Blind Sampling Clock Offset Estimation in OFDM Systems Based on Second Order Statistics

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Abstract— **Synchronization mismatches are of an outstanding harm to the performance of an OFDM receiver. More specifically, Sampling Clock Offset (SCO), if not compensated, could lead to an unacceptable BER increase. In this paper we exploit the second order statistics of the received OFDM samples to provide a blind estimation of the SCO. The proposed method gives satisfactory results without the need of a synchronization symbol, thus achieving high spectral efficiency.**

I. Introduction

Due to its high spectral efficiency and robustness to multipath fading channels, OFDM is becoming the technique of choice for high data rates transmission. OFDM is already used in digital audio broadcasting (DAB) [1], digital video broadcasting (DVB) [2], Wireless Local Area Networks (WLAN) [3], and other high speed data applications for both wireless and wired communications. OFDM is also a serious candidate to be in the standard for 4th generation (4G) mobile communication systems.

Despite its promises, OFDM is very sensitive to synchronization errors and, in the presence of such inaccuracies, the performance of an OFDM system can be greatly degraded. Two mismatches are particularly harmful to the OFDM system; the carrier frequency offset [4] and the sampling clock offset [5]. These two offsets induce the same effects; amplitude reduction and phase rotation of the demodulated data, in addition, they destroy the orthogonality between the subcarriers, or, in other words, they generate Inter Carrier Interference (ICI). The ICI will behave like an additional noise and will therefore degrade the SNR and, a fortiori, the performance of the OFDM system. Consequently, these errors should be estimated and compensated before demodulating the data with the Discrete Fourier Transform (DFT).

A huge amount of methods have been proposed to combat the carrier frequency offset (CFO). Both data-aided methods [6]-[8] and blind schemes [9]-[11] have been developed. However research devoted to the estimation of the sampling clock offset is rather scarce, this is surprising since, as discussed above, the SCO can be as deleterious to the OFDM system as the CFO.

Few data-aided approaches have been introduced in the literature for SCO estimation. Those methods rely on the transmission of a known synchronization symbol [12] or in the insertion of pilot symbols within the transmitted OFDM symbol [13]. While those methods provide a reliable estimation, the insertion of training sequences will inevitably trim down the spectral efficiency of OFDM systems. From this perspective, blind methods represent an interesting alternative.

In this paper, hinging on the second order statistics of the received OFDM samples, we derive a blind SCO estimator that will provide good results for a wide range of SNR values. Second order statistics were also used successfully for carrier frequency offset and symbol timing offset estimation in OFDM systems (see [15] and [16]). Which suggests a possible combination of these methods and the proposed one to provide full synchronization for OFDM systems.

The rest of the paper is organized as follows. In Section II, the OFDM system model is introduced. The synchronization method is described in section III. Numerical examples are illustrated in Section IV. Conclusions are given in Section V.

II. System Model

As shown in Fig.1, we consider a discrete-time OFDM system with N subcarriers. At the transmitter, N complex-valued symbols X_k, that belong to a QAM or PSK constellation, modulate N orthogonal subcarriers using the Inverse Fast Fourier Transform (IFFT). Before transmission, a cyclic prefix is appended at the beginning of the signal, yielding:

$$s(n) = \begin{cases} x(n+N), & \text{if } \ -N_g \leq n < 0, \\ x(n), & \text{if } 0 \leq n \leq N-1, \end{cases}$$

where $x(n) = \frac{1}{N}\sum_{k=0}^{N-1} X_k e^{\frac{j2\pi kn}{N}}, n = 0, \ldots, N-1$. At the receiver, the cyclic prefix is discarded leading to the following received samples:

$$\begin{aligned} r(n) &= \sum_{l=0}^{N_c-1} h_l s((\varepsilon+1)n - l) + \mu(n) \qquad (1) \\ &= \frac{1}{N}\sum_{l=0}^{N_c-1} h_l \left(\sum_{k=0}^{N-1} X_k e^{\frac{j2\pi k(n(\varepsilon+1)-l)}{N}}\right) + \mu(n) \\ &= \frac{1}{N}\sum_{k=0}^{N-1} X_k \left(\sum_{l=0}^{N_c-1} h_l e^{\frac{-j2\pi kl}{N}}\right) e^{\frac{j2\pi kn(\varepsilon+1)}{N}} + \mu(n) \\ &= \frac{1}{N}\sum_{k=0}^{N-1} X_k H_k e^{\frac{j2\pi kn(\varepsilon+1)}{N}} + \mu(n) \\ &= y(n) + \mu(n), \quad n = 0, \ldots, N-1, \qquad (2) \end{aligned}$$

