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Asymptotic Analysis and Design of Multiuser Cooperative DS-CDMA Systems

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***Abstract*— The performance of a cooperative multiuser direct-sequence code-division multiple-access (DS-CDMA) system is analyzed in the asymptotic regime where both the spreading codes and the number of users grow large with the same ratio. A simple signal-to-interference-plus-noise ratio (SINR) expression is derived that is independent from the spreading codes and explicitly accounts for the effects of the multiple-access interference (MAI) and the relay noise. The so-obtained SINR expression is then computed based entirely on the available local information. The results obtained above are then used to optimally design the cooperative system. In particular, it is shown how the amount of cooperation between each collaborating pair can be adjusted to simultaneously achieve a pre-assigned target SINR for both users. Based on the local information, the globally optimal amount of the relay power is also obtained that maximizes the achieved SINR at the access point.**

I. Introduction

In scenarios where practical constraints such as size, cost, and complexity prohibit the use of multiple collocated antennas, cooperative schemes wherein single-antenna terminals relay other users' information are considered as reliable alternative means to provide spatial diversity [1]-[6].

We consider the cooperative diversity in multiuser direct-sequence code-division multiple-access (DS-CDMA) systems. A cooperative DS-CDMA scheme was introduced in [1] for a two-user system with orthogonal spreading codes. The design of this cooperative DS-CDMA system was then generalized to the multiuser scenario with nonorthogonal user spreading codes in, for instance, [2]-[4]. Other approaches to the design of cooperative DS-CDMA systems include the techniques proposed in [5] wherein idle terminals are recruited as relays and jointly used along with the active terminals to form a cooperative multiuser system and the scheme developed in [6] where the information from a single source is relayed using multiple DS-CDMA terminals.

Similar to [2] and [3], we consider here a cooperative multiuser DS-CDMA scheme wherein each user is paired with a unique terminal which, besides transmitting its own information, estimates and relays the data transmitted from its partner. However, unlike most of the previous contributions, we study the practical scenario in which either member of each pair is constrained by a limited power resource a part of which is used to transmit the user's own signal while the rest is spent to relay for its partner. A major issue that should be properly addressed in such a scenario is how the amount of cooperation between the pair can be adjusted to meet a particular design objective. For example, taking into account the multiple-access interference (MAI) from the other users and the relay noise, is it possible to simultaneously obtain a given target signal-to-interference-plus-noise ratio (SINR) pair at the access point just by adjusting the power quota at which the two partners transmit their own signals? If so, what is then the optimal power quota pair that results in the given target SINRs?

Finding a solution to a problem such as the one above mainly depends on the existence of a simple expression that clearly explains the impact of the parameters influencing the receiver SINR performance. However, even when a simple matched-filter (MF) is used at the access point, it can be shown that the SINR expression of the multiuser cooperative system is an extremely tedious function of all users' spreading codes. This, in turn, results in obscuring the effects of other parameters of interest on the receiver performance.

To get around this hurdle, we adopt the large system analysis approach [7], [8] that has proven its effectiveness in tackling quite similar problems in the conventional non-cooperative DS-CDMA systems. Under the common assumption that the spreading factor and the number of users grow large with the same rate, we obtain a simple SINR expression that is independent of the spreading codes while it explicitly explains the effects of the parameters such as MAI and the relay noise. Then, we show how to compute each user's SINR based entirely on the local information available to that user, its partner, and the access point without requiring any feedback from the interferences. Using the above results, we present a simple technique that solely relies on the available local information to determine the particular amount of cooperation between the partnering pair that simultaneously obtains the given target SINRs for the two partners. Based on the available local information, we also derive the globally optimal amount of cooperation that should be offered from each user's partner to maximize the user's SINR. It is shown that, due to the presence of MAI and noise, the user's SINR is not always an increasing function of the amount of cooperation offered by its partner, and, even in some scenarios, the partner may have a deteriorating effect on the performance at the access point. Sufficient conditions under which the cooperation from the partner decreases or increases the user's SINR are derived. Simulations are used to verify the accuracy of the results in practical systems with typical number of users and common values of spreading factor.

The paper is organized as follows. Section II presents the signal and the receiver models and describes the cooperative protocol. Section III includes the receiver performance analysis and Section IV shows how the so-obtained analytical results can be used to optimize the cooperative communication scheme. Computer simulations are demonstrated in Section V.

Conclusions are drawn in Section VI.

Notation: Uppercase and lowercase bold letters denote matrices and vectors, respectively. $[\cdot]_{\bullet l}$ and $[\cdot]_{il}$ are the lth column and the (i,l)th entry of a matrix, respectively. $\mathbf{A}_{\bar{k}}$ and $\mathbf{a}_{\bar{k}}$ are a matrix obtained by deleting the k-th column of $\mathbf{A}$ and a vector achieved by deleting the k-th entry of $\mathbf{a}$, respectively. $\mathbf{I}$ is the identity matrix while $\mathrm{diag}\{\cdot\}$ stands for a diagonal matrix. $(\cdot)^*$, $(\cdot)^T$, and $(\cdot)^H$ are the conjugate, the transpose, and the Hermitian transpose, respectively. $\|\cdot\|$ is the 2-norm of a vector or a matrix. $\mathrm{E}\{\cdot\}$ and $\mathrm{E}\{\cdot|\cdot\}$ denote the statistical expectation and the conditional statistical expectation, respectively. $|\cdot|$ denotes the absolute value while δ_{ij} is the Kronecker delta. $\xrightarrow{a.s.}$ denotes the almost sure convergence.

II. System Description

Consider a K-user synchronous[1] cooperative DS-CDMA system [1]-[3], [6] wherein each user $k \in \{1,\ldots,K\}$ is coupled with a partner $f(k) \in \{1,\ldots,k-1,k+1,\ldots,K\}$ which, in addition to transmitting its own data, exclusively relays for user k. Note that relaying is not necessarily bidirectional and $f(f(k))$ may be different from k [2]. We assume that the users operate in a full-duplex mode and perform perfect echo cancelation [1], [2], [4]. The following two-phase protocol for the cooperative signal transmission is used [1], [2]: During the odd transmission periods all users transmit their own data. In this phase, each user also uses an MF to estimate the symbol transmitted from its partner. During the even transmission periods, all users act as relays and transmit the normalized version of the symbol estimate of their partners.

