A decision-directed SNR estimator for QAM signals over time-varying SIMO channels

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***Abstract*— The problem of signal-to-noise ratio (SNR) estimation over time-varying flat fading single-input multiple-output (SIMO) channels, for non constant envelope signals, is addressed in this article when the signal is corrupted by complex additive white Gaussian noise (AWGN). It is based on the use of periodically transmitted pilot symbols to facilitate the estimation process. It relies also on the detection of the remaining symbols, and is therefore a decision directed (DD) procedure. The normalized root mean square error (NRMSE) is used as a measure of performance by Monte Carlo simulations that validate our approach and its performance over a wide SNR range.**

I. Introduction

In recent years, there has been a lot of research interest in SNR estimation. In fact, the development of some new applications for modern communication systems has motivated work on SNR estimation techniques. The SNR knowledge is often a requirement when dealing with, for instance, transmit power control, adaptive modulation, handoff, dynamic allocation of resources or soft decoding procedures [1, 2]. A variety of methods have therefore been developed. Blind techniques, that do not use the a priori knowledge of the transmitted symbols, are called non data-aided (NDA). Those which base the estimation process on the knowledge of the transmitted signal are called data-aided (DA) and they have the drawback of limiting the system throughput due to the transmission of known data. There are other methods that rely on the detected transmitted data and they are qualified as decision-directed (DD).

There are many articles that deal with SNR estimation over flat fading [3, 4] and frequency-selective [5, 6] channels that can be assumed constant over the estimation interval. However, most of the derived methods are not applicable when the channel is time-varying. In fact, even small time variations over the estimation interval can dramatically degrade the performance of traditional, constant channel, SNR estimators. A recent technique for SNR estimation over flat fading time-varying channels was introduced by Wiesel and Messer-Yaron [7]. Nevertheless, this method is only applicable for constant modulus constellations, i.e. for phase shift keying (PSK) signals. In fact, to the best of our knowledge, the estimation/tracking of the instantaneous SNR over time-varying channels for non-constant envelope constellations has never been addressed before. Hence, we will present, in this paper, a new technique for SNR estimation over flat fading time-varying SIMO channels, with quadrature amplitude modulation (QAM) signals.

The remainder of this paper is organized as follows. We will begin by deriving the new SNR estimator. Then, via Monte Carlo simulations and using the NRMSE as a performance parameter, we will study the performance of our new DD method. We will see that the new technique can estimate accurately the SNR, and that, for a wide SNR range, it exhibits a performance similar to the one that could be achieved if all symbols were ideally known to the receiver.

II. System model

Consider a digital communication system with a single-input multiple-output configuration. We assume the channel to be time-varying and frequency-flat fading. We also assume that the transmitted data are corrupted by AWGN. All the noise components are supposed to be of equal average power σ^2 and mutually uncorrelated across the different antenna elements. Assuming an ideal receiver with perfect synchronization, and considering the antenna element i, the input-output baseband relationship can be written as

$$y_i(n) = a(n)h_i(n) + w_i(n), \quad n = 1, 2, \ldots, N, \tag{1}$$

where, at time index n and for each antenna element i, $a(n)$ is the n^{th} transmitted symbol and $y_i(n)$ is the corresponding received sample. $h_i(n)$ is the time-varying channel gain and $w_i(n)$ is a realization of a zero-mean AWGN of variance σ^2. These received samples can be conveniently written in the following $N \times 1$ vector form:

$$\boldsymbol{y}_i = \boldsymbol{A}\boldsymbol{h}_i + \boldsymbol{w}_i, \quad i = 1, 2 \cdots, N_a, \tag{2}$$

where

$$\begin{aligned}
\boldsymbol{y}_i &= [y_i(1), y_i(2), \ldots, y_i(N)]^T, && (3)\\
\boldsymbol{h}_i &= [h_i(1), h_i(2), \ldots, h_i(N)]^T, && (4)\\
\boldsymbol{w}_i &= [w_i(1), w_i(2), \ldots, w_i(N)]^T, && (5)\\
\boldsymbol{A} &= \text{diag}\{a(1), a(2), \ldots, a(N)\}. && (6)
\end{aligned}$$

