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Distributed processing techniques for beamforming in wireless sensor networks

(Invited Paper)

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***Abstract*—A main task in distributed beamforming (DBF) techniques for wireless sensor networks (WSNs) is to maximize the received signal power at the access point (AP) while inflicting small interfering effect on unintended receivers. When the DBF nodes are unaware of the directions of unintended receivers, interference at the latter receivers may be substantially reduced by forming a beampattern with a narrow mainlobe that is pointed towards the AP. However, such an approach requires the DBF nodes to be sporadically scattered over a large area and, hence, increases the probability of the loss of their inter-connection. Assuming that nodes are uniformly distributed in the network, we show how the DBF nodes can be intelligently selected to ameliorate the network disconnectivity problem.**

In turn, when the directions of unintended receivers are known, one may aim to apply the so-called null-steering beamforming approach to effectively nullify the received power at those directions. However, it can be shown that implementing a null-steering beamformer in WSNs requires each node to be aware of the locations of all other nodes in the network; a requirement that opposes the distributed nature of WSNs. For such a scenario, we approximate the null-steering beamformer with another beamformer that is amenable to a distributed implementation.

***Index Terms*—Beampattern, Distributed beamforming, Null-steering beamforming, Wireless sensor networks**

I. Introduction

A major challenge in wireless sensor networks (WSNs) is to establish a reliable communication link from battery-powered sensor nodes to an access point (AP) that can be far beyond the nodes' transmission range. As a promising approach to deal with this challenge, distributed beamforming (DBF) [1]-[5] uses selected nodes to transmit a common message with proper weights such that their transmitted signals are coherently combined in the direction of the AP. As a result, the nodes aggregate transmission range in the latter direction is substantially increased without amplifying the nodes total transmit power.

In many practical scenarios, not only is it necessary to increase the received power at the AP, but also it is essential to avoid inflicting an interfering effect on unintended receivers. When the directions of unintended receivers are unknown, a conventional approach to accomplish the latter task is to form a narrow beampattern mainlobe such that most of the transmitted power concentrates towards the direction of the AP while dissipating only a negligible power in other directions. Note that a narrow beampattern mainlobe requires the DBF nodes (the nodes that participate in DBF) to be scattered in a large area within the network [1]. This, in turn, results in the loss of the DBF nodes inter-connections and, consequently, in impeding the implementation of DBF in practice. Assuming that the nodes are uniformly distributed on a plane, we propose a simple DBF node selection technique that results in a narrow beampattern mainlobe with a much better connectivity condition than that of its conventional counterpart.

When the directions of unintended receivers are known, a popular technique to avoid inducing interference on the latter receivers is to use the so-called null-steering beamformer [6], [7] that sets the transmission weights such that the transmitted signals are coherently combined in the direction of the AP and destructively merged in the directions of unintended receivers. Unfortunately, the null-steering beamformer may not be directly applied in WSNs as it requires each DBF node to know the exact locations of all other DBF nodes in the network. This requirement does not conform with the distributed nature of WSNs and, moreover, is not scalable with a growing number of DBF nodes.

When the nodes are uniformly distributed, we show how the statistical knowledge about the nodes locations can be used to introduce a novel null-steering beamformer that is applicable to WSNs in which each node is oblivious to all other nodes' locations. The average beampattern expression of the proposed beamformer is then obtained and it is further proved that the average gain of the beamformer in the directions of unintended receivers is inversely proportional to the number of DBF nodes.

This paper is organized as follows. Section II presents a background on the problem and the signal model. Section III studies the conventional DBF and presents our node selection scheme. Section IV presents the proposed distributed version of the null-steering beamformer and Section V includes the concluding remarks.

