstract— We propose a maximum *a posteriori* metric for the optimum detection of the primary synchronization channel of the UMTS/FDD mode. This metric is derived assuming a perfect knowledge of the small-scale propagation channel statistics at the mobile station. It is then extended to take into account time-switched transmit diversity at the base station. The performance of this optimum metric in terms of false detection rate is compared to that of the non-coherent averaging metric used in the evaluation process carried out by 3GPP. The simulation results show a performance gain of up to 3 dB for critical indoor, outdoor-to-indoor and pedestrian propagation environments which are characterized by small time and multipath diversity.

I. Introduction

When the UMTS/FDD mode mobile station is switched on, it starts by acquiring slot synchronization to the best received base station by detecting the primary synchronization channel (PSC). The mobile station detects the PSC timing by matched-filtering the received signal and combining the resulting metrics over several consecutive time slot durations. A simple non-coherent combining metric has been used by the 3GPP in the evaluation of the primary synchronization detection performance. This metric is asymptotically optimum for rapidly moving mobile stations. However, it is inappropriate for slowly moving mobile stations, because of the strong correlation of the channel paths over several consecutive time slots.

Under the assumption of perfectly known small-scale propagation channel statistics, we propose in this paper an optimum Maximum *A Posteriori* (MAP) metric for the detection of the PSC. This metric is then extended to take into account Time-Switched Transmit Diversity (TSTD) at the base station.

The paper is organized as follows. In Section II, the system model is introduced. In Section III, the basic MAP metric is derived and extended to take into account transmit diversity at the base station. In Section IV, the PSC detection process is explained. Numerical and simulation results are provided in Section V. Our conclusions are given in Section VI.

II. System model

II.1. Transmitted signal model

Let $(\cdot)^T$ denote transposition. The PSC transmits a hierarchical synchronization sequence $\mathbf{s} = (s_0, s_1, \ldots, s_{F-1})^T$, with length F and good aperiodic auto-correlation function, at the beginning of each time slot. This synchronization sequence is common to all cells in the system. Its QPSK-modulated components are assumed here to be normalized to 1 in module.

Let $g(t)$ denote the impulse response of the root raised-cosine transmit filter adopted in the UMTS system. Let also T_c, T_s and E_c denote respectively the chip period, the time slot duration and the transmitted energy per chip. The transmitted signal of the PSC can be expressed as

$$x(t) = \sum_k s(t - kT_s - t_0),$$

where t_0 is the transmission time reference and

$$s(t) = \sqrt{E_c} \sum_{k=0}^{F-1} s_k g(t - kT_c)$$

is the waveform associated to the synchronization sequence.

II.2. Channel model

Let $\delta(t)$ denote the Dirac function. The equivalent base-band propagation channel is described by the time-varying impulse response

$$c(t;\tau) = \sum_{l=0}^{L-1} \alpha_l(t)\, \delta(\tau - \tau_l(t))$$

where $\alpha_l(t)$ and $\tau_l(t)$ are respectively the attenuation and delay of the l-th path and L is the total number of discrete multipath components.

Let $E[\cdot]$ denote expectation. The autocorrelation of the time-varying transfer function

$$C(f;t) = \int c(\tau;t) e^{-j2\pi f \tau} d\tau = \sum_{l=0}^{L-1} \alpha_l(t)\, e^{-j2\pi f \tau_l(t)}$$

is given by

$$\phi_C(\Delta t) = E[C^*(f;t) C(f;t+\Delta t)] = \phi_C(0)\, J_0(2\pi f_d \Delta t)$$

where $J_0(\cdot)$ is the zero-order Bessel function of the first kind and f_d is the maximum Doppler frequency.