The superscript T denotes the transpose operator and N stands for the total number of received symbols. The SNR estimation problem may be stated as follows. Given some known symbols (pilot symbols) and the received samples $\boldsymbol{y}_i$, estimate the SNR, over each antenna element i, which is expressed as

$$\rho_i = \frac{\sum_{n=1}^{N} |h_i(n)|^2 |a(n)|^2}{N\sigma^2}. \quad (7)$$

Most of the existing SNR estimation methods suppose that the channel is highly correlated and can be assumed constant during the observation interval. However, as shown in [7], even small variations from these assumptions can dramatically degrade their performance. A more appropriate model for the time-variations of the channel coefficients is polynomial in time [8, 9]. Accordingly, using Taylor's series expansion, the channel coefficients for a given antenna element i can be written as

$$h_i(n) = \sum_{m=0}^{L_c-1} c_{i;m} t_n^m + R_{i;L_c}(n), \quad (8)$$

where $c_{i;m}$ is the m^{th} polynomial coefficient for the channel at antenna branch i and t_n is the time index of the n^{th} sample (relative to the beginning of the estimation interval). The mean-squared value of the remainder, $R_{i;L_c}(n)$, approaches zero as $L_c \rightarrow +\infty$ or as $(\frac{f_d}{f_s})N \rightarrow 0$ [9], where f_d is the maximum Doppler frequency and f_s is the sampling rate. Therefore, in practice, for L_c sufficiently high or for $(\frac{f_d}{f_s})N \ll 1$, the channel can be reasonably approximated by a polynomial-in-time model, and one can write:

$$h_i(n) = \sum_{m=0}^{L_c-1} c_{i;m} t_n^m, \quad n = 1, 2, \ldots, N. \quad (9)$$

Using eq. (9) and considering the entire observation interval, the channel can be conveniently represented in the following $(N \times 1)$ column vector form:

$$\boldsymbol{h}_i = \boldsymbol{T}_{L_c} \boldsymbol{c}_i, \quad (10)$$

where

$$\boldsymbol{T}_{L_c} = \begin{pmatrix} 1 & t_1 & \cdots & t_1^{L_c-1} \\ 1 & t_2 & \cdots & t_2^{L_c-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & t_N & \cdots & t_N^{L_c-1} \end{pmatrix} \boldsymbol{c}_i = \begin{pmatrix} c_{i;0} \\ c_{i;1} \\ \vdots \\ c_{i;L_c-1} \end{pmatrix}. \quad (11)$$

Using eqs. (7) and (10), the SNR that we want to estimate can be written as

$$\rho_i = \frac{\boldsymbol{c}_i^H \boldsymbol{T}_{L_c}^T \boldsymbol{A}^H \boldsymbol{A} \boldsymbol{T}_{L_c} \boldsymbol{c}_i}{N\sigma^2}, \quad (12)$$

where the superscript H stands for the Hermitian operator. Moreover, from now on, for ease of notation, the subscript L_c will be omitted and we will simply refer to the matrix $\boldsymbol{T}_{L_c}$ by $\boldsymbol{T}$. We will also only consider cases for which the number of received symbols N is chosen such that $N > L_c$.

III. Formulation of the new SNR estimator

Our approach, in this paper, resembles the one presented in [7], but QAM signals are considered instead of only PSK, and the presence of pilot symbols is exploited to provide us with a non-iterative solution. The approach also exploits the presence of an array of antenna elements, at the reception, instead of one antenna element. The idea behind the use of a SIMO configuration is to exploit the intercorrelation between the received data over all the antenna elements in order to estimate the SNR on a given antenna branch. Indeed, using an array of antenna elements has the major advantage to provide us with a number of equations that can be sufficient to find all the desired unknowns in eq. (12).