Notation: Uppercase and lowercase bold letters denote matrices and vectors, respectively. $[\cdot]_{il}$ and $[\cdot]_i$ are the (i,l)-th entry of a matrix and i-th entry of a vector, respectively. $j \triangleq \sqrt{-1}$ and $\mathbf{I}$ is the identity matrix. $(\cdot)^T$, $(\cdot)^*$, and $(\cdot)^H$ denote the transpose, the conjugate, and the Hermitian transpose, respectively. $\|\cdot\|$ is the 2-norm of a vector and $|\cdot|$ is the absolute value. $\mathrm{E}\{\cdot\}$ stands for the statistical expectation and ($\xrightarrow{ep1}$) $\xrightarrow{p1}$ denotes (element-wise) convergence with probability one. $J_n(\cdot)$ stands for the n-th order Bessel function of the first kind.

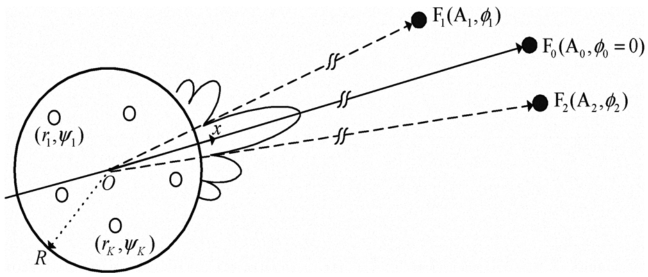

Fig. 1. The structure of the DBF network and the receiver terminals.

II. Background and Signal Model

Consider a WSN with N nodes uniformly distributed on $D(O, R_{\max})$, a disc centered at O with radius $R_{\max}$, and assume that the AP as well as L unintended receivers are located on the same plane containing $D(O, R_{\max})$. Let O be the pole and the ray towards the AP be the polar axis of a polar coordinate system and denote the coordinates of the n-th node as (r_n, ψ_n). Assume that $K \leq N$ nodes are randomly selected from $D(O, R)$ ($R \leq R_{\max}$) to participate in DBF. As all WSN nodes are uniformly distributed, $\mathbf{r} \triangleq [r_1, \ldots, r_K]^T$ and $\boldsymbol{\psi} \triangleq [\psi_1, \ldots, \psi_K]^T$ are comprised of i.i.d. random variables with the following probability density functions (PDFs) [1]:

$$f_{r_k}(r) = \frac{2r}{R^2} \qquad f_{\psi_k}(\psi) = \frac{1}{2\pi}. \tag{1}$$

Denote the locations of the AP and the L unintended receivers as $F_0(A_0, \phi_0 = 0)$ and $F_l(A_l, \phi_l)$, $l = 1, \ldots, L$, respectively. All DBF nodes know the direction of the AP while, due to the distributed nature of WSNs, **A1)** each DBF node is only aware of its own coordinates while being unaware of the locations of all other DBF nodes. The following assumptions are also common in the literature of array processing for planar waves [8], [9] and are adopted in this work: **A2)** All receivers are in the far field such that $A_l \gg R$ for $l = 0, 1, \ldots, L$ and the signals transmitted from any two arbitrary cluster nodes and received at one of the above terminals have equal path losses; **A3)** The signal transmitted from the DBF nodes is narrow-band; **A4)** The effect of scattering or signal reflection is negligible and, therefore, there is no multipath fading or shadowing; **A5)** All DBF nodes can be perfectly synchronized[1].

Two different scenarios regarding the nodes' level of knowledge about unintended receivers are studied in this work: **B1)** DBF nodes are oblivious to the directions of unintended receivers; **B1′)** DBF nodes know the direction of unintended receivers ϕ_l, $l = 1, \ldots, L$.

Let

$$s_k(t) = w_k^* z(t) e^{j2\pi f t} \tag{2}$$

denote the transmitted signal from the k-th DBF node where $z(t)$ is the zero-mean unit-norm information-bearing signal, w_k is the k-th node beamforming weight, and f is the carrier frequency. The received signal at an arbitrary far-field point $F_\bullet(A_\bullet, \phi_\bullet)$ from the latter node is given by

$$y_{F_\bullet,k}(t) = \beta_{F_\bullet} w_k^* z\left(t - \frac{d_{F_\bullet,k}}{c}\right) e^{-j\frac{2\pi d_{F_\bullet,k}}{\lambda}} e^{j2\pi f t} \tag{3}$$