II.3. Received signal model before and after matched filtering

Prior to matched filtering, the received signal is expressed as

$$y(t) = \int c(\tau;t) x(t-\tau) d\tau + n(t)$$

where $n(t)$ is a complex AWGN with two-sided power spectral density σ^2. To simplify the derivation of the MAP metric used in the detection of the PSC, the receiver assumes a flat fading channel with $L = 1$. The received signal model adopted by the receiver is therefore given by

$$y(t) = \alpha_0(t)\, x(t - \tau_0(t)) + n(t).$$

Moreover, to simplify the expression of the matched-filtered received signal, the receiver assumes the attenuation factor $\alpha_0(t)$ and delay $\tau_0(t)$ to be slowly varying over several chip periods and time slot durations, respectively. This assumption is generally valid for a mobile speed below 120 km/h.

For the detection of the PSC, the mobile receiver makes two main hypotheses, $\mathcal{H}_0$ and $\mathcal{H}_1$, corresponding respectively to the absence and the presence of a PSC.

Let τ_0 denote the approximate value taken by the slowly varying propagation delay $\tau_0(t)$ over several consecutive time slots. The second hypothesis $\mathcal{H}_1$ consists of an infinity of elementary hypothesis, denoted by $\mathcal{H}_1^{\Delta t}$, with $\Delta t \in [0, T_s[$ representing an additional hypothesis on the receiving time reference $t_0 + \tau_0$.

Conditional on hypothesis $\mathcal{H}_1^{\Delta t}$, the matched-filtered received signal sampled at time instants $\Delta t + kT_c$, $k \in \mathbb{Z}$, provides sufficient statistics. The time interval over which the received signal is observed and analyzed by the mobile station for time slot synchronization is given by $[0, KT_s + FT_c[$, where K is a non negative integer. The integer K should not be chosen too large in order to reduce the synchronization time and should not be chosen too small in order to reduce false synchronization probability. Let $\Delta k = \lfloor \Delta t / T_c \rfloor$ denote the discrete equivalent of the hypothetical receiving time reference Δt corresponding to hypothesis $\mathcal{H}_1^{\Delta t}$. According to hypothesis $\mathcal{H}_1^{\Delta t}$, the $KN + F$ components of the vector of sampled matched filter outputs $\mathbf{y} = (y_0, y_1, \ldots, y_{KN+F-1})^T$ are given by

$$y_k = \begin{cases} n_k & k \in \Sigma_0^{\Delta k} \\ \sqrt{E_c}\, c_k s_{(k-\Delta k) \bmod N} + n_k & k \in \Sigma_1^{\Delta k} \end{cases},$$

where $c_k = \alpha_0(\Delta t + (k - \Delta k)T_c)$ is the propagation attenuation factor corresponding to sample y_k, $\Sigma_1^{\Delta k} = \{\Delta k + i + jN\}_{i=0,j=0}^{F-1,K-1}$ is the set of indices in front of a transmitted synchronization sequence, $\Sigma_0^{\Delta k} = \{i\}_{i=0}^{KN+F-1} \setminus \Sigma_1^{\Delta k}$ is the set of indices corresponding exclusively to noise and n_k is a discrete additive decorrelated Gaussian noise with variance σ^2.

In case hypothesis $\mathcal{H}_0$ is true, the components of the vector $\mathbf{y}$, associated to receiving time reference hypothesis Δt, are made exclusively from noise. In both hypotheses cases, the complex vector $\mathbf{y}$ remains Gaussian distributed.

III. MAP detection metric

In a first stage of the derivation of the MAP detection metric, we assume a perfect knowledge of the propagation channel statistics. We also assume no drift in base station and mobile station local oscillators. In a second stage, we extend the obtained detection metric to take into account TSTD at the base station.

III.1. Basic MAP detection metric

Assuming always Δt as a time reference hypothesis, we can split the vector $\mathbf{y}$ into two complementary decorrelated vectors $\mathbf{y}_0$ and $\mathbf{y}_1$ with component indices arranged in increasing order in $\Sigma_0^{\Delta k}$ and $\Sigma_1^{\Delta k}$, respectively.