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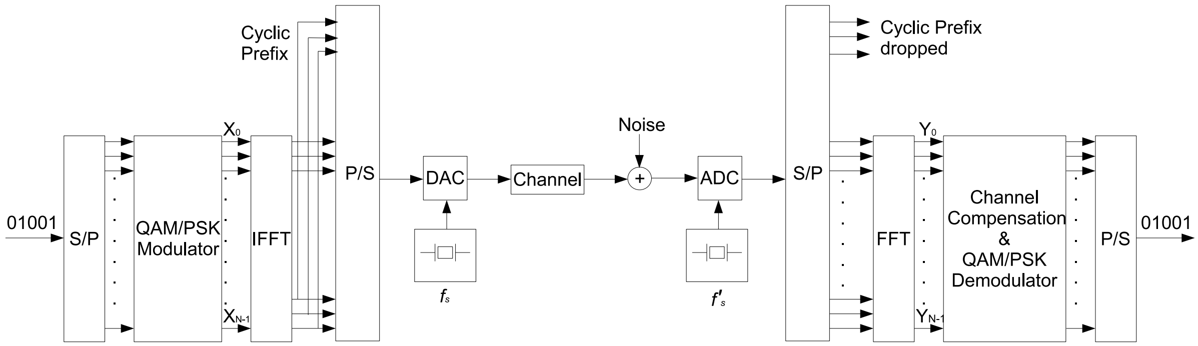

Fig. 1. OFDM transceiver.

where $H_k = \sum_{l=0}^{N_c-1} h_l e^{-j2\pi kl/N}$ is the transfer function of the channel at the frequency of the kth subcarrier, N_c corresponds to the channel length, ε is the relative sampling clock offset (the ratio of the actual offset to the sampling period), and $\mu(n)$ is an additive noise. If we process directly the received samples via the FFT without a prior estimation and cancelation of the Sampling Clock Offset, then the demodulated signal will be as follows:

$$Y_k = \sum_{m=0}^{N-1} r(m)\exp(-\frac{j2\pi km}{N}) \quad (3)$$

$$= \frac{1}{N}\sum_{l=0}^{N-1} X_l H_l \sum_{m=0}^{N-1} e^{\frac{j2\pi(l\varepsilon+l-k)m}{N}} + \sum_{m=0}^{N-1}\mu(m)e^{-\frac{j2\pi km}{N}} \quad (4)$$

$$= \frac{1}{N}\sum_{l=0}^{N-1} X_l H_l \left(\frac{1-e^{j2\pi l\varepsilon}}{1-e^{\frac{j2\pi(l\varepsilon+l-k)}{N}}}\right) + N_k \quad (5)$$

$$= X_k H_k e^{j\pi k\varepsilon\frac{N-1}{N}} \frac{\sin(k\pi\varepsilon)}{N\sin(\frac{\pi k\varepsilon}{N})} + ICI(k) + N_k. \quad (6)$$

where the $ICI(k)$ refers to the Inter Carrier Interference generated by the sampling clock offset and is given by

$$ICI(k) = \sum_{l=0, l\neq k}^{N-1} X_l H_l e^{j\pi l\varepsilon\frac{N-1}{N}} e^{-\frac{j\pi(l-k)}{N}} \frac{\sin(l\pi\varepsilon)}{N\sin(\frac{\pi(l\varepsilon+l-k)}{N})} \quad (7)$$

Note that the effect of the sampling clock offset is threefold; an amplitude reduction and a phase rotation of the signal $X_k H_k$, and the appearance of ICI which signals the loss of the orthogonality between the subcarriers. All these three effects will contribute to the degradation of the OFDM system performance. Thus, in order to keep the smart benefits offered by OFDM we need to estimate and compensate the sampling clock offset.