where $\beta_{F_\bullet}$ stands for the signal path loss at $F_\bullet$, λ is the carrier wavelength, c is the speed of an electromagnetic wave, and

$$d_{F_\bullet,k} = \sqrt{A_\bullet^2 + r_k^2 - 2A_\bullet r_k \cos(\phi_\bullet - \psi_k)} \approx A_\bullet - r_k \cos(\phi_\bullet - \psi_k) \tag{4}$$

is the distance between the k-th DBF node and $F_\bullet$. Using (4) in (3), we have that

$$y_{F_\bullet,k}(t) \approx \beta_{F_\bullet} e^{-j\frac{2\pi A_\bullet}{\lambda}} z\left(t - \frac{d_{F_\bullet,k}}{c}\right) w_k^* e^{j\frac{2\pi}{\lambda} r_k \cos(\phi_\bullet - \psi_k)} e^{j2\pi f t}. \tag{5}$$

Introducing $\mathbf{a}_{\phi_\bullet} \triangleq [e^{j\frac{2\pi}{\lambda} r_1 \cos(\phi_\bullet - \psi_1)} \ \ldots \ e^{j\frac{2\pi}{\lambda} r_K \cos(\phi_\bullet - \psi_K)}]^T$ and $\mathbf{w} \triangleq [w_1 \ \ldots \ w_K]^T$, the received signal at $F_\bullet$ from all DBF nodes is then approximately equal to

$$y_{F_\bullet}(t) = \beta_{F_\bullet} e^{-j\frac{2\pi}{\lambda} A_\bullet} z\left(t - \frac{d_{F_\bullet,k}}{c}\right) e^{j2\pi f t} \mathbf{w}^H \mathbf{a}_{\phi_\bullet} \tag{6}$$

and the received power at $F_\bullet$ is $\xi_{F_\bullet}(\phi_\bullet, \mathbf{r}, \boldsymbol{\psi}) = \beta_{F_\bullet}^2 |\mathbf{w}^H \mathbf{a}_{\phi_\bullet}|^2$ where the latter two arguments in $\xi_{F_\bullet}(\phi_\bullet, \mathbf{r}, \boldsymbol{\psi})$ are used to stress that the received power at $F_\bullet$ is a random variable due to its dependency to the random vectors $\mathbf{r}$ and $\boldsymbol{\psi}$. In the array processing literature, $\xi_{F_\bullet}(\phi_\bullet, \mathbf{r}, \boldsymbol{\psi})$ is usually normalized by $K\beta_{F_\bullet}^2$ to form the beampattern

$$P(\phi_\bullet, \mathbf{r}, \boldsymbol{\psi}) = \frac{1}{K} |\mathbf{w}^H \mathbf{a}_{\phi_\bullet}|^2. \tag{7}$$

Given the total transmission power budget $\|\mathbf{w}\|^2 \leq 1$, the primary aim of DBF is to maximize $\xi_{F_0}(0, \mathbf{r}, \boldsymbol{\psi})$, or, equivalently, to maximize $P(0, \mathbf{r}, \boldsymbol{\psi})$. Moreover, it is also desired to have $P(\phi_l, \mathbf{r}, \boldsymbol{\psi})$, $l = 1, \ldots, L$ as small as possible. Note that since $P(\phi_\bullet, \mathbf{r}, \boldsymbol{\psi})$ is a random variable, it is usually more practical to try to reduce the average beampattern $\tilde{P}(\phi_\bullet) \triangleq \mathrm{E}_{\mathbf{r},\boldsymbol{\psi}}\{P(\phi_\bullet, \mathbf{r}, \boldsymbol{\psi})\}$ at $\phi_\bullet = \phi_1, \ldots, \phi_L$. It is also noteworthy that, as the entries of $\mathbf{r}$ and $\boldsymbol{\psi}$ are i.i.d., we have

$$\lim_{K \to \infty} P(\phi_\bullet, \mathbf{r}, \boldsymbol{\psi}) \xrightarrow{p1} \tilde{P}(\phi_\bullet) \tag{8}$$

for any arbitrary realization of $\mathbf{r}$ and $\boldsymbol{\psi}$. It can be inferred from the above fact that, when K is large enough, the difference between $P(\phi_\bullet, \mathbf{r}, \boldsymbol{\psi})$ and $\tilde{P}(\phi_\bullet)$ is negligible at all directions. This further justifies the practical importance of $\tilde{P}(\phi_\bullet)$.