The probability distribution $p(\mathbf{y}_0 \mid \mathcal{H}_i)$ of $\mathbf{y}_0$ is independent of hypothesis $\mathcal{H}_i$, $i = 0$ or 1. Conditional on hypothesis $\mathcal{H}_0$, the probability density of $\mathbf{y}_1$ is given by

$$p(\mathbf{y}_1 \mid \mathcal{H}_0) = (\pi\sigma^2)^{-KF} \exp(-\mathbf{y}_1^{*T}\mathbf{y}_1 / \sigma^2). \qquad (1)$$

Let $(\cdot)^\dagger$ and $\otimes$ denote respectively Hermitian transposition and matrix Kronecker product. Let also $\mathbf{I}_D$ denote the D-dimensional identity matrix. Conditional on hypothesis $\mathcal{H}_1^{\Delta t}$, the probability density of $\mathbf{y}_1$ is given by

$$p(\mathbf{y}_1 \mid \mathcal{H}_1^{\Delta t}) = (\pi\sigma^2)^{-KF} \det(\mathbf{Q}^{-1}) \exp(-\mathbf{z}^\dagger \mathbf{Q}^{-1} \mathbf{z} / \sigma^2), \qquad (2)$$

where $\mathbf{Q} = E_c \mathbf{R} / \sigma^2 + \mathbf{I}_{KF}$, $\mathbf{R}$ is the covariance matrix of the attenuation factors $\alpha_0(t)$ of the propagation channel in front of the transmitted synchronization channel sequences, $\mathbf{z} = \mathbf{M}^\dagger \mathbf{y}_1$ is the modulation compensated version of $\mathbf{y}_1$, $\mathbf{M} = \mathbf{I}_K \otimes \mathbf{S}$ is a diagonal unitary matrix, where $\mathbf{S}$ is a diagonal matrix with the components of the synchronization sequence $\mathbf{s}$ as diagonal entries.

Taking into account the independence of $\mathbf{y}_0$ and $\mathbf{y}_1$ and using (1) and (2), we end up with

$$p(\mathbf{y} \mid \mathcal{H}_1^{\Delta t}) / p(\mathbf{y} \mid \mathcal{H}_0) = \det(\mathbf{Q}^{-1}) \exp(\mathbf{z}^\dagger (\mathbf{I}_{KF} - \mathbf{Q}^{-1}) \mathbf{z} / \sigma^2),$$

where we recall that $p(\mathbf{y} \mid \mathcal{H}_0)$ generally depends on the time reference hypothesis Δt through the sampling instants leading to the components of $\mathbf{y}$. If $\mathbf{R}$ is nonsingular then the previous expression simplifies to

$$\frac{p(\mathbf{y} \mid \mathcal{H}_1^{\Delta t})}{p(\mathbf{y} \mid \mathcal{H}_0)} = \det(\mathbf{Q}^{-1}) \exp(\mathbf{z}^\dagger (\mathbf{I}_{KF} + \sigma^2 \mathbf{R}^{-1} / E_c)^{-1} \mathbf{z} / \sigma^2).$$

If the time resolution used in the determination of Δt is equal to the chip period T_c, then the conditional probability $p(\mathbf{y} \mid \mathcal{H}_0)$ is independent of Δt. In this case, we can use the MAP metric

$$m_{MAP}(\mathbf{y} \mid \mathcal{H}_1^{\Delta t}) = \mathbf{z}^\dagger (\mathbf{I}_{KF} + \sigma^2 \mathbf{R}^{-1} / E_c)^{-1} \mathbf{z} \qquad (3)$$

in the determination of the most likely hypothesis among hypothesis $\mathcal{H}_0$ and $\mathcal{H}_1^{\Delta t}$, where $\Delta t \in \{\Delta t_0 + kT_c\}_{k=0}^{N-1}$ for some initial sampling instant $\Delta t_0 \in [0, T_c[$.

