Joint Carrier and Sampling Frequency Offset Estimation Based on Harmonic Retrieval

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Abstract— **The performance of an OFDM system is highly sensitive to both carrier and sampling frequency offsets. This paper proposes a new joint carrier and sampling frequency offset estimation scheme for OFDM communications suitable for a frequency-selective fading channel. The joint carrier frequency offset (CFO) and sampling frequency offset (SFO) estimation in OFDM is reformulated and resolved as a harmonic retrieval problem. Accurate estimation can be reached even when only one OFDM symbol is used, hence making this method appropriate for offsets with a high level of variability in time. Extension to block processing is presented. Moreover the presented method proved to be robust against large CFO-SFO values, a claim that can not be made for other joint estimation techniques.**

I. Introduction

Due to its high spectral efficiency and robustness to multipath fading channels, OFDM is becoming the technique of choice for high data rates transmission. OFDM has been already used for digital audio broadcasting (DAB), digital video broadcasting (DVB), Wireless Local Area Networks (WLAN), and other high speed data applications for both wireless and wired communications. OFDM is also a serious candidate to be the standard for 4th generation (4G) mobile communication systems. Despite its promises, OFDM is very sensitive to synchronization errors and in the presence of such inaccuracies, the performance of an OFDM system and all the smart benefits of OFDM are almost lost [1]. Consequently, both carrier and sampling frequency offsets should be estimated and compensated before demodulating the data with the Discrete Fourier Transform (DFT).

CFO estimation has been thoroughly investigated in several papers. Both data-aided schemes [2]-[4] and blind schemes [5]-[7] were developed. SFO estimation has also received some interest in [8]-[10].

These carrier and sampling frequency offset estimators were derived assuming perfect conditions. Indeed, they assumed zero SFO when dealing with CFO estimation and vice versa. In practice, these two imperfections coexist, which will in return degrade these estimator's accuracy. A nonzero CFO or SFO will generate Inter Carrier Interference (ICI) that makes the estimators inaccurate. Therefore joint estimation schemes are of great interest in practice.

Few methods appraise the joint CFO-SFO estimation problem. These methods hinge on the repetition of two or more OFDM symbols [11],[12] or on pilot subcarriers inserted in multiple successive OFDM symbols [13]. The fact, that these methods make use of multiple OFDM blocks in their estimation scheme, will introduce an ambiguity in the choice of the number of OFDM symbols to be used. In fact this choice has to be taken under the consideration of two constraints: (1) the delay requirements; (2) the duration for which the frequency offsets can be assumed to be constant, which can be constricted to a small number of OFDM symbols in an environment presenting fast varying Doppler shift. These methods work well when both the CFO and SFO are small enough, but their estimation accuracy trims down when one of these parameters increases.

In our technique, accurate estimation is reached by using only one OFDM symbol. It is therefore suitable for situations with strict delay requirements. The case where the frequency offsets are constant over more than one OFDM symbol is also covered. In such a case, block processing is proposed, thereby allowing higher estimation accuracy. In addition our technique is immune against the effects of large CFOs and SFOs.

In this paper, we address frequency synchronization by reformulating the joint CFO-SFO estimation into a harmonic retrieval problem. In fact the received OFDM signal is nothing but a sum of superimposed complex exponentials whose frequencies are a function of the CFO and the SFO. Thus retrieving those frequencies offers us the possibility to estimate both the CFO and SFO and compensate them. This reformulation allows the use of a wide range of harmonic retrieval methods. We chose to apply two of the most known of them, namely the Tufts-Kumaresan (TK) [14] and the Matrix Pencil (MP) [15] algorithms.

The paper is organized as follows. In Section II, the OFDM system model is briefly described and CFO-SFO estimation is reformulated as a harmonic retrieval problem. The estimation procedures based on the TK and the MP methods are presented in Section III. Numerical examples are illustrated in Section IV.