In fact, on one hand, the unknowns of the problem are the N_a vectors $\{\boldsymbol{c}_i\}_{i=1,2,\ldots,N_a}$ and the N transmitted symbols $\{a(n)\}_{n=1,2,\ldots,N}$. Note that each vector $\boldsymbol{c}_i$ contains L_c unknowns $\{c_{i;m}\}_{m=1,2,\ldots,L_c}$, the coefficients of the channel corresponding to the antenna element i. The total number of unknowns is therefore $N_a L_c + N$. On the other hand, considering eq. (2) for $i = 1, 2, \ldots, N_a$, we see that we have NN_a independent equations. Hence, to be able to find all the unknowns, we need $NN_a \geq N_a L_c + N$, which means $N_a \geq \frac{N}{N-L_c} > 1$ since, in practice, N can be chosen sufficiently high. This is what justifies the effectiveness of the SIMO configuration choice in our procedure.

Using eqs. (2) and (10), the input-output baseband relationship can be extended, with the presence of N_a antenna branches, to the following $(N \times N_a)$ matrix form:

$$\boldsymbol{Y} = \boldsymbol{A}\boldsymbol{T}\boldsymbol{C} + \boldsymbol{W}, \quad (13)$$

where

$$\begin{aligned} \boldsymbol{Y} &= [\boldsymbol{y}_1, \cdots, \boldsymbol{y}_{N_a}], & (14) \\ \boldsymbol{C} &= [\boldsymbol{c}_1, \cdots, \boldsymbol{c}_{N_a}], & (15) \\ \boldsymbol{W} &= [\boldsymbol{w}_1, \cdots, \boldsymbol{w}_{N_a}]. & (16) \end{aligned}$$

From eq. (12), one can immediately see that the estimation of the SNR ρ_i, for each antenna element i, requires the estimation of the matrices $\boldsymbol{C}$ and $\boldsymbol{A}$. The matrix $\boldsymbol{T}$ is known to the receiver and does not need to be estimated. In practice, some data called pilot symbols, are often known to the receiver. Contrarily to the method introduced in [7], we will exploit the presence of $N_p > L_c$ such symbols in order to provide a non-iterative solution. Let $\boldsymbol{A}_p$ denote the diagonal matrix that contains such N_p symbols and let $\boldsymbol{T}_p$ be the corresponding time matrix which has the same form as matrix $\boldsymbol{T}$. By writing $\boldsymbol{\Phi}_p = \boldsymbol{A}_p \boldsymbol{T}_p$, eq. (13) reduces simply to

$$\begin{aligned} \boldsymbol{Y}_p &= \boldsymbol{A}_p \boldsymbol{T}_p \boldsymbol{C} + \boldsymbol{W}_p, & (17) \\ &= \boldsymbol{\Phi}_p \boldsymbol{C} + \boldsymbol{W}_p, & (18) \end{aligned}$$

where $\boldsymbol{Y}_p$ and $\boldsymbol{W}_p$ are, respectively, the received data and the noise components corresponding to the pilot symbols. In the least square (LS) sense, an estimate $\widehat{\boldsymbol{C}}_p$ of $\boldsymbol{C}$ is given by

$$\widehat{\boldsymbol{C}}_p = (\boldsymbol{\Phi}_p^H \boldsymbol{\Phi}_p)^{-1} \boldsymbol{\Phi}_p^H \boldsymbol{Y}_p. \quad (19)$$

Injecting this $\widehat{\boldsymbol{C}}_p$ in eq. (13), we can now estimate the matrix $\boldsymbol{A}$. In fact, by considering $\boldsymbol{\Phi}'_p = \boldsymbol{T}\widehat{\boldsymbol{C}}_p$, eq. (13) can be simply written as

$$\boldsymbol{Y} = \boldsymbol{A}\boldsymbol{\Phi}'_p + \bar{\boldsymbol{W}}. \quad (20)$$

One should notice that we no longer have the same noise components. In fact, $\bar{\boldsymbol{W}}$ contains the original noise components $\boldsymbol{W}$ and an additional noise which is due to the replacement of the matrix $\boldsymbol{C}$ by its estimate $\widehat{\boldsymbol{C}}_p$. In the LS sense, an estimate of $\boldsymbol{A}$ can be easily deduced from the transconjugate of eq. (20) which is given by

$$\boldsymbol{Y}^H = \boldsymbol{\Phi}_p'^H \boldsymbol{A}^H + \bar{\boldsymbol{W}}^H. \tag{21}$$