In almost all practical situations, a time resolution of $T_c / 2$ or below is used. In this case, the previous maximum likelihood metric in (3) should be augmented by the metric correction term

$$\Delta m_{MAP}(\mathbf{y} \mid \mathcal{H}_1^{\Delta t}) = -\mathbf{y}^\dagger \mathbf{y} = -\mathbf{z}^\dagger \mathbf{z}. \qquad (4)$$

For an oversampling factor Q and a time resolution T_c / Q, this metric correction takes only Q different values corresponding to initial sampling instants $\{\Delta t_0 + qT_c / Q\}_{q=0}^{Q-1}$ with $\Delta t_0 \in [0, T_c / Q[$. In order to simplify metric computation, we can use the simple metric in (3) as it is without any incorporation of the correction term in (4).

For a mobile speed below 120 km/h, the attenuation factor $\alpha_0(t)$ could be considered to be constant over the synchronization sequence duration. Hence, $\mathbf{R}$ becomes asymptotically singular and can be expressed as

$$\mathbf{R} = F\mathbf{H} \otimes \mathbf{P}$$

where $\mathbf{P} = \mathbf{p}_F \mathbf{p}_F^\dagger$ is a projector over the F-dimensional vector $\mathbf{p}_F = (1, 1, \ldots, 1)^T / \sqrt{F}$ and $\mathbf{H}$ is the covariance matrix of the attenuation factors $\alpha_0(t)$ of the propagation channel corresponding to the middle of the transmitted synchronization channel sequences. The entries of $\mathbf{H}$ are given explicitly by $H_{mn} = \phi_C((m-n)T_s)$, $m,n =$ 0,1,..., $K-1$.

Let $\mathbf{P}^{\perp} = \mathbf{I}_F - \mathbf{P}$ denote the projector over the orthogonal space of $\mathbf{p}_F$. Based on the previous considerations, we can rewrite the MAP metric as

$$m_{MAP}(\mathbf{y} \mid \mathcal{H}_1^{\Delta t}) = \mathbf{z}^{\dagger}((\mathbf{I}_K + \sigma^2 \mathbf{H}^{-1} / FE_c)^{-1} \otimes \mathbf{P})\mathbf{z}\,, \quad (5)$$

where we have assumed the covariance matrix $\mathbf{H}$ to be nonsingular, which is generally the case.

Let $\mathbf{z}_k$ denote the contribution of the k-th time slot duration to $\mathbf{z}$. The vector $\mathbf{z}$ can be rewritten as

$$\mathbf{z} = (\mathbf{z}_0^T, \mathbf{z}_1^T, \ldots, \mathbf{z}_{K-1}^T)^T\,.$$

Let $\mathbf{u} = (u_0, u_1, \ldots, u_{K-1})^T$ denote the vector with as k-th component the normalized correlator output

$$u_k = \mathbf{p}_F^{\dagger} \mathbf{z}_k\,, \quad (6)$$

at starting correlation time $\Delta t + kT_s$. The MAP metric in (5) could be further simplified into

$$m_{MAP}(\mathbf{y} \mid \mathcal{H}_1^{\Delta t}) = \mathbf{u}^{\dagger}(\mathbf{I}_K + \sigma^2 \mathbf{H}^{-1} / FE_c)^{-1}\mathbf{u}\,. \quad (7)$$