Let us denote the access point by $k = 0$. The received signals during the odd transmission periods at the terminals $k = 0,\ldots,K$ and the even transmission periods at the access point are given by

$$\mathbf{r}_k(2i-1) = \sum_{p=1,p\neq k}^{K} \sqrt{\rho_p} h_{p,k}\mathbf{c}_p b_p(i) + \mathbf{v}_k(2i-1) \qquad (1)$$

$$\mathbf{r}_0(2i) = \sum_{p=1}^{K} \sqrt{1-\rho_{f(p)}} h_{f(p),0}\mathbf{c}_{f(p)}\hat{b}_p(i) + \mathbf{v}_0(2i) \qquad (2)$$

where $h_{p,k} = \sqrt{P_p} g_{p,k}$ is the channel link from terminal p to terminal k with $g_{p,k}$ and $P_p/2$ representing the fading coefficient between terminals p and k and the average power transmitted from the p-th user during two consecutive transmission periods, respectively. The weighting factor $\rho_p \in [0,1]$ is the ratio of the p-th user transmitted energy during the odd symbol periods to the total energy transmitted from this user in two consecutive transmission periods, $\mathbf{c}_p$ is the $N\times 1$ unit-energy spreading vector of user p, and $b_p(i)$ is the i-th zero-mean unit-variance i.i.d. symbol of the p-th user while $\hat{b}_p(i)$ is the normalized soft estimate of $b_p(i)$ obtained at $f(p)$. Note that, $\hat{b}_p(i)$ is normalized to avoid an over-quota power transmission in the relaying period. Finally, $\mathbf{v}_k(i)$ is the zero-mean noise at terminal k with $\mathrm{E}\{\mathbf{v}_k(i)\mathbf{v}_l(j)^H\} = \delta_{ij}\delta_{kl}\sigma_k^2\mathbf{I}$, that is, the noise is spatially and temporary white with possibly different powers at different terminals.

[1]The synchronous assumption is made mainly to avoid cumbersome notations and our results can be readily extended to the asynchronous scenario.

Note that the flat fading channel assumption is used to isolate the effect of the user cooperation on the receiver performance (see [1]- [4] for a similar treatment) and our analysis can be readily generalized to the frequency selective fading scenario. Denoting the length of each transmission period as T_s, it should also be noticed that the p-th user transmitted energy during the odd and even transmission periods are equal to $\rho_p P_p T_s$ and $(1-\rho_p)P_pT_s$, and, therefore, the total transmitted energy during two consecutive transmission periods is given by P_pT_s. In fact, increasing ρ_p in the interval $[0,1]$, the p-th user allocated power for the transmission of its own symbol increases at the cost of decreasing the power level at which this terminal relays for its partner.

Let us denote $\mathbf{C} \triangleq [\mathbf{c}_1\ \mathbf{c}_2\ \ldots\ \mathbf{c}_K]$, $\mathbf{\Theta} \triangleq \mathrm{diag}\{\rho_1,\ldots,\rho_K\}$, $\mathbf{b}(i) \triangleq [b_1(i)\ \ldots\ b_K(i)]^T$, and $\mathbf{H}^{(k)} \triangleq \mathrm{diag}\{h_{1,k},\ldots,h_{K,k}\}$, where $h_{k,k} = 0$. Equation (1) can be represented as

$$\mathbf{r}_k(2i-1) = \mathbf{C}\mathbf{H}^{(k)}\mathbf{\Theta}^{1/2}\mathbf{b}(i) + \mathbf{v}_k(2i-1). \qquad (3)$$

Assuming that $f(k)$ has the knowledge of ρ_k, $h_{k,f(k)}$, and $\mathbf{c}_k$, the filter matched at this terminal to the signal received from terminal k is equal to $\mathbf{f}_{k,f(k)} = \sqrt{\rho_k} h_{k,f(k)}\mathbf{c}_k$, and, therefore, the un-normalized soft estimate of $b_k(i)$ is given by $\check{b}_k(i) = \mathbf{f}_{k,f(k)}^H \mathbf{r}_{f(k)}(2i-1)$. Denoting $\xi_k \triangleq \left(\mathrm{E}\{|\check{b}_k(i)|^2\}\right)^{1/2}$, it is then straightforward to show that

$$\xi_k^2 = \rho_k |h_{k,f(k)}|^2 \mathbf{c}_k^H\left(\mathbf{C}\mathbf{H}^{(f(k))}\mathbf{\Theta}{\mathbf{H}^{(f(k))}}^H\mathbf{C}^H + \sigma_{f(k)}^2\mathbf{I}\right)\mathbf{c}_k. \qquad (4)$$

Hence, $\hat{b}_k(i) = \check{b}_k(i)/\xi_k = \beta_k b_k(i) + \epsilon_k(i)$ where $\beta_k \triangleq \rho_k|h_{k,f(k)}|^2\mathbf{c}_k^H\mathbf{c}_k/\xi_k$ and

$$\epsilon_k(i) \triangleq \frac{\sqrt{\rho_k}}{\xi_k} h_{k,f(k)}^* \mathbf{c}_k^H\left(\mathbf{C}_{\bar{k}}\mathbf{H}_{\bar{k}}^{(f(k))}\mathbf{\Theta}_{\bar{k}}^{1/2}\mathbf{b}_{\bar{k}}(i) + \mathbf{v}_{f(k)}(2i-1)\right). \qquad (5)$$

It is obvious that

$$\beta_k^2 + \mathrm{E}\{|\epsilon_k(i)|^2\} = \mathrm{E}\{|\hat{b}_k(i)|^2\} = 1. \qquad (6)$$

Note that $\hat{b}_k(i)$ is a scaled version of $b_k(i)$ by the fix factor of β_k plus a corrupting random term $\epsilon_k(i)$. The latter term is the normalized version of the estimation error at the relay $f(k)$ due to the noise and the residual MAI from the terminals other than $f(k)$ and k.

Let $\tilde{\mathbf{C}} \triangleq \left[\mathbf{c}_{f(1)}, \mathbf{c}_{f(2)}, \ldots, \mathbf{c}_{f(K)}\right]$, $\mathbf{\Phi} \triangleq \mathrm{diag}\{\beta_1 \ldots, \beta_K\}$, $\boldsymbol{\epsilon}(i) \triangleq [\epsilon_1(i), \ldots, \epsilon_K(i)]^T$, $\tilde{\mathbf{\Theta}} \triangleq \mathbf{I} - \mathrm{diag}\left\{\rho_{f(1)}, \ldots, \rho_{f(K)}\right\}$, and $\tilde{\mathbf{H}}^{(0)} \triangleq \mathrm{diag}\left\{h_{f(1),0}, \ldots, h_{f(K),0}\right\}$. From (2) we have

$$\mathbf{r}_0(2i) = \tilde{\mathbf{C}}\tilde{\mathbf{H}}^{(0)}\tilde{\mathbf{\Theta}}^{1/2}(\mathbf{\Phi}\mathbf{b}(i) + \boldsymbol{\epsilon}(i)) + \mathbf{v}_0(2i). \qquad (7)$$