II. System Model

We consider a discrete time OFDM system, which is not fully loaded, i.e., not all available subcarriers are used for transmitting data (this is usually the case for multicarrier systems). At the transmitter, t complex-valued symbols X_k, modulate t out of N orthogonal subcarriers using the IDFT transform:

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$$x(n) = \frac{1}{N} \sum_{k \in K} X_k e^{j2\pi kn/N}, \quad n = 0, \ldots, N-1 \quad (1)$$

where $K = \{k_1 \ldots k_t\}$ designates the set of active subcarriers. Before transmitting the signal, a cyclic prefix is added. The transmitted signal will have consequently the following periodic shape:

$$s(n) = \begin{cases} x(n+N) & \text{if } -N_g \leq n < 0, \\ x(n) & \text{if } 0 \leq n \leq N-1, \end{cases}$$

where the cyclic prefix length N_g is assumed to be greater than the channel length. This signal $s(n)$ is sent through a bandpass channel. At the receiver, the first N_g samples are discarded as they are contaminated by Inter Symbol Interference (ISI), leading to:

$$r(n) = \frac{1}{N} \sum_{k \in K} X_k H_k e^{\frac{j2\pi(k(1+\varepsilon_s)+\varepsilon_c)n}{N}} + w(n), n = 0, \ldots, N-1 \quad (2)$$

where $H_k = \sum_{l=0}^{N_c-1} h_l e^{-j2\pi kl/N}$ is the transfer function of the channel at the frequency of the kth carrier, ε_c is the relative carrier frequency offset (normalized to the subcarrier spacing), ε_s is the relative sampling frequency offset (normalized to the sampling frequency interval) and $w(n)$ is an additive white Gaussian noise.

One can notice that the received signal consists of a sum of superimposed complex exponentials embedded in noise, i.e.,

$$r(n) = \sum_{i=1}^{t} a_i e^{j2\pi f_i n} + w(n), \quad n = 0, \ldots, N-1 \quad (3)$$

where $a_i = \frac{1}{N} X_{k_i} H_{k_i}$ and $f_i = \frac{1}{N}(k_i(1+\varepsilon_s) + \varepsilon_c)$ for $i = 1 \ldots t$. The frequencies f_i are closely related to the CFO ε_c and the SFO ε_s and so recovering these frequencies offers us the opportunity to estimate both the CFO and the SFO. The original problem of joint CFO-SFO estimation therefore reduces to that of determining the frequencies of complex superimposed sinusoids.

Note that (3) represents a system of N equations with $2t$ unknown variables $\{f_i, a_i\}_{i=1\ldots t}$. Thus to solve this system, we have to satisfy the constraint $2t \leq N$. This constraint requires that at least half of the subcarriers must be virtual ones. The fact that half of the subcarriers are virtual will inevitably affect the bandwidth efficiency. However, other joint estimation schemes will exhibit the same limitation. In deed, as an example, the method in [11] will send twice the same OFDM symbol, which then will translate into the same bandwidth efficiency as in our method.

The estimation accuracy can be considerably improved if both the CFO and SFO can be considered to be constant during more than one symbol. This is actually the case in some applications like in OFDM-Based WLAN systems where multiple OFDM systems are used in the synchronization phase. Indeed, let us introduce:

$$r_c(m) = E[r(n) r^*(n-m)], \qquad m = 0, \ldots, N-1. \quad (4)$$

Under the assumptions that the noise and the data samples are both independent and identically distributed (i.i.d.) and mutually independent, we can easily show that $\{r_c(m), m = 0, \ldots, N-1\}$ exhibits the same structure in that it is also a sum of superimposed complex sinusoids:

$$r_c(m) = E\left[\left(\frac{1}{N}\sum_{k \in K} X_k H_k e^{j2\pi(k(1+\varepsilon_s)+\varepsilon_c)n/N} + w(n)\right)\right.$$
$$\left.\left(\frac{1}{N}\sum_{l \in K} X_l^* H_l^* e^{-j2\pi(l(1+\varepsilon_s)+\varepsilon_c)(n-m)/N} + w^*(n-m)\right)\right]$$
$$= \frac{1}{N^2}\sum_{k \in K} E[|X_k|^2] E[|H_k|^2] e^{j2\pi(k(1+\varepsilon_s)+\varepsilon_c)m/N} + \delta(m)\sigma_w^2$$
$$m = 0, \ldots, N-1.$$

Thus the same algorithms could be applied to both the instant received samples and the autocorrelation sequence. To summarize, we have reformulated the joint CFO-SFO estimation into a harmonic retrieval problem. We have shown that two approaches may be used: one based on the instant received samples when estimation is carried out over one OFDM symbol and another based on the autocorrelation terms over many symbols. In the next section we will see how harmonic retrieval algorithms can be used for joint CFO-SFO estimation.