For an additional simplification of this MAP metric, we introduce the spectral factorization

$$\mathbf{H} / \phi_C(0) = \mathbf{W}\mathbf{\Lambda}\mathbf{W}^{\dagger} = \sum_{k=0}^{K-1} \lambda_k \mathbf{w}_k \mathbf{w}_k^{\dagger} \quad (8)$$

of the normalized version of $\mathbf{H}$, where $\lambda_0, \lambda_1, \ldots, \lambda_{K-1}$, arranged in decreasing order, are the normalized eigenvalues of $\mathbf{H}$ and the column vectors $\mathbf{w}_0, \mathbf{w}_1, \ldots, \mathbf{w}_{K-1}$, of the unitary matrix $\mathbf{W}$ are the corresponding orthonormal eigenvectors. Using (8), we can simply rewrite the metric in (7) as

$$m_{MAP}(\mathbf{y} \mid \mathcal{H}_1^{\Delta t}) = \sum_{k=0}^{K-1} \omega_k \left|\mathbf{w}_k^{\dagger}\mathbf{u}\right|^2, \quad (9)$$

where the weighting factors ω_k are given by

$$\omega_k = (1 + \sigma^2 / (F\bar{E}_c \lambda_k))^{-1}, \quad k = 0,1,\ldots,K-1\,,$$

and $\bar{E}_c = \phi_C(0)E_c$ is the average received energy per synchronization sequence chip.

The number of significant weighting factors ω_k decreases with any reduction in the mobile speed, in the chip energy to noise ratio $\bar{E}_c / \sigma^2$ or in the number of analyzed time slot durations K. Let κ denote the number of non negligible weighting factors ω_k with respect to unity. With an unnoticeable degradation in detection performance, we can further simplify the MAP metric in (9) into

$$m_{MAP}(\mathbf{y} \mid \mathcal{H}_1^{\Delta t}) = \sum_{k=0}^{K-1} \omega_k \left|\mathbf{w}_k^{\dagger}\mathbf{u}\right|^2. \quad (10)$$

When the mobile speed goes to 0, all eigenvalues vanish except λ_0 which goes to K. In this case $\mathbf{w}_0$ goes to $\mathbf{p}_K = (1,1,\ldots,1)^T / \sqrt{K}$ and the metric in (10) becomes

$$m_H^1(\mathbf{y} \mid \mathcal{H}_1^{\Delta t}) = \omega_0 \left|\mathbf{p}_K^{\dagger}\mathbf{u}\right|^2. \quad (11)$$

For large mobile speeds, all weighting factors are approximately equal to unity. In this case, the metric in (10) is transformed into

$$m_H^K(\mathbf{y} \mid \mathcal{H}_1^{\Delta t}) = \omega_0 \mathbf{u}^{\dagger}\mathbf{u} = \omega_0 \left\|\mathbf{u}\right\|^2, \quad (12)$$

where $\|\cdot\|$ denotes the Euclidean norm.

The latter limit of the MAP metric has been used in the evaluation of the detection performance of the PSC of the UMTS FDD mode [1]. Both metrics limits, in (11) and (12), represent extreme cases of a class of heuristic metrics, known as Segmented Replica Correlation (SRC) metrics [2].

III.2. MAP detection with time-switched transmit diversity

When TSTD is used in the base station, the synchronization sequence of the PSC is alternately transmitted on one of two spatially decorrelated transmit antennas.

For the determination of the MAP detection metric with TSTD, we assume the parameter K to be even. For any hypothetical receiving time reference $\Delta t \in [0, T_s[$, all correlators outputs u_k in (6) with common parity come from the same antenna. Let $\mathbf{u}^e = (u_0, u_2, \ldots, u_{K-2})^T$ and $\mathbf{u}^o = (u_1, u_3, \ldots, u_{K-1})^T$ denote the vectors of normalized correlator's outputs with even and odd indices, respectively. The spatial decorrelation of the propagation channels associated to the two transmit antennas lead to the detection metric

$$m_{MAP}(\mathbf{y} \mid \mathcal{H}_1^{\Delta t}) = \sum_{k=0}^{K/2-1} \tilde{\omega}_k (\left|\tilde{\mathbf{w}}_k^{*}\mathbf{u}^e\right|^2 + \left|\tilde{\mathbf{w}}_k^{*}\mathbf{u}^o\right|^2)\,,$$