Straightforward development yields the following expression for the estimate $\widehat{\boldsymbol{A}}$ of $\boldsymbol{A}$

$$\widehat{\boldsymbol{A}} = \boldsymbol{Y}\widehat{\boldsymbol{C}}_p^H(\widehat{\boldsymbol{C}}_p\widehat{\boldsymbol{C}}_p^H)^{-1}(\boldsymbol{T}^T\boldsymbol{T})^{-1}\boldsymbol{T}^T, \tag{22}$$

which, by writing $\boldsymbol{\Phi}'' = \widehat{\boldsymbol{A}}\boldsymbol{T}$, allows the recomputation of a refined estimate $\widehat{\boldsymbol{C}}$ of $\boldsymbol{C}$:

$$\widehat{\boldsymbol{C}} = (\boldsymbol{\Phi}''^H\boldsymbol{\Phi}'')^{-1}\boldsymbol{\Phi}''^H\boldsymbol{Y}. \tag{23}$$

The SNR estimate $\widehat{\rho}_i$, on the i^{th} antenna, is therefore given by

$$\widehat{\rho}_i = \frac{\widehat{\boldsymbol{c}}_i^H\boldsymbol{T}^T\widehat{\boldsymbol{A}}^H\widehat{\boldsymbol{A}}\boldsymbol{T}\widehat{\boldsymbol{c}}_i}{N\widehat{\sigma}^2}, \tag{24}$$

which, using eqs (22) and (23), can be simply reduced to

$$\widehat{\rho}_i = \frac{\boldsymbol{y}_i^H\boldsymbol{P}\boldsymbol{y}_i}{N\widehat{\sigma}^2}, \tag{25}$$

where

$$\begin{aligned} \boldsymbol{P} &= \boldsymbol{\Phi}''(\boldsymbol{\Phi}''^H\boldsymbol{\Phi}'')^{-1}\boldsymbol{\Phi}''^H, && (26)\\ &= \widehat{\boldsymbol{A}}\boldsymbol{T}(\boldsymbol{T}^T\widehat{\boldsymbol{A}}^H\widehat{\boldsymbol{A}}\boldsymbol{T})^{-1}\boldsymbol{T}^T\widehat{\boldsymbol{A}}^H. && (27)\end{aligned}$$

Moreover, by writting

$$\bar{\boldsymbol{W}} = [\bar{\boldsymbol{w}}_1, \bar{\boldsymbol{w}}_2, \ldots, \bar{\boldsymbol{w}}_{N_a}], \tag{28}$$

then, from eq. (20) noise components are given by

$$\begin{aligned} \bar{\boldsymbol{w}}_i &= \boldsymbol{y}_i - \hat{\boldsymbol{A}}\boldsymbol{T}\hat{\boldsymbol{c}}_i, && (29)\\ &= \boldsymbol{y}_i - \boldsymbol{\Phi}''\hat{\mathbf{c}}_i. && (30)\end{aligned}$$

Therfore, the noise power estimate is given by

$$\begin{aligned} N\hat{\sigma}^2 &= (\boldsymbol{y}_i - \boldsymbol{\Phi}''\hat{\mathbf{c}}_i)^H(\boldsymbol{y}_i - \boldsymbol{\Phi}''\hat{\mathbf{c}}_i), && (31)\\ &= (\boldsymbol{y}_i - \boldsymbol{P}\boldsymbol{y}_i)^H(\boldsymbol{y}_i - \boldsymbol{P}\boldsymbol{y}_i), && (32)\\ &= \boldsymbol{y}_i^H(\boldsymbol{I}-\boldsymbol{P})^H(\boldsymbol{I}-\boldsymbol{P})\boldsymbol{y}_i, && (33)\\ &= \boldsymbol{y}_i^H(\boldsymbol{I}-\boldsymbol{P})\boldsymbol{y}_i, && (34)\\ &= \boldsymbol{y}_i^H\boldsymbol{P}^\perp\boldsymbol{y}_i. && (35)\end{aligned}$$

The SNR estimate $\widehat{\rho}_i$, on the i^{th} antenna element, reduces simply to

$$\widehat{\rho}_i = \frac{\boldsymbol{y}_i^H\boldsymbol{P}\boldsymbol{y}_i}{\boldsymbol{y}_i^H\boldsymbol{P}^\perp\boldsymbol{y}_i}. \tag{36}$$

It should be noted that, in a geometric representation, $\boldsymbol{P}$ and $\boldsymbol{P}^\perp = \boldsymbol{I} - \boldsymbol{P}$ are projection matrices onto the "signal-plus-noise" and "noise" subspaces, respectively. Note that, since, for each antenna branch, the noise components are zero-mean and are independent from the channel realization and the transmitted symbols, these two subspaces are orthogonal.