III. Conventional DBF

A. Average beampattern expression

When **B1** holds, that is, there is no knowledge about the directions of the unintend receivers, one has to resort to apply the conventional beamforming vector $\mathbf{w}_{\mathrm{c}} = (1/\sqrt{K})\mathbf{a}_0$. In such a case we have $P_{\mathrm{c}}(\phi_\bullet, \mathbf{r}, \boldsymbol{\psi}) = (1/K^2)|\mathbf{a}_0^H \mathbf{a}_{\phi_\bullet}|^2$ where the subscript "c" is used to show that the beampattern expression is associated with the conventional beamformer $\mathbf{w}_{\mathrm{c}}$. Note that $P_{\mathrm{c}}(0, \mathbf{r}, \boldsymbol{\psi}) = 1$ for any arbitrary realization of $\mathbf{r}$ and $\boldsymbol{\psi}$ while it can be shown that [1]

$$\tilde{P}_{\mathrm{c}}(\phi_\bullet) = \frac{1}{K} + \left(1 - \frac{1}{K}\right)\left(2\frac{J_1(R\beta(\phi_\bullet))}{R\beta(\phi_\bullet)}\right)^2 \tag{9}$$

[1]See, e.g., [2], [10] for distributed synchronization techniques in WSNs.

where $\beta(\phi_\bullet) \triangleq (4\pi/\lambda)\sin(\phi_\bullet/2)$. As the directions of unintended receivers $\phi_l,\ l = 1,\ldots,L$ are unknown, there is no systematic approach to reduce $\tilde{P}_c(\phi_l),\ l = 1,\ldots,L$. However, the chance of having a small $\tilde{P}_c(\phi_l),\ l = 1,\ldots,L$ is increased if $\tilde{P}_c(\phi_\bullet)$ has a very narrow mainlobe and rapidly drops towards $1/K$ for small values of $\phi_\bullet$. It can be observed from (9) that a rapid decay of $\tilde{P}_c(\phi_\bullet)$ towards $1/K$ requires a large R. In fact, it has been shown in [1] that the width of the average beampattern mainlobe is almost inversely proportional to R. At first glance, it may seem that, as long as $R \leq R_{\max}$, a large R does not pose any practical challenge when implementing the DBF. Our discussion in the following subsection shows otherwise.

B. The effect of R on the network connectivity

A successful implementation of a DBF technique in a real world scenario is typically contingent upon resolving two main challenges: First, as the DBF nodes in WSNs are independent sensing units, their common information-bearing signal $z(t)$ may need to be generated through a rumor spreading or a consensus process [11], [12] among the nodes. Second, as each WSN node has its own clock, the DBF nodes do not share a common time reference. Therefore, they should distributively synchronize with one another [1], [2], [10] to be able to transmit their common signal $z(t)$ in a coordinated manner. These required preliminary steps necessitate intensive inter-node communications prior to the actual DBF process. As such, it is essential that the DBF nodes form a connected network such that a signal originating from any of them can eventually reach all others[2].

The above discussion along with the discussion at the end of Subsection III-A show that a narrow mainlobe and the network connectivity can be two conflicting requirements in general: A narrow mainlobe requires a large R and, hence, an increased DBF network area. A larger DBF network area causes that the K randomly-selected DBF nodes have larger distances, and, eventually, get disconnected from one another. In fact, it can be shown that when R is much larger than the nodes transmission range R_f, the probability of having no isolated nodes[3] is approximately equal to [4]

$$\bar{I}_c(R) = \left(1 - \left(1 - (R_f^2/R^2)\right)^{K-1}\right)^K. \qquad (10)$$

Note that $\bar{I}_c(R)$ is an upper bound on the probability of the network connectivity and the gap between the latter two probabilities is very small when both R and K are large [13], [14]. It can be observed from (10) that $\bar{I}_c(R)$ rapidly converges to zero as R increases.