Using (3) and (7), it follows that the received data vector at the access point corresponding to two consecutive transmission periods is equal to

$$\mathbf{r}(i) \triangleq \begin{bmatrix} \mathbf{r}_0(2i-1) \\ \mathbf{r}_0(2i) \end{bmatrix} = \mathbf{S}\mathbf{b}(i) + \mathbf{\Psi}\boldsymbol{\epsilon}(i) + \mathbf{v}(i), \qquad (8)$$

where $\mathbf{S} \triangleq \begin{bmatrix} \mathbf{C}\mathbf{H}^{(0)}\mathbf{\Theta}^{1/2} \\ \tilde{\mathbf{C}}\tilde{\mathbf{H}}^{(0)}\tilde{\mathbf{\Theta}}^{1/2}\mathbf{\Phi} \end{bmatrix}$, $\mathbf{\Psi} \triangleq \begin{bmatrix} \mathbf{0} \\ \tilde{\mathbf{C}}\tilde{\mathbf{H}}^{(0)}\tilde{\mathbf{\Theta}}^{1/2} \end{bmatrix}$, and $\mathbf{v}(i) \triangleq \begin{bmatrix} \mathbf{v}_0(2i-1) \\ \mathbf{v}_0(2i) \end{bmatrix}$.

Denoting $\mathbf{\Lambda}_{\boldsymbol{\epsilon}\mathbf{b}} \triangleq \mathrm{E}\{\boldsymbol{\epsilon}(i)\mathbf{b}(i)^H\}$, it can be shown from (5) that $[\mathbf{\Lambda}_{\boldsymbol{\epsilon}\mathbf{b}}]_{mn} = \sqrt{\rho_m\rho_n} h_{m,f(m)}^* h_{n,f(m)} \mathbf{c}_m^H\mathbf{c}_n\left(1-\delta_{mn}\right)/\xi_m$.

Note that $[\mathbf{\Lambda}_{\boldsymbol{\epsilon}\mathbf{b}}]_{mn}$ is, in fact, the residual MAI inflicted by user n at the MF output of the relay $f(m)$.

Due to the access point's limited processing power and the practical complications in computing the minimum mean-squared error multiuser receiver in a cooperative DS-CDMA system (see [2] for a more detailed discussion), similar to [1] and [5], we consider the case wherein the access point uses the suboptimal but more practical MF to detect $b_k(i)$ from $\mathbf{r}(i)$. Denoting $\boldsymbol{\lambda}_k \triangleq [\mathbf{\Lambda}_{\boldsymbol{\epsilon}\mathbf{b}}]_{\bullet k}$ and $\mathbf{s}_k \triangleq [\mathbf{S}]_{\bullet k}$, the MF receiver vector is given by

$$\mathbf{f}_k = \mathrm{E}\{\mathbf{r}b_k^*(i)\} = \mathbf{s}_k + \mathbf{\Psi}\boldsymbol{\lambda}_k, \tag{9}$$

where the first term at the right-hand side of (9) is the vector matched to the signal carrying the main part of the energy of $b_k(i)$ and transmitting from users k and $f(k)$ while the second term is due to the residual interference induced by the k-th user at the MF outputs of all terminals other than k and $f(k)$. It can be observed from (9) and the definition of $\boldsymbol{\lambda}_k$ that the access point requires the knowledge of all $\xi_m,\ h_{m,f(m)},\ h_{k,f(m)},\ m = 1, \ldots, K$ to capture the residual MAI energy of the k-th user. While the value of $h_{m,f(m)}$ may be fed back from $f(m)$ to the access point, none of the terminals is able to estimate the inter-terminal interfering channel links $h_{k,f(m)},\ m \neq k$. The above practical concern suggests to ignore the residual MAI energy of $b_k(i)$ contained in the signals sent from the users other than k and $f(k)$ and use

$$\mathbf{s}_k = \begin{bmatrix} \sqrt{\rho_k} h_{k,0}\mathbf{c}_k \\ \sqrt{1-\rho_{f(k)}} h_{f(k),0}\beta_k \mathbf{c}_{f(k)} \end{bmatrix} \tag{10}$$

as the approximated MF at the access point (see [2] for a similar treatment). Doing so, the energy of the k-th user at the access point output $\varrho \triangleq \mathbf{s}_k^H \mathbf{r}(i)$ is given by

$$|\mathrm{E}\{\varrho | b_k(i)\}|^2 = |\mathbf{s}_k^H \mathbf{f}_k|^2 = |\mathbf{s}_k^H \mathbf{s}_k + \mathbf{s}_k^H \mathbf{\Psi}\boldsymbol{\lambda}_k|^2. \tag{11}$$

Denoting the k-th user's SINR as η_k, from (8) and (11) we have that

$$\eta_k = \frac{|\mathbf{s}_k^H \mathbf{s}_k + \mathbf{s}_k^H \mathbf{\Psi}\boldsymbol{\lambda}_k|^2}{\mathbf{s}_k^H \mathbf{R}\mathbf{s}_k - |\mathbf{s}_k^H \mathbf{s}_k + \mathbf{s}_k^H \mathbf{\Psi}\boldsymbol{\lambda}_k|^2} \tag{12}$$

where $\mathbf{R} \triangleq \mathrm{E}\{\mathbf{r}(i)\mathbf{r}(i)^H\}$, which, following (8), is given by

$$\mathbf{R} = \mathbf{S}\mathbf{S}^H + \sigma_0^2\mathbf{I} + \underbrace{\mathbf{\Psi}\mathbf{\Lambda}_{\boldsymbol{\epsilon}\boldsymbol{\epsilon}}\mathbf{\Psi}^H + \mathbf{S}\mathbf{\Lambda}_{\boldsymbol{\epsilon}\mathbf{b}}^H\mathbf{\Psi}^H + \mathbf{\Psi}\mathbf{\Lambda}_{\boldsymbol{\epsilon}\mathbf{b}}\mathbf{S}^H}_{\mathbf{\Xi}_{\boldsymbol{\epsilon}}} \tag{13}$$