III. CFO-SFO ESTIMATION PROCEDURES

To estimate the frequencies of these complex exponentials, many harmonic retrieval algorithms could be used. We have chosen two of the most known of them, namely the Tufts-Kumaresan (TK) [14] and the Matrix Pencil (MP) [15] algorithms.

First let us define the forward and backward matrices that will be used in the algorithms as

$$\mathbf{R}_{jFB} = \begin{bmatrix} \mathbf{R}_{jF} \\ \mathbf{R}_{jB} \end{bmatrix}, \quad j = \{0, 1\} \quad (5)$$

where

$$\mathbf{R}_{jF} = [\mathbf{r}_{L-1+j}, \mathbf{r}_{L-2+j}, \ldots, \mathbf{r}_j], \quad (6)$$

$$\mathbf{R}_{jB} = [\mathbf{r}_{1-j}, \mathbf{r}_{2-j}, \ldots, \mathbf{r}_{L-j}]^*, \quad (7)$$

and

$$\mathbf{r}_i = [r(i), r(i+1), \ldots, r(i+N-L-1)]^T. \quad (8)$$

The superscripts T and $*$ denote, respectively, transpose and complex conjugate; and L should satisfy $t \leq L \leq N-t$, ($t \leq L \leq N - \lceil \frac{t}{2} \rceil$ for the TK-based method, where $\lceil \frac{t}{2} \rceil$ stands for an integer rounded to the next larger integer value if $\frac{t}{2}$ is not an integer). The estimation algorithm can now be derived as follows:

Step1: obtain $\{z_i = e^{j2\pi f_i}, i = 1, 2, \ldots, t\}$ using the TK or the MP method (see Sections III.A and B) or any other desired harmonic retrieval method.

Step2: Compute the CFO and SFO estimates using the following formula:

$$\hat{\varepsilon}_s = \frac{1}{2(t-1)\pi} \sum_{i=2}^{t} \frac{\angle z_i^N - \angle z_{i-1}^N}{k_i - k_{i-1}}. \quad (9)$$

$$\hat{\varepsilon}_c = \frac{1}{t}\sum_{i=1}^{t}\left(\frac{\angle z_i^N}{2\pi} - k_i\hat{\varepsilon_s}\right). \quad (10)$$

where $\angle z_i$ stands for the argument of z_i. The acquisition range of this algorithm is $|\varepsilon_c| < 0.5$. We will now detail the steps to obtain $\{z_i, i = 1, \ldots, t\}$ using the TK and the MP methods.

A. TK method

The TK method was introduced by Tufts and Kumaresan [14]. When applied to the CFO-SFO estimation problem, this method can be summarized in the following steps:

Step1: Form the data vector $\mathbf{h}_{FB} = [\mathbf{r}_L^T, \mathbf{r}_0^H]^T$, where the superscript H denotes the Hermitian.

Step2: Form the coefficients vector of the polynomial $\mathbf{g}$ of order L as

$$\mathbf{g} = -[\mathbf{R}_{0FB}]_T^{\dagger}.\mathbf{h}_{FB}, \quad (11)$$

where $[\mathbf{R}_{0FB}]_T^{\dagger}$ is the "truncated rank-t" pseudoinverse of $\mathbf{R}_{0FB}$, which can be expressed as

$$[\mathbf{R}_{0FB}]_T^{\dagger} = \sum_{i=1}^{t}\frac{1}{\sigma_i}\mathbf{v}_{0i}\mathbf{u}_{0i}^H = \mathbf{V}_0\mathbf{A}^{-1}\mathbf{U}_0^H, \quad (12)$$

where $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_t \geq \ldots$ are the t largest singular values of $\mathbf{R}_{0FB}$. $\mathbf{u}_{0i}$ and $\mathbf{v}_{0i}$ are, respectively, the corresponding right and left singular vectors; $\mathbf{V}_0 = [\mathbf{v}_{01}, \ldots, \mathbf{v}_{0t}]$; $\mathbf{U}_0 = [\mathbf{u}_{01}, \ldots, \mathbf{u}_{0t}]$ and $\mathbf{A} = diag\{\sigma_1, \ldots, \sigma_t\}$.

Step3: Find the roots of the polynomial $1 + \sum_{l=1}^{L} g_l z^{-l}$, where g_l is the l^{th} element of $\mathbf{g}$. The t zeros $(z_i, i = 1, 2, \ldots, t)$ which are the nearest to the unit circle, are taken as the estimates of $z_i = e^{j2\pi f_i}$.