IV. Simulation Results

We will now assess the performance of our new estimator by Monte Carlo simulations over 1000 realizations. The NRMSE, given by eq. (37), will be used as performance measure.

$$\text{NRMSE}(\rho) = \frac{\sqrt{\text{E}\{(\rho-\hat{\rho})^2\}}}{\rho}. \tag{37}$$

The number of antenna branches will be set to $N_a = 8$. The DA method will refer to the case where all the transmitted symbols are supposed to be known to the receiver ($N_p = N$), while the DD method will refer to the case where only a subset of the N_p transmitted symbols is known to the receiver. Fig. 1 shows the empirical NRMSE as a function of the

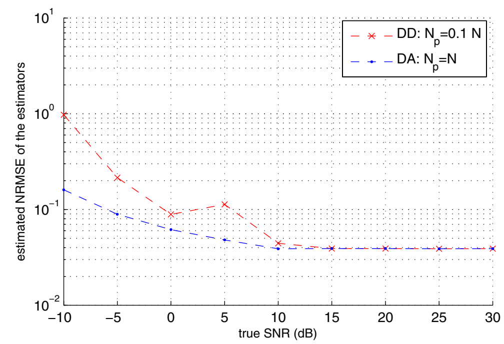

Fig. 1. SNR NRMSE on one of the 8 antenna elements, 16-QAM, $f_s = 243$ kHz, $f_d = 100$ Hz, $L_c = 5$, $N = 1000$.

true SNR, under a Rayleigh time-varying fading channel, for both the DA and DD scenarios. We notice that our LS-based method performs well over the entire SNR range, more so over the practical medium range of SNR values. We also notice that our DD method performs the same as the DA method when the SNR exceeds 10 dB.

Next, we will show that the estimation accuracy of our new LS-based method is primarily dependant on the size of the observation interval N.

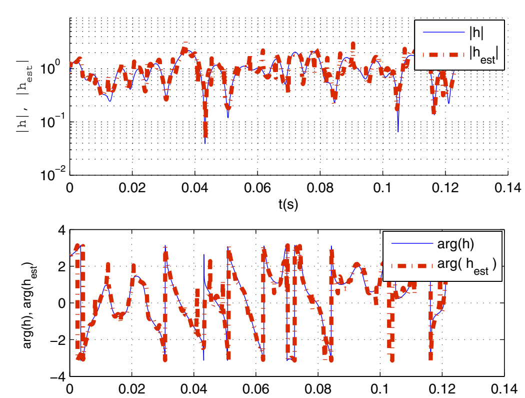

Fig. 2. Channel modulus and phase argument, and their estimates, on one of the 8 antenna elements, SNR = 10 dB, 16-QAM, $f_s = 243$ kHz, $f_d = 1000$ Hz, $L_c = 5$, $N = 100$, $N_p = 0.1N$.

Figs. 2 and 3 depict the estimation accuracy of the coefficients ($\boldsymbol{c}_i$) of a complex channel. The maximum Doppler

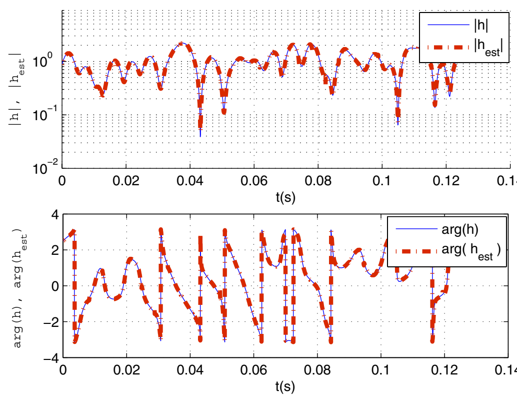

Fig. 3. Channel modulus and phase argument, and their estimates, on one of the 8 antenna elements, SNR $= 25$ dB, 16-QAM, $f_s = 243$ kHz, $f_d = 1000$ Hz, $L_c = 5$, $N = 100$, $N_p = 0.1N$.

frequency is set to $f_d = 1000$ Hz. Only 100 transmitted symbols are considered, which will be shown to be sufficient to estimate the signal power but not enough to precisely affaiblissement de parcours power σ^2.