III. Sampling clock offset estimation

In this section we use the following assumptions

- $\{X_k, k = 0...N-1\}$ is a zero mean i.i.d. sequence, i.e.,

$$\begin{cases} E[X_k] = 0, \\ E[X_k X_l^*] = \delta_{k,l}\sigma_X^2. \end{cases} \quad (8)$$

- $\{\mu_k, k = 0...N-1\}$ is a zero mean i.i.d. sequence, i.e.,

$$\begin{cases} E[\mu_k] = 0, \\ E[\mu_k \mu_l^*] = \delta_{k,l}\sigma_\mu^2. \end{cases} \quad (9)$$

- $\{X_k, k = 0...N-1\}$ and $\{\mu_k, k = 0...N-1\}$ are uncorrelated, i.e.,

$$E[\mu_k X_n] = 0, (k,n) \in \{0...N-1\}^2. \quad (10)$$

- We make use of subcarrier weighting, i.e, each symbol on the kth subcarrier X_k is multiplied by a weight w_k. This is equivalent to say that the subcarriers are transmitted with different powers. In the receiver the demodulated symbol corresponding to the kth subcarrier has to be divided by the weight w_k. The utility of subcarrier weighting will be highlighted later.

Let us consider the autocorrelation function of the received sequence

$$r_c(m) = E[r(n)r^*(n-m)], \qquad m = 1, \ldots, N-1. \quad (11)$$

Since the noise and the data are uncorrelated, then

$$r_c(m) = E[y(n)y^*(n-m)] = y_c(m), \quad (12)$$

where now, due to the introduction of the subcarrier weighting, $y(n)$ is given by

$$y(n) = \frac{1}{N}\sum_{k=0}^{N-1} w_k X_k H_k e^{\frac{j2\pi kn(\varepsilon+1)}{N}}. \quad (13)$$

It is easy to see that

$$y_c(m) = \frac{1}{N^2}\sum_{k=0}^{N-1}\sum_{l=0}^{N-1} w_k w_l^* E[X_k X_l^*] E[H_k H_l^*] \times e^{\frac{j2\pi(1+\varepsilon)[n(k-l)+lm]}{N}}. \quad (14)$$

As a consequence of the assumptions made earlier, we have that

$$y_c(m) = \frac{1}{N^2}\sigma_X^2 \sum_{k=0}^{N-1} |w_k|^2 E[|H_k|^2] e^{\frac{j2\pi k(1+\varepsilon)m}{N}}. \quad (15)$$

Assuming that the channel path gains $h_l, (l = 0...N_c-1)$ are independent zero-mean Gaussian random variables (this describes a frequency selective Rayleigh fading channel), we can easily show that

$$E[|H_k|^2] = \sum_{l=0}^{N_c-1} E[|h_l|^2] = \sigma_H^2. \quad (16)$$

Therefore the autocorrelation function of the received sequence is given by

$$r_c(m) = \frac{1}{N^2}\sigma_X^2\sigma_H^2 \sum_{k=0}^{N-1} |w_k|^2 e^{\frac{j2\pi k(1+\varepsilon)m}{N}}. \quad (17)$$

Now since $N\varepsilon$ is small, the following approximation holds

$$e^{\frac{j2\pi k\varepsilon m}{N}} \approx 1 + \frac{j2\pi k\varepsilon m}{N}. \quad (18)$$

Consequently

$$r_c(m) \approx \frac{1}{N^2}\sigma_X^2\sigma_H^2 \sum_{k=0}^{N-1} |w_k|^2 e^{\frac{j2\pi km}{N}}(1 + \frac{j2\pi k\varepsilon m}{N}). \quad (19)$$