Step4: Compute the SFO and the CFO estimates by (9) and (10).

B. MP method

The Matrix Pencil was developed for the harmonic retrieval problem in [15]. The procedure to estimate the CFO and the SFO using this method is summarized in the next steps.

Step1: Form the "truncated rank-t" pseudoinverse of $\mathbf{R}_{0FB}$, $[\mathbf{R}_{0FB}]_T^{\dagger}$, as in the previous method.

Step2: Compute the t eigenvalues of $\mathbf{A}^{-1}\mathbf{U}_0^H\mathbf{R}_{1FB}\mathbf{V}_0$, which correspond to the estimates of $z_i = e^{j2\pi f_i}$.

Step3: Compute the SFO and the CFO estimates by (9) and (10).

IV. Numerical Examples

In this section, we will illustrate some numerical examples that will show the accuracy of our estimators. The results of our estimators were compared to the estimator recently presented in [13]. As a measure of performance, the mean square error (MSE) was plotted for different values of the SNR:

$$MSE_{c,s} = \frac{1}{M_c}\sum_{i=1}^{M_c}(\hat{\varepsilon}_{c,s}(i) - \varepsilon_{c,s})^2, \quad (13)$$

where M_c denotes the number of Monte Carlo trials.

The simulations were performed for an OFDM system with $N = 32$ subcarriers and a cyclic prefix length equal to 8. The active subcarriers correspond to the set $\{2i, i = 0 \ldots 15\}$ (i.e., $t = 16$) and L was fixed to 16. For the method presented in [13] we used 2 OFDM symbols having each 16 pilots. The figures were obtained by running 5000 independent realizations (i.e., $M_c = 5000$). Finally the simulations were conducted in a multipath environment having four paths with path delays of 0, 3, 5 and 7 samples. The amplitude h_i of the ith path varies independently of the others according to a Rayleigh distribution with exponential power delay profile, i.e., $E[|h_i|^2] = exp(-\tau_i)$ where τ_i is the delay of the ith path.

Fig. 1 depicts the results obtained for an actual carrier frequency offset equal to 0.4 and a sampling frequency offset selected equal to 100 ppm (parts per million). For the CFO estimation, it can be seen that our methods clearly outperform the method in [13]. For the SFO estimation, our methods will outperform [13] for a wide range of SNR values. This figure shows also that the MP method gives better results

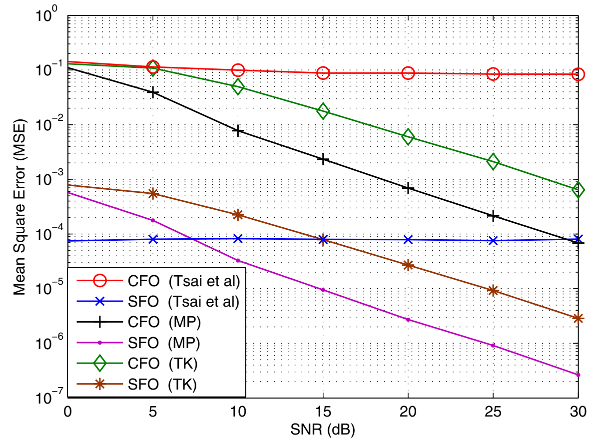

Fig. 1. Mean square error as a function of SNR ($\varepsilon_c = 0.4$, $\varepsilon_s = 100$ppm).

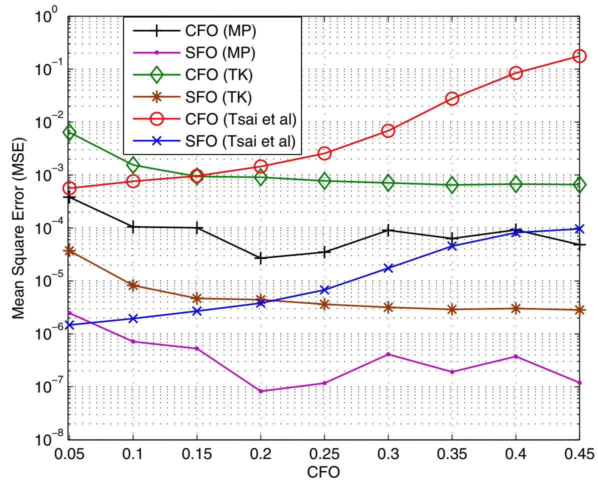

Fig. 2. Mean square error as a function of CFO, SNR=30 dB ($\varepsilon_s = 100$ppm).