with $\mathbf{\Lambda}_{\boldsymbol{\epsilon}\boldsymbol{\epsilon}} \triangleq \mathrm{E}\{\boldsymbol{\epsilon}(i)\boldsymbol{\epsilon}(i)^H\}$. Note that $\mathbf{\Xi}_{\boldsymbol{\epsilon}}$ is due to the effect of the residual MAI after despreading at the output of the users $1, \ldots, K$. From (5), it can be readily proven for $n \neq m$ that $[\mathbf{\Lambda}_{\boldsymbol{\epsilon}\boldsymbol{\epsilon}}]_{mn} = \mu_{mn}\mathbf{c}_m^H \mathbf{C}_{\bar{m},\bar{n}} \mathbf{H}_{\bar{m},\bar{n}}^{(f(m))} \mathbf{\Theta}_{\bar{m},\bar{n}} \mathbf{H}_{\bar{m},\bar{n}}^{(f(n))^H} \mathbf{C}_{\bar{m},\bar{n}}^H \mathbf{c}_n$ where $\mu_{mn} = \sqrt{\rho_m\rho_n} h_{m,f(m)}^* h_{n,f(n)}/\xi_m\xi_n$ while, according to (6), $[\mathbf{\Lambda}_{\boldsymbol{\epsilon}\boldsymbol{\epsilon}}]_{mm} = \mathrm{E}\left\{|\epsilon_m(i)|^2\right\} = 1 - \beta_m^2$. As it can be observed from (12) and (13) and the so-obtained expressions of $\mathbf{\Lambda}_{\boldsymbol{\epsilon}\mathbf{b}}$ and $\mathbf{\Lambda}_{\boldsymbol{\epsilon}\boldsymbol{\epsilon}}$, η_k depends on all the system parameters, that is, all $\mathbf{c}_p$, ρ_p, $h_{p,0}$, $p = 1, \ldots, K$, and all σ_p^2, $p = 0, \ldots, K$, as well as all $h_{p,k}$, $p, k = 1, \ldots, K, k \neq p$ which are unknown unless p and k are partnering users. This very complicated expression of η_k hampers acquiring any clear insight into the performance of the cooperative system and prohibits analyzing the isolated effect of individual parameters on the users' SINR. This fact also poses a major challenge when one intends to adjust ρ_k and $\rho_{f(k)}$ to achieve pre-assigned SINR objectives for terminals k and $f(k)$.

In the next section, the approach to analyze large DS-CDMA systems with random spreading codes [7], [8] will be adopted to derive a simple and reliable approximate SINR expression that depends only on a few parameters. It will be shown that the number of these parameters does not grow with the number of terminals, and, moreover, all of the parameters are known or can be locally estimated.

III. Receiver Performance Analysis

In this section, we first analyze the SINR performance of the cooperative DS-CDMA system when N and K grow to infinity with the same rate. Then, we propose a technique to compute the so-obtained SINR expression based only on the local information distributed among the access point, the user of interest, and its partner. Two common assumptions [7], [8] will be used throughout the rest of this paper: **A1)** $\mathbf{c}_k = \frac{1}{\sqrt{N}}\check{\mathbf{c}}_k$ for $k = 1, \ldots, K$ where the entries of $\check{\mathbf{c}}_k$ are zero-mean unit-variance i.i.d. random variables with a finite eighth-order moment. **A2)** There is a constant B such that $|h_{p,k}| \leq B,\ p, k = 0, \ldots, K$.

The following lemma is of critical importance in our later developments. For the proof of this lemma and all the following theorems refer to [9].

Lemma 1: As N goes to infinity with $\frac{K}{N} \to \alpha$,

$$\beta_k^2 \xrightarrow{a.s.} \bar{\beta}_k^2 \triangleq \frac{\rho_k \left|h_{k,f(k)}\right|^2}{\rho_k \left|h_{k,f(k)}\right|^2 + \gamma_{f(k)}} \tag{14}$$

where $\gamma_{f(k)} \triangleq \sigma_{f(k)}^2 + \frac{1}{N}\sum_{p=1}^K \rho_p \left|h_{p,f(k)}\right|^2$. Moreover, we have that $[\mathbf{\Lambda}_{\boldsymbol{\epsilon}\boldsymbol{\epsilon}}]_{mm} \xrightarrow{a.s.} 1 - \bar{\beta}_m^2$, while for $m \neq n$, $[\mathbf{\Lambda}_{\boldsymbol{\epsilon}\boldsymbol{\epsilon}}]_{mn} \xrightarrow{a.s.} 0$ and $[\mathbf{\Lambda}_{\boldsymbol{\epsilon}\mathbf{b}}]_{mn} \xrightarrow{a.s.} 0$.

Let us now turn our attention to obtaining η_k as N approaches infinity while $\frac{K}{N} \to \alpha$. As it follows from (12) and (13), obtaining the limiting value of η_k requires computing the asymptotic value of $\mathbf{s}_k^H \mathbf{\Xi}_{\boldsymbol{\epsilon}} \mathbf{s}_k$. It should be noticed that, due to the dependency of $\mathbf{\Xi}_{\boldsymbol{\epsilon}}$ on $\mathbf{\Lambda}_{\boldsymbol{\epsilon}\boldsymbol{\epsilon}}$ and $\mathbf{\Lambda}_{\boldsymbol{\epsilon}\mathbf{b}}$, $\mathbf{s}_k^H \mathbf{\Xi}_{\boldsymbol{\epsilon}} \mathbf{s}_k$ comprises of up to the eighth-order terms of the entries of the spreading codes. Note that using classical techniques such as the Borel-Cantelli lemma [7] to establish the almost-sure convergence of $\mathbf{s}_k^H \mathbf{\Xi}_{\boldsymbol{\epsilon}} \mathbf{s}_k$ necessitates computing the fourth-order moment of this quantity, which, in turn, requires obtaining up to the 32nd-order cross-moments of the entries of the spreading codes. To avoid such an extremely complicated computation, we use Lemma 1 along with the fact that $[\mathbf{\Lambda}_{\boldsymbol{\epsilon}\mathbf{b}}]_{mm} = 0$ to approximate $\mathbf{\Lambda}_{\boldsymbol{\epsilon}\mathbf{b}} \approx \mathbf{0}$ and $\mathbf{\Lambda}_{\boldsymbol{\epsilon}\boldsymbol{\epsilon}} \approx \hat{\mathbf{\Lambda}}_{\boldsymbol{\epsilon}\boldsymbol{\epsilon}} \triangleq \mathrm{diag}\left\{1 - \bar{\beta}_1^2, \ldots, 1 - \bar{\beta}_K^2\right\}$. Using the latter approximations in (13), it directly follows from (12) that η_k can be well-approximated by

$$\hat{\eta}_k = \frac{|\mathbf{s}_k^H \mathbf{s}_k + \mathbf{s}_k^H \mathbf{\Psi}\boldsymbol{\lambda}_k|^2}{\mathbf{s}_k^H \left(\mathbf{S}\mathbf{S}^H + \sigma_0^2\mathbf{I} + \mathbf{\Psi}\hat{\mathbf{\Lambda}}_{\boldsymbol{\epsilon}\boldsymbol{\epsilon}}\mathbf{\Psi}^H\right)\mathbf{s}_k - |\mathbf{s}_k^H \mathbf{s}_k + \mathbf{s}_k^H \mathbf{\Psi}\boldsymbol{\lambda}_k|^2}. \tag{15}$$

Note that $\eta_k - \hat{\eta}_k$ does not necessarily converge to zero in the exact mathematical sense. However, simulation results in

Section V demonstrate that, even for moderate values of N and K, the difference between η_k and $\hat{\eta}_k$ is in the range of a small fraction of dB.