where $\tilde{\mathbf{w}}_0, \tilde{\mathbf{w}}_1, \ldots, \tilde{\mathbf{w}}_{K/2-1}$ are the eigenvectors of the normalized version $\tilde{\mathbf{H}} / \phi_C(0)$ of the covariance matrix $\tilde{\mathbf{H}}$ with entries $\tilde{H}_{mn} = H_{2m,2n}$, $m,n = 0,1,\ldots,K/2-1$, and the weighting factors $\tilde{\omega}_k$ are given by

$$\tilde{\omega}_k = (1 + \sigma^2 / (F\bar{E}_c \tilde{\lambda}_k))^{-1}, \quad k = 0,1,\ldots,K/2-1\,,$$

where $\tilde{\lambda}_0, \tilde{\lambda}_1, \ldots, \tilde{\lambda}_{K/2-1}$, arranged in decreasing order, are the normalized eigenvalues of $\tilde{\mathbf{H}}$ associated to the eigenvectors $\tilde{\mathbf{w}}_0, \tilde{\mathbf{w}}_1, \ldots, \tilde{\mathbf{w}}_{K/2-1}$.

IV. Primary synchronization channel detection

For a fixed oversampling factor Q and an observation and analysis time interval $[0, KT_s + FT_c[$, the receiver carries KNQ sliding correlations. The receiver computes the metric $m(\mathbf{y} \mid \mathcal{H}_1^{\Delta t})$ for all NQ hypothetical receiving reference times in the set $\Omega_{\Delta t} = \{kT_c / Q\}_{k=0}^{NQ-1}$. The most likely estimate of the reference receiving time for detection metric $m(\mathbf{y} \mid \mathcal{H}_1^{\Delta t})$ is determined by

$$\widehat{\Delta t} = \arg\max_{\Delta t \in \Omega_{\Delta t}} m(y \mid \mathcal{H}_1^{\Delta t})\,.$$

The time slot synchronization is declared to be correctly achieved if the reference receiving time estimate $\widehat{\Delta t}$ is within less than $T_c / 2Q$ of one of the L main paths of the time-varying impulse response $c(t;\tau)$. Otherwise, a false time slot synchronization is accounted for.

V. Numerical and simulation results

V.1. Numerical results

We show in figure 1 the behavior of the weighting coefficients ω_k as a function of k, for $K = 16$, a mobile speed of 120 km/h and $\bar{E}_c / \sigma^2$ ranging from -25 to -5 dB. As expected, the number of significant weighting factors is small (respectively, large) for small (respectively, large) values of the mobile speed and $\overline{E}_c/\sigma^2$. With an unnoticeable performance degradation, only $\kappa = 2$ eigenvectors $\mathbf{w}_k$ need to be considered in the simplified metric in (10) for a mobile speed of 3 km/h. However, as much as $\kappa = 8$ eigenvectors should be taken into account for a mobile speed of 120 km/h, in order to have an unnoticeable degradation in performance.

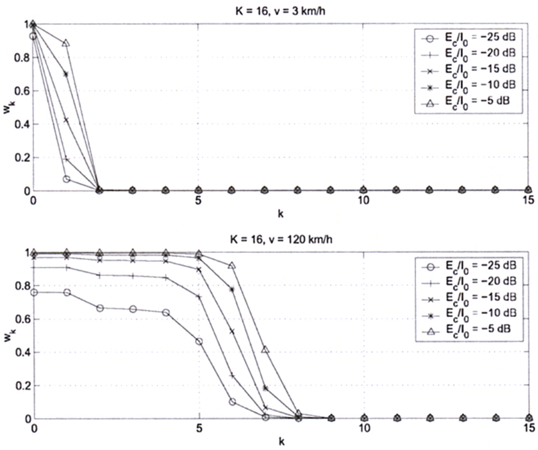

Figure 1: Behavior of the weighting factors ω_k for several values of $\overline{E}_c/\sigma^2$.