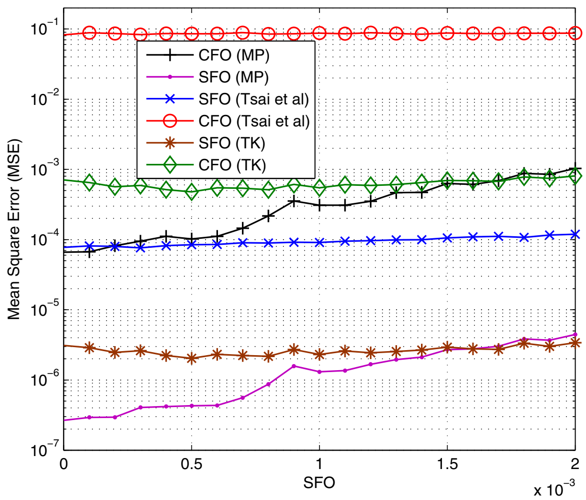

Fig. 3. Mean square error as a function of SFO, SNR=30 dB ($\varepsilon_c = 0.4$).

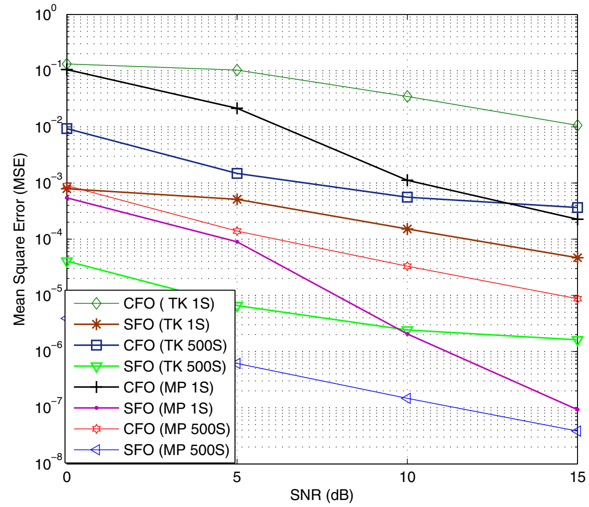

Fig. 4. Mean square error as a function of SNR ($\varepsilon_c = 0.4$, $\varepsilon_s = 100$ppm).

when compared to the TK method. One can notice that the performance of the method [13] does not really improve when the SNR increases, this can be explained by the fact that since the CFO has a relatively large value (0.4) the Inter Carrier Interference (ICI) power will prevail on the SNR power. Notice that our methods are immune against this effect and that they are still accurate even for large CFO values.

Fig. 2 illustrates the simulations obtained for different values of the CFO going from 0 to 0.45 and for a sampling frequency offset set at 100 ppm. The SNR was fixed here at 30 dB. Again our estimators provide a noticeable advantage for a wide range of CFO values when compared to the method in [13]. The MP method has always the best overall performance of the three methods. Again one can notice that our methods will perform well even at high CFO. This is not the case for the method in [13], indeed as shown in this figure the performance of this method will begin to degrade at CFOs superior to 0.25.

Fig. 3 illustrates the simulations obtained for different values of the SFO going from 0 ppm to 2000 ppm and for a carrier frequency offset set at 0.4. The SNR was fixed at 30 dB. One can notice that our methods outperform the method in [13] for all SFO values.

To assess the effect of block processing on the behavior of our estimators, comparaison between the performance of the data-based version (which uses instantly the received samples of one OFDM symbol) and the one using the autocorrelation sequence (we used 500 OFDM symbols to calculate the autocorrelation sequence) are depicted in fig. 4. It can be seen that the estimation accuracy is increased when using the version based on the autocorrelation sequence. But this assumes that both the CFO and SFO are constant during these symbols.

V. Conclusion

We have reformulated the joint carrier and sampling frequency offset estimation as a harmonic retrieval problem. Based on this approach, two new techniques for the joint estimation of the CFO and SFO for OFDM systems suitable for a frequency-selective fading channel were presented. One of the significant characteristics of the proposed methods is data-efficiency. In fact only one OFDM symbol is used to produce accurate estimation. Averaging over a block of symbols results in even higher accuracy when the CFO and the SFO are constant over more than one symbol. Simulations have shown that our estimators provide better performance than the one presented in [13].

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