The following theorem derives the limiting value of $\hat{\eta}_k$ in the asymptotic regime of our concern.

Theorem 1: As N goes to infinity with $K/N \to \alpha$, we have $\hat{\eta}_k \xrightarrow{a.s.} \bar{\eta}_k$ where $\bar{\eta}_k$ is given in (16) at the top of the next page. In (16), $\tilde{\gamma}_0 \triangleq \sigma_0^2 + \frac{1}{N}\sum_{p=1}^{K}(1-\rho_p)\left|h_{p,0}\right|^2$ and $\gamma_0 \triangleq \sigma_0^2 + \frac{1}{N}\sum_{p=1}^{K}\rho_p\left|h_{p,0}\right|^2$.

As it can be observed from (16) and the definitions of γ_0, $\tilde{\gamma}_0$, and $\bar{\beta}_k$, the limiting value of $\hat{\eta}_k$ is independent from all spreading codes and all σ_p, $p \neq 0, f(k)$. More importantly, $\bar{\eta}_k$ depends only on $h_{k,0}$, $h_{f(k),0}$, $h_{k,f(k)}$ and some weighted averages of $\left|h_{p,0}\right|^2$ and $\left|h_{p,f(k)}\right|^2$, $p = 1,\ldots,K$, while it is completely independent of all other channel links. Note that as ρ_p and $\left|h_{p,0}\right|^2$, $p = 1,\ldots,K$ are known at the access point, γ_0 and $\tilde{\gamma}_0$ may be directly computed at this terminal. However, due to practical reasons, it is more appealing if one does not need to rely on the knowledge of all $h_{p,0}$, $p = 1,\ldots,K$ to obtain γ_0 and $\tilde{\gamma}_0$. In addition to the above fact, a major practical hurdle in computing $\bar{\eta}_k$ is due to the need for determining $\bar{\beta}_k$. According to (14), the latter parameter depends on $\gamma_{f(k)}$, which, itself is dependent on a weighted average of unknown $\left|h_{p,f(k)}\right|^2$, $p = 1,\ldots,K$. The following theorem addresses the above concerns.

Theorem 2: As N goes to infinity with $\frac{K}{N} \to \alpha$, we have $1/N\left\|\mathbf{r}_{f(k)}(2i-1)\right\|^2 \xrightarrow{a.s.} \gamma_{f(k)}$, $1/N\left\|\mathbf{r}_0(2i-1)\right\|^2 \xrightarrow{a.s.} \gamma_0$, and $1/N\left\|\mathbf{r}_{0,\mathrm{nc}}(2i-1)\right\|^2 \xrightarrow{a.s.} \tilde{\gamma}_0 + \gamma_0 - \sigma_0^2$ where $\mathbf{r}_{0,\mathrm{nc}}$ is the received signal at the access point in the non-cooperative mode, that is, when $\rho_p = 1$ for $p = 1,\ldots,K$.

Theorem 2 shows that $\gamma_{f(k)}$ and γ_0 can be computed, respectively, from the signal received at the terminal $f(k)$ and the access point during only one odd transmission period. Similarly, it follows from Theorem 2 that, when γ_0 becomes known at the access point, only one non-cooperative symbol transmission is sufficient to compute $\tilde{\gamma}_0$ at this terminal.

From Theorem 2, (14), and (16), it follows that if only $\mathcal{B}_k = \left\{\rho_k, \rho_{f(k)}, h_{k,0}, h_{f(k),0}, h_{k,f(k)}, \sigma_0, \mathbf{r}_0, \mathbf{r}_{f(k)}, \mathbf{r}_{0,\mathrm{nc}}\right\}$ (for the sake of simplicity, the time index $2i-1$ is dropped from the receiver vectors) is known, then $\bar{\eta}_k$ can be computed from (16). This is in contrast to the non-asymptotic case in which η_k is a function of all parameters in the system. To elaborate on this property, let us introduce $\mathcal{G}_{k,f(k)}$ as the subnetwork comprising the access point and the terminals k, $f(k)$, and $\mathcal{G}_{k,f(k)}^c$ as its complement comprising all other terminals $p = 1,\ldots,K$, $p \neq k, f(k)$. It can be easily verified that all the elements of $\mathcal{B}_k$, and, therefore, $\bar{\eta}_k$, can be autonomously obtained by $\mathcal{G}_{k,f(k)}$ using a very limited information exchange among its three members and without requiring any feedback from the members of $\mathcal{G}_{k,f(k)}^c$.

Note also that if the terminal k relays for $f(k)$, that is, $f(f(k)) = k$, then, the expression for $\bar{\eta}_{f(k)}$ is obtained simply by permuting k and $f(k)$ in (16). In such a case, $\bar{\eta}_{f(k)}$ depends only on the elements of $\mathcal{B}_{f(k)}$, the set obtained by substituting $h_{f(k),k}$ and $\mathbf{r}_k$ in $\mathcal{B}_k$ in lieu of $h_{k,f(k)}$ and $\mathbf{r}_{f(k)}$. Therefore, $\bar{\eta}_{f(k)}$ can also be determined solely using the information distributed among the members of $\mathcal{G}_{k,f(k)}$. As will be discussed in Section IV, the fact that $\mathcal{G}_{k,f(k)}$ can autonomously compute $\bar{\eta}_k$ and $\bar{\eta}_{f(k)}$ is very useful specially when one intends to optimize ρ_k and $\rho_{f(k)}$.

IV. Design of the Weighting Factors

Assume that $\mathcal{G}_{k,f(k)}$ aims to solve the following problem:

P1: Is it possible to obtain a target SINR pair $(\eta_k^{\dagger}, \eta_{f(k)}^{\dagger})$ just by adjusting ρ_k and $\rho_{f(k)}$? If so, what is the weighting pair $(\rho_k^{\dagger}, \rho_{f(k)}^{\dagger})$ that achieves $(\eta_k^{\dagger}, \eta_{f(k)}^{\dagger})$?