Or equivalently

$$r_c(m) \approx \frac{1}{N^2}\sigma_X^2\sigma_H^2(A(m) + \varepsilon B(m)), \quad (20)$$

where

$$A(m) = \sum_{k=0}^{N-1} |w_k|^2 e^{\frac{j2\pi km}{N}} \quad (21)$$

and

$$B(m) = \frac{2j\pi m}{N} \sum_{k=0}^{N-1} |w_k|^2 k e^{\frac{j2\pi km}{N}}. \quad (22)$$

The information about ε can be retrieved from Eq. (20,) but this requires the knowledge of σ_H^2. In order to overcome this problem we will propose two approaches; in the first one we consider the ratio $\frac{r_c(m+1)}{r_c(m)}$ and in the second one we consider the ratio $\frac{r_c(m)}{r_c(0)}$.

A. First Approach

Consider the following ratio

$$\frac{r_c(m+1)}{r_c(m)} \approx \frac{A(m+1) - \varepsilon B(m+1)}{A(m) - \varepsilon B(m)}. \quad (23)$$

This ratio contains all the necessary information about ε i.e.,

$$\varepsilon \approx \frac{r_c(m+1)A(m) - r_c(m)A(m+1)}{r_c(m+1)B(m) - r_c(m)B(m+1)}. \quad (24)$$

The final value of the sampling clock offset is obtained by averaging as follows

$$\hat{\varepsilon} = \frac{1}{N-2}\sum_{m=1}^{N-2}\left(\frac{\hat{r}_c(m+1)A(m) - \hat{r}_c(m)A(m+1)}{\hat{r}_c(m+1)B(m) - \hat{r}_c(m)B(m+1)}\right), \quad (25)$$

where the estimates of the autocorrelation sequence are given by

$$\hat{r}_c(m) = \frac{1}{N-m}\sum_{l=m}^{N-1} r(l)r^*(l-m). \quad (26)$$

Note that if no subcarrier weighting is used, i.e., $w_k = 1$, then we would have

$$A(m) = \sum_{k=0}^{N-1} e^{\frac{j2\pi km}{N}} = 0, \quad (27)$$

and therefore the previous estimator will be useless, since the ratio $\frac{r_c(m+1)}{r_c(m)}$ would have no information about ε.

B. Second Approach

We have that $r_c(0) = \frac{1}{N^2}\sigma_X^2\sigma_H^2\sum_{k=0}^{N-1}|w_k|^2 + \sigma_\mu^2$. Now consider this ratio

$$\frac{r_c(m)}{r_c(0)} = \frac{\frac{1}{N^2}\sigma_X^2\sigma_H^2(A(m)+\varepsilon B(m))}{\frac{1}{N^2}\sigma_X^2\sigma_H^2\sum_{k=0}^{N-1}|w_k|^2 + \sigma_\mu^2}. \quad (28)$$

For high SNR one can neglect σ_μ^2 and therefore we will have that

$$\frac{r_c(m)}{r_c(0)} \approx \frac{(A(m)+\varepsilon B(m))}{A(0)}. \quad (29)$$

The SCO will be given by

$$\varepsilon \approx \frac{r_c(m)A(0) - r_c(0)A(m)}{r_c(0)B(m)}. \quad (30)$$

As it has been done before we average over all the possible values of m as follows

$$\hat{\varepsilon} = \frac{1}{N-1}\sum_{m=1}^{N-1}\left(\frac{\hat{r}_c(m)A(0) - \hat{r}_c(0)A(m)}{\hat{r}_c(0)B(m)}\right). \quad (31)$$

Note that the use of weights is not mandatory for this algorithm, however we will see that the use of subcarrier weighting can enhance significantly the estimation accuracy. The next section will compare the performance of these two estimators. We will see that the approximation made in Eq.(29) does not hinder the performance of the second estimator at low SNR.

IV. Numerical examples

For the simulation we consider an OFDM system with 64 subcarriers fed by BPSK symbols. The SCO was fixed to 100 ppm (parts per million) and 5000 independent trials were performed to obtain the mean-square error estimates. Simulations were conducted in a multipath environment having four paths with relative path delays of 0, 4, 6 and 8 samples. The amplitude h_i of the ith path varies independently of the others according to an exponential power delay profile, i.e., $E[h_i^2] = exp(-\tau_i)$, where τ_i is the delay of the ith path.