When a large R is chosen to shrink the mainlobe, one may suggest using idle nodes in the network as relays to reestablish the DBF network connectivity. However, this can cause a substantial increase in the network energy consumption and the earlier depletion of the nodes' valuable energy resources [4]. In what follows, we develop an alternative DBF node selection approach that provides a narrow average beampattern mainlobe while avoiding a fast decrease in the network connectivity probability.

[2]When the nodes' common signal $z(t)$ is obtained by decoding the transmitted signal from a source, one should still retain the nodes connectivity for synchronization purposes.

[3]An isolated node is a node that is disconnected from the rest of the DBF network

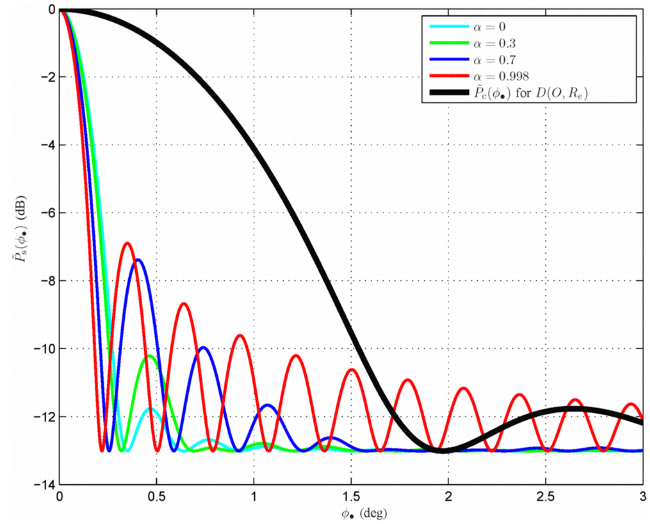

Fig. 2. $\tilde{P}_s(\phi_\bullet)$ (dB) versus $\phi_\bullet$ (deg) for different α.

C. Selecting the nodes from a perimeter's vicinity

As discussed in Subsections III-A and III-B, a large DBF network radius results in a narrow mainlobe while a large DBF network area causes the nodes disconnectivity. In light of the above facts, it seems reasonable to select the DBF nodes from a region with large dimensions, yet with a small area. Therefore, instead of selecting the DBF nodes from $D(O,R)$, we suggest to select them from a close vicinity of the perimeter of $D(O,R)$, that is, the ring $S(O,\alpha R,R)$ where αR and R are the inner and the outer radii of the ring, respectively. Regardless of R, α can be selected such that the area of $S(O,\alpha R,R)$ is equal to a predetermined value $A_S \leq \pi R^2$. When the nodes are selected from $S(O,\alpha R,R)$, the average beampattern expression is [4]

$$\tilde{P}_s(\phi_\bullet) = \frac{1}{K} + \left(1 - \frac{1}{K}\right)\left(2\frac{J_1(R\beta(\phi_\bullet)) - \alpha J_1(\alpha R\beta(\phi_\bullet))}{(1-\alpha^2)R\beta(\phi_\bullet)}\right)^2. \qquad (11)$$

Similar to the case when the DBF nodes are selected from $D(O,R)$, it can be observed from (11) that if R is large, $\tilde{P}_s(\phi_\bullet)$ rapidly drops to $1/K$ for small values of $\phi_\bullet$. Note that if A_S is kept fixed, R can be chosen large enough such that $\alpha \approx 1$. In such a case, the probability of having no isolated nodes is approximately given by [4]

$$\bar{I}_s(R) = \left(1 - \left(1 - (R_f/\pi R)\right)^{K-1}\right)^K. \qquad (12)$$

For practical values of R_f/R and K, $\bar{I}_s(R)$ can be up to several orders of magnitude larger than $\bar{I}_c(R)$ [4].