As $\eta_k^{\dagger}$ and $\eta_{f(k)}^{\dagger}$ depend on all system parameters, to solve **P1**, $\mathcal{G}_{k,f(k)}$ needs to acquire the knowledge of all of the parameters in the system many of which cannot be estimated in practice. To avoid this hurdle, $\mathcal{G}_{k,f(k)}$ can set $(\bar{\eta}_k^{\ddagger}, \bar{\eta}_{f(k)}^{\ddagger}) = (\eta_k^{\dagger}, \eta_{f(k)}^{\dagger})$ and, instead, solve the following problem:

P2: Is it possible to obtain a target asymptotic SINR pair $(\bar{\eta}_k^{\ddagger}, \bar{\eta}_{f(k)}^{\ddagger})$ just by adjusting ρ_k and $\rho_{f(k)}$? If so, what is the weighting pair $(\rho_k^{\ddagger}, \rho_{f(k)}^{\ddagger})$ that achieves $(\bar{\eta}_k^{\ddagger}, \bar{\eta}_{f(k)}^{\ddagger})$?

The approach to solve **P2** is straightforward: Consider ρ_k and $\rho_{f(k)}$ as the unknown variables of interest. Having the knowledge of all other members of $\mathcal{B}_k$, the right-hand side of (16) can be considered as a rational function of ρ_k and $\rho_{f(k)}$. Denote it by $h_1\left(\rho_k, \rho_{f(k)}\right)$. Similarly, knowing all the members of $\mathcal{B}_{f(k)}$ but ρ_k and $\rho_{f(k)}$, the so-obtained expression of $\bar{\eta}_{f(k)}$ can be viewed as another rational function of the two unknown variables of ρ_k and $\rho_{f(k)}$. Denote the latter expression as $h_2\left(\rho_k, \rho_{f(k)}\right)$. Then, $\mathcal{G}_{k,f(k)}$ needs to solve the following set of equations

$$\bar{\eta}_k^{\ddagger} = h_1\left(\rho_k, \rho_{f(k)}\right), \qquad \bar{\eta}_{f(k)}^{\ddagger} = h_2\left(\rho_k, \rho_{f(k)}\right) \tag{17}$$

for ρ_k and $\rho_{f(k)}$. Since $h_1\left(\rho_k, \rho_{f(k)}\right)$ and $h_2\left(\rho_k, \rho_{f(k)}\right)$ are rational functions, (17) can be transformed into a pair of polynomial equations and then solved using standard numerical techniques. If among the pairs of solutions to (17) there is no pair both of whose entries are real and in the interval $[0,1]$, then $(\bar{\eta}_k^{\ddagger}, \bar{\eta}_{f(k)}^{\ddagger})$ cannot be achieved. In turn, if there is such a solution, then, it is $(\rho_k^{\ddagger}, \rho_{f(k)}^{\ddagger})$.

Note that **P1** and **P2** tend to become equivalent problems when N and K are large enough. Hence, if $(\bar{\eta}_k^{\ddagger}, \bar{\eta}_{f(k)}^{\ddagger})$ is not possible to achieve, so is not $(\eta_k^{\dagger}, \eta_{f(k)}^{\dagger})$. On the other hand, when achieving $(\bar{\eta}_k^{\ddagger}, \bar{\eta}_{f(k)}^{\ddagger})$ is feasible, $(\eta_k^{\dagger}, \eta_{f(k)}^{\dagger})$ should also be achievable and, moreover, $(\rho_k^{\dagger}, \rho_{f(k)}^{\dagger}) \approx (\rho_k^{\ddagger}, \rho_{f(k)}^{\ddagger})$.

Using the analysis in Section III, we can also study of the effect of $\rho_{f(k)}$ on the SINR performance of user k. In particular, it is shown below that the SINR performance of terminal k does not necessarily improve if terminal $f(k)$ reduces $\rho_{f(k)}$ to increase the relaying power for its partner at the cost of decreasing the transmission power of its own symbols. To investigate this issue in more details, we take the derivative of $\bar{\eta}_k$ with respect to $\rho_{f(k)}$ to obtain $\partial\bar{\eta}_k/\partial\rho_{f(k)} = g(\rho_{f(k)})(\rho_{f(k)} - a_1)(a_2\rho_{f(k)} - (a_2 - a_3))$ where $g(\rho_{f(k)}) < 0$ is a function of $\rho_{f(k)}$, $a_1 = 1 + (\rho_k|h_{k,0}|^2/|h_{f(k),0}|^2\bar{\beta}_k^2)$, $a_2 = |h_{f(k),0}|^2\bar{\beta}_k^2\tilde{\gamma}_0 - 2\rho_k|h_{f(k),0}|^2|h_{k,0}|^2(1-\bar{\beta}_k^2)$, while $a_3 = \rho_k|h_{k,0}|^2\tilde{\gamma}_0 - 2\rho_k|h_{k,0}|^2\gamma_0$.

$$\bar{\eta}_k = \frac{\left(\rho_k \left|h_{k,0}\right|^2 + \left(1-\rho_{f(k)}\right)\left|h_{f(k),0}\right|^2 \bar{\beta}_k^2\right)^2}{\rho_k \left|h_{k,0}\right|^2 \gamma_0 + \left(1-\rho_{f(k)}\right)\left|h_{f(k),0}\right|^2 \bar{\beta}_k^2 \tilde{\gamma}_0 + \left(1-\rho_{f(k)}\right)^2 \left|h_{f(k),0}\right|^4 \bar{\beta}_k^2 \left(1-\bar{\beta}_k^2\right)} \tag{16}$$

	$a_2 < 0$	$a_2 > 0$
$\kappa_2 < 0$	$\partial\bar{\eta}_k/\partial\rho_{f(k)} > 0 \rightarrow \rho^\star_{f(k)} = 0$	$\partial\bar{\eta}_k/\partial\rho_{f(k)} < 0 \rightarrow \rho^\star_{f(k)} = 1$
$0 \le \kappa_2 \le 1$	$\begin{matrix}\partial\bar{\eta}_k/\partial\rho_{f(k)} > 0 \text{ if } \rho_{f(k)} < \kappa_2 \\ \partial\bar{\eta}_k/\partial\rho_{f(k)} < 0 \text{ if } \rho_{f(k)} > \kappa_2\end{matrix} \rightarrow \rho^\star_{f(k)} = \kappa_2$	$\begin{matrix}\partial\bar{\eta}_k/\partial\rho_{f(k)} < 0 \text{ if } \rho_{f(k)} < \kappa_2 \\ \partial\bar{\eta}_k/\partial\rho_{f(k)} > 0 \text{ if } \rho_{f(k)} > \kappa_2\end{matrix} \rightarrow \rho^\star_{f(k)} = \begin{cases} 0 \text{ if } \bar{\eta}_k(0) \ge \bar{\eta}_k(1) \\ 1 \text{ else}\end{cases}$
$\kappa_2 > 1$	$\partial\bar{\eta}_k/\partial\rho_{f(k)} > 0 \rightarrow \rho^\star_{f(k)} = 1$	$\partial\bar{\eta}_k/\partial\rho_{f(k)} < 0 \rightarrow \rho^\star_{f(k)} = 0$

TABLE I
THE BEHAVIOR OF $\bar{\eta}_k$ AS A FUNCTION OF $\rho_{f(k)} \in [0, 1]$.