In fact, from Figs 2 and 3, we see that the channel coefficients, for the considered the high SNR values (10 dB and 25 dB), are estimated with relatively high accuracy. This results in accurate estimates for the matrices $\boldsymbol{C}$ and $\boldsymbol{A}$. Consequently, the signal power $P_i = \widehat{\boldsymbol{c}}_i^H \boldsymbol{T}^T \widehat{\boldsymbol{A}}^H \widehat{\boldsymbol{A}} \boldsymbol{T} \widehat{\boldsymbol{c}}_i$ is estimated quite accurately and the main cause for performance degradation, in the SNR estimation process, stems from the inaccurate estimation of the noise power. A behavior that is better illustrated in Fig 4.

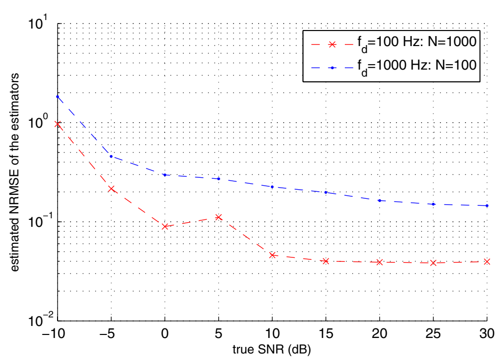

Fig. 4. SNR NRMSE on one of the 8 antenna elements for different Doppler frequencies, 16-QAM, $L_c = 5$, $f_s = 243$ kHz, $N_p = 0.1N$.

Indeed, this figure depicts the effect of varying the size of the observation interval N on the performance of our new estimator. It shows the NRMSE for the DD approach over a Rayleigh channel, when $f_d = 100$ Hz and $f_d = 1000$ Hz. In fact, the size of the estimation interval N can not be arbitrarily chosen. This is because we should always respect the condition $(\frac{f_d}{f_s})N \ll 1$ so that the channel can be accurately approximated by a polynomial in time. Indeed, for a constant sampling rate, the value of N depends on the value of the maximum Doppler frequency f_d.

From Fig. 4, a significant difference in performance is observed. In fact, when $f_d = 1000$ Hz, to respect the condition $(\frac{f_d}{f_s})N \ll 1$, we have selected $N = 100$ symbols. Consequently, with such a relatively low value of N, the noise power σ^2 can not be very accurately estimated. However, a lower Doppler frequency ($f_d = 100$ Hz) allows us to increase the number of received symbols used in the estimation process to $N = 1000$ symbols. Thus, we obtain a more accurate local estimate of the noise power and consequently a more accurate estimate of the SNR. In fact, the possibility of increasing the number of samples over the observation interval obviously increases the estimation accuracy over the entire SNR range. Finally, it should be noted that the optimal choice of N depends on the SNR value, as well as on the ratio $(\frac{f_d}{f_s})$.

V. Conclusion

In this paper, the estimation of the instantaneous SNR, under time-varying flat fading SIMO channels, for QAM signals was considered. We developed a novel LS-based SNR estimator. Polynomial fitting is used to locally approximate the channel. Our DD method bases the estimation process on the use of pilot symbols to provide a non-iterative solution. The new estimator was shown to have a satisfactory performance over the entire SNR range. Also, for a wide range of reasonably high SNR values, our DD method was shown to exhibit a performance similar to the one that could be achieved if all symbols were known to the receiver. The impact of changing the size of the observation interval on the performance of our new estimator was also studied. It was shown that the accuracy of the our new technique can be clearly improved by increasing the number of samples in the estimation interval, provided that the size of the observation interval still allows for a proper fit of the channel variations using our polynomial model.

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