In the first simulation, the window or subcarriers weights are selected according to a Bartlett window. Fig. 2 shows the results of the simulation. Note that even for the low SNR region the estimator based on the second approach (second estimator) behaves better than the estimator based on the first one (first estimator) and this trend is preserved at high SNR. This can be explained by the fact that the estimation of $r_c(0)$ is always more accurate than the estimation of $r_c(m)$ since more samples are involved in the estimation of the former.

This figure depicts also a comparison with the recent method in [14] which applies harmonic retrieval algorithms to estimate

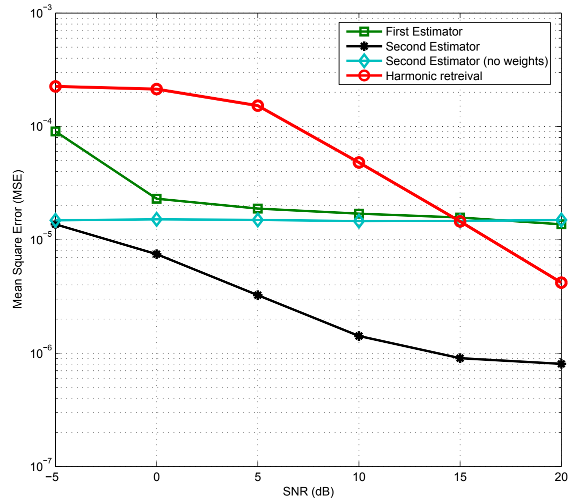

Fig. 2. Mean Square Error versus SNR over a multipath channel (Bartlett weighting window).

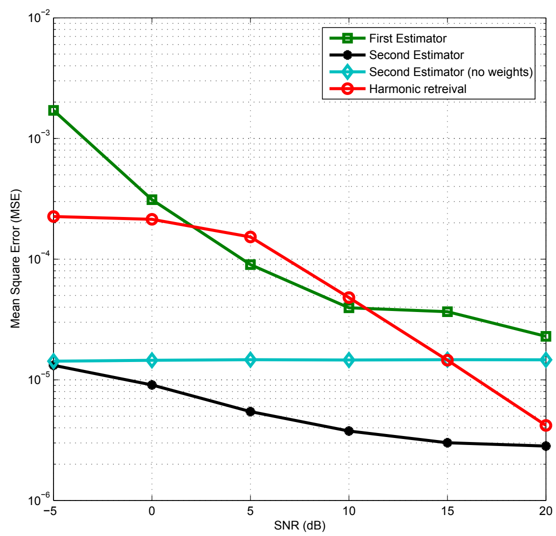

Fig. 3. Mean Square Error versus SNR over a multipath channel (Tukey weighting window).

the sampling clock offset (and the carrier frequency offset). This figure shows that our two estimators offer reliable and accurate performance for a wide range of SNR values, which makes it very attractive especially since the algorithm in [14] is more demanding from a complexity standpoint. Note also that the use of weights improves the estimation accuracy.
Fig. 3 shows the same estimators but for an other set of weights, this time chosen according to a Tukey window (with parameter $\alpha = 0.25$). Even though the general behavior is maintained, we can see that the performance achieved by the two estimators depends on the choice of the weighs. This hints to the eventuality that optimal weights (optimal in terms of estimation accuracy) may exist. In order to derive the optimal weights, a performance analysis of the estimators is necessary; specifically, we need to calculate the variance of the proposed estimators and find the weights that minimize this variance. Further investigation is needed to determine if and how it is possible to find optimal weights.

V. Conclusion

In this paper we have presented a new estimation scheme of the sampling clock offset in OFDM systems. The method is totally blind and does not require any training sequence. We have exploited the information provided by the second order statistics of the received OFDM samples. Simulations over frequency selective channels proved the robustness of the presented scheme over a wide range of SNR values. In a future work, we will extend the presented approach to cover the joint estimation of the sampling clock offset and the carrier frequency offset. We will determine as well the optimal subcarrier weights for the estimation.

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