We are interested to investigate the behavior of $\bar{\eta}_k$ as a function of $\rho_{f(k)} \in [0, 1]$ and obtain $\rho^\star_{f(k)}$, the value of $\rho_{f(k)}$ that maximizes $\bar{\eta}_k$. It should be noticed that $\partial\bar{\eta}_k/\partial\rho_{f(k)}$ has two roots at $\kappa_1 = a_1 > 1$ and $\kappa_2 = 1 - (a_3/a_2)$ and the behavior of $\bar{\eta}_k$ when $\rho_{f(k)}$ moves from 0 to 1 depends only on the value of κ_2 and the sign of a_2. For example, when $a_2 < 0$ and $\kappa_2 \in [0, 1]$, $\partial\bar{\eta}_k/\partial\rho_{f(k)} > 0$ for $\rho_{f(k)} \in [0, \kappa_2)$ and $\partial\bar{\eta}_k/\partial\rho_{f(k)} < 0$ for $\rho_{f(k)} \in (\kappa_2, 1]$, and, consequently, $\rho^\star_{f(k)} = \kappa_2$. Depending on the sign of a_2 and the value of κ_2, the behavior of $\bar{\eta}_k$ can be classified into six different categories summarized in Table 1. It should be stressed that $\mathcal{G}_{k,f(k)}$ can use its available local information to compute κ_2 and a_2, and, then, obtain $\rho^\star_{f(k)}$ from Table I.

In the absence of MAI and relay noise, it can be readily shown that $a_2 > 0$ and $\kappa_2 > 1$, and, therefore, according to Table I, $\rho^\star_{f(k)} = 0$. In practical scenarios where MAI and noise are present, it is quite difficult in general to find straightforward physical interpretations for all of the six cases shown in Table I. However, Table I can still be used to gain significant insight into the impact of physical parameters on $\rho^\star_{f(k)}$. The following theorem is obtained based on the results given in Table I.

Theorem 3: Let $\bar{\rho} \triangleq \sum_{p=1}^{K} \rho_p P_p / \sum_{p=1}^{K} P_p$ and denote $\varphi_{k,f(k)} \triangleq |g_{k,f(k)}|^2/|g_{k,0}|^2$. Assume that $\mathrm{E}\{|g_{p,0}|^2\} = \theta_0$ while $\mathrm{E}\{|g_{p,f(k)}|^2\} = \theta_{f(k)}$ for $p = 1, \ldots, K$ and let $\bar{\varphi}_{f(k)} \triangleq \theta_{f(k)}/\theta_0$. Then, in the high SNR regime as σ_0 and $\sigma_{f(k)}$ converge to zero, if

$$\varphi_{k,f(k)} < \bar{\varphi}_{f(k)} \cdot (2\bar{\rho}/1 - \bar{\rho}) \qquad \text{and} \qquad \bar{\rho} < 1/3 \tag{18}$$

we have that $\rho^\star_{f(k)} = 1$. In turn, if

$$\varphi_{k,f(k)} > \bar{\varphi}_{f(k)} \cdot (2\bar{\rho}/1 - \bar{\rho}) \qquad \text{and} \qquad \bar{\rho} > 1/3 \tag{19}$$

then $\rho^\star_{f(k)} = 0$.

Note that $\bar{\rho} \in [0, 1]$ measures the amount of cooperation among the terminal pairs. In average, the smaller the $\bar{\rho}$, the higher the amount of the relaying power that terminals offer to their partners. $\varphi_{k,f(k)}$ measures the quality of the channel between terminal k and its partner relative to the quality of the channel between terminal k and the access point. Finally, $\bar{\varphi}_{f(k)}$ measures the average quality of the channel between an arbitrary user and terminal $f(k)$ relative to the average quality of the channel between that user and the access point. Note also that, if $g_{k,0}$ and $g_{k,f(k)}$ are independent random variables, then $\mathrm{E}\{\varphi_{k,f(k)}\} = \bar{\varphi}_{f(k)}$. Taking into account the above interpretations of $\bar{\rho}$, $\varphi_{k,f(k)}$, and $\bar{\varphi}_{f(k)}$, it follows from (18) that if the channel quality between the k-th user and its relay is so low that $\varphi_{k,f(k)}$ is less than $2\bar{\rho}/1-\bar{\rho}$ times $\mathrm{E}\{\varphi_{k,f(k)}\}$ and, moreover, the average amount of the collaboration between the pairs is so high that the relayed signal from $f(k)$ in the even transmission periods is subject to a considerable amount of MAI, then, $\rho^\star_{f(k)} = 1$. In such a case, it is better that $f(k)$ does not relay for user k at all. On the contrary, if the average cooperation among the pairs is not excessively high and, in addition, the channel quality between the k-th terminal and its relay is good enough such that both inequalities in (19) hold, then the SINR of the k-th user becomes maximal if $f(k)$ spends all of its power to transmit the estimate of $b_k(i)$.

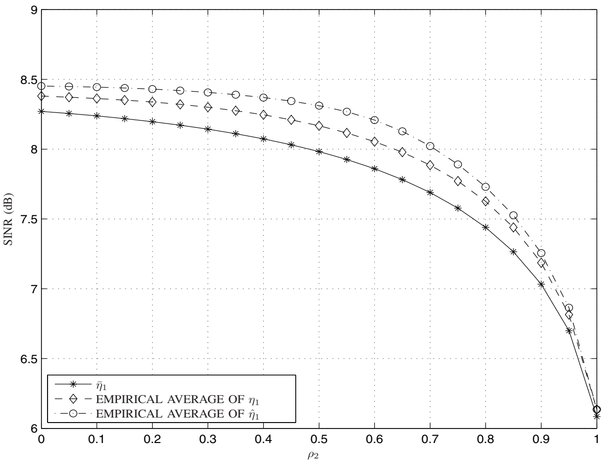

Fig. 1. SINR results versus ρ_2 when $\rho_1 = 1$, $\rho_p = 0.5$, $p = 3, \ldots, K$.

Table I can also be used to show the following result concerning the scenario in which $f(k)$ is the only relaying terminal in the system.