V.2. Simulation results

Figures 2 provides a characterization of the potential enhancement in PSC detection performance when the optimum metric (OM) is used instead of the heuristic metric (HM). In this characterization, we have considered the ITU Outdoor-to-Indoor and Pedestrian A channel model with a small mobile speed of 3 km/h. We notice here an enhancement in performance following any increase in the parameter K. For a false detection rate of 10^{-2}, less than 1 dB in performance gain is observed for $K = 2$. However, as much as 3 dB are gained for $K = 16$. Moreover, with $K = 8$, the OM achieves the same performance as the HM, with $K = 16$.

In Figure 3, we compare the performances of the OM and the HM when TSTD is used at the base station. This comparison is carried for a mobile speed of 3 km/h and the ITU Indoor A channel model. We observe here a reduced enhancement in performance with respect to the case where no transmit diversity is used at the base station. For instance, only a 2.5 dB gain is observed with TSTD for $K = 16$ at a false detection ratio of 10^{-2}. This reduced gain is due to the decorrelation of the two channels observed by the even and odd numbered time slots.

VI. Conclusion

We have proposed a maximum *a posteriori* metric for the detection of the primary synchronization channel of the UMTS/FDD mode. In the derivation of this optimum metric, we have assumed perfect small-scale propagation channel statistics at the mobile station. Then, we have extended this metric to take into account time-switched transmit diversity at the base station.

The performance of this optimum metric in terms of false detection rate has been compared to that of the heuristic non-coherent averaging metric used in the evaluation process of the 3GPP. We have observed larger gains with both smaller mobile station speeds and larger values of the number of processed time slot periods. However, we have noticed a reduction in the resulting gains with the use of time switched transmit diversity.

For an extra reduction in complexity, our metric can be conditionally application to contending synchronization time instants obtained by the non-coherent heuristic metric. Our approach could be profitably used in the derivation of an optimum metric for the detection of the secondary synchronization channel of the UMTS/FDD mode.

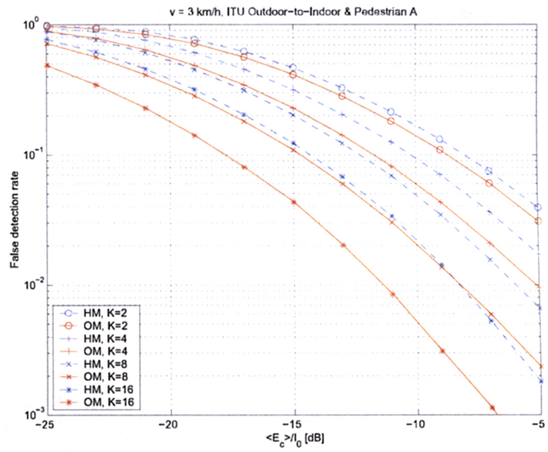

Figure 2: False detection rates for the Heuristic Metric (HM) and the Optimum Metric (OM) without TSTD.

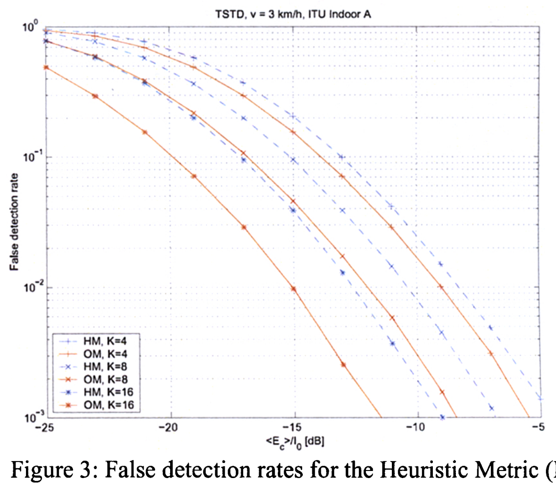

Figure 3: False detection rates for the Heuristic Metric (HM) and the Optimum Metric (OM) with TSTD.

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