Theorem 4: Assume that the only relaying terminal in the system is $f(k)$, that is, $\rho_p = 1$ for all $p \neq f(k)$. Then, as σ_0 converges to zero, we have $\rho^\star_{f(k)} = \max\{0, \vartheta\}$ where $\vartheta \triangleq 1 - (\gamma_0/|h_{f(k),0}|^2(1 - \bar{\beta}_k^2))$.
Note that, through $\left|h_{f(k),0}\right|^2$, ϑ depends on $P_f(k)$ and, hence, $\rho^\star_{f(k)}$ can be changed by adjusting $P_f(k)$.

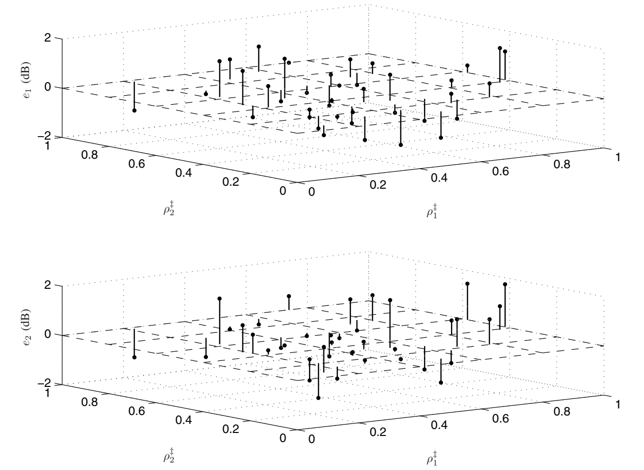

Fig. 2. $e_1 = \eta_1 - \bar{\eta}_1^\ddagger$ and $e_2 = \eta_2 - \bar{\eta}_2^\ddagger$ versus $(\rho_1^\ddagger, \rho_2^\ddagger)$ for $\bar{\eta}_1^\ddagger = 7$ (dB) and $\bar{\eta}_2^\ddagger = 5$ (dB).

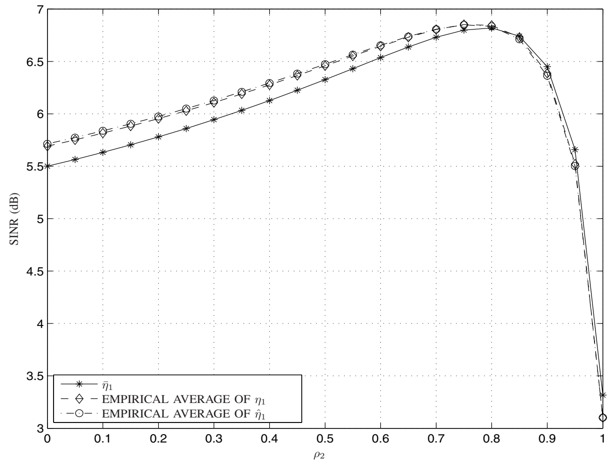

Fig. 3. SINR results versus ρ_2 when $\rho_p = 1$ for $p \neq 2$ and $\vartheta = 0.8$.

V. Simulations

To show the accuracy of our analysis in practical scenarios with bounded N and K, $N = 128$ and $K = 32$ are considered in all examples. Moreover, following **A1**, the entries of the spreading codes are randomly generated from $\pm 1/\sqrt{N}$. It is assumed that $\sigma_p = \sigma_0$ and all inter-terminal channel links $g_{p,k}$ are randomly and independently drawn from the zero-mean unit-variance circular Gaussian distribution and, unless otherwise stated, $P_p = P$ for $p = 1, \ldots, K$ with $P/\sigma_0^2 = 15$ (dB). For the sake of simplicity, we consider the case where $f(f(p)) = p$ and choose the first user as the user of interest and the second user as its partner.

Fig. 1 displays the curve of $\bar{\eta}_1$ as well as the empirical average curves of η_1 and $\hat{\eta}_1$ versus ρ_2 for $\rho_1 = 1$ and $\rho_p = 0.5,\ p = 3, \ldots, K$. The empirical average results are obtained by averaging over 100 realizations of the inter-terminal fading coefficients. As it can be observed from Fig. 1, all three curves remain very close over the whole range of $\rho_2 \in [0, 1]$. This verifies that our asymptotic results can accurately predict the performance of the cooperative system in the nonasymptotic regime. Fig. 1 also shows that, for the aforementioned values of ρ_p and the i.i.d. fading coefficients, η_1 is a decreasing function of ρ_2.

The next example examines how one can simultaneously achieve a target SINR pair for the first and the second users just by adjusting ρ_1 and ρ_2. Choosing $\rho_p = 0.5,\ p = 3, \ldots, K$, $\bar{\eta}_1^\ddagger = 7$ (dB), and $\bar{\eta}_2^\ddagger = 5$ (dB), 100 sets of fading coefficients realizations are generated, and, then, for each set, Equation (17) is numerically solved to obtain ρ_1 and ρ_2. If both entries of the so-obtained pair are in the interval $[0, 1]$, the pair is feasible and is regarded as the target weighting pair $(\rho_1^\ddagger, \rho_2^\ddagger)$. Then, to assess how accurately $(\rho_1^\ddagger, \rho_2^\ddagger)$ can deliver the target SINR pair, each of the achieved target weighting pairs is used in (12) to obtain the corresponding values of η_1 and η_2. Fig. 2 shows $e_1 = \eta_1 - \bar{\eta}_1^\ddagger$ and $e_2 = \eta_2 - \bar{\eta}_2^\ddagger$ for all pairs of $(\rho_1^\ddagger, \rho_2^\ddagger)$. The small values of e_1 and e_2 show the effectiveness of the proposed technique to obtain the target SINRs.

Fig. 3 shows the SINR results versus ρ_2 when $\rho_p = 1$ for $p \neq 2$. In this example, P_2 is determined such that ϑ in Theorem 4 is equal to 0.8. As we can observe from Fig. 3, the experimental SINR curves corroborate their analytical counterpart, and, in particular, the maximal value of η_1 occurs around $\rho_2 = 0.8$. This verifies the result of Theorem 4 and shows that $\rho_2^\star$ can in fact be adjusted by tuning P_2.

VI. Conclusions

A simple user SINR expression for multiuser cooperative DS-CDMA systems was obtained that is independent from the spreading codes and can be computed based only on the local information available at the user of interest, its partner, and the access point. The so-obtained expression was then used to derive the cooperating weight factors required to simultaneously achieve a given target SINR pair for each partnering users. The optimal amount of the relaying power that maximizes the SINR of each user was also obtained. It was shown that, due to relay estimation errors, increasing the amount of cooperation from the relay does not necessarily improve the reception performance.

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