Fig. 2 shows $\tilde{P}_s(\phi_\bullet)$ versus $\phi_\bullet$ in the case that $K = 20$ nodes are randomly selected from $S(O, \alpha R, R)$ with $R = 100\lambda$ and four different values of $\alpha = 0,\ 0.3,\ 0.7,\ 0.998$. Assuming that $R_f = \lambda$, we have obtained R_e such that $\bar{I}_c(R_e) = \bar{I}_s(R)$. For the sake of comparison, we have also plotted $\tilde{P}_c(\phi_\bullet)$ in the case that the nodes are selected from $D(O, R_e)$. As can be observed from Fig. 2, increasing α shrinks the average beampattern mainlobe. Moreover, the directivity of $\tilde{P}_s(\phi_\bullet)$ when the DBF nodes are selected from $S(O, 0.998R, R)$ is much higher than that of $\tilde{P}_c(\phi_\bullet)$ when the nodes are selected from $D(O, R_e)$. Note though that the nodes isolation probabilities are approximately equal in both scenarios.

IV. Null-Steering DBF

If $\mathbf{B1}'$ holds, it is possible to find a beamforming vector $\mathbf{w}$ that maximizes $P(0, \mathbf{r}, \boldsymbol{\psi})$ while 1) upholding the total transmission power constraint $\|\mathbf{w}\|^2 \le 1$ and; 2) nulling out all unintended receivers by having $P(\phi_l, \mathbf{r}, \boldsymbol{\psi}) = 0$ for $l = 1, \dots, L$. Such a beamforming vector $\mathbf{w}$ is the solution to

$$\max_{\mathbf{w}} |\mathbf{w}^H \mathbf{a}_0|^2 \qquad \text{subject to} \quad \begin{array}{rcl} \mathbf{w}^H \mathbf{A} & = & \mathbf{0} \\ \mathbf{w}^H \mathbf{w} & \le & 1 \end{array}, \tag{13}$$

where $\mathbf{A} \triangleq [\mathbf{a}_{\phi_1} \ \dots \ \mathbf{a}_{\phi_L}]$. The solution to (13) is

$$\mathbf{w}_{\text{ns}} = \frac{(\mathbf{I} - \mathbf{P}_{\mathbf{A}})\, \mathbf{a}_0}{\|(\mathbf{I} - \mathbf{P}_{\mathbf{A}})\, \mathbf{a}_0\|} \tag{14}$$

where $\mathbf{P}_{\mathbf{A}} \triangleq \mathbf{A}(\mathbf{A}^H\mathbf{A})^{-1}\mathbf{A}^H$ is the orthogonal projection matrix onto the subspace spanned by the columns of $\mathbf{A}$. The solution to (13) is often called the null-steering beamformer. This justifies the subscript of $\mathbf{w}_{\text{ns}}$. Defining the $L \times L$ matrix $\mathbf{E} \triangleq (1/K)\mathbf{A}^H\mathbf{A}$ and the $L \times 1$ vector $\mathbf{g}_{\phi_\bullet} \triangleq (1/K)\mathbf{A}^H\mathbf{a}_{\phi_\bullet}$, (14) can be equivalently represented as

$$\mathbf{w}_{\text{ns}} = \frac{\mathbf{a}_0 - \mathbf{A}\mathbf{E}^{-1}\mathbf{g}_0}{\sqrt{K\left(1 - \mathbf{g}_0^H\mathbf{E}^{-1}\mathbf{g}_0\right)}}. \tag{15}$$

It follows from (15) that the k-th node null-steering beamforming weight is

$$\begin{aligned} w_k = [\mathbf{w}_{\text{ns}}]_k &= \frac{[\mathbf{a}_0]_k - \sum_{i=1}^{L}[\mathbf{A}]_{ki}[\mathbf{E}^{-1}\mathbf{g}_0]_i}{\sqrt{K\left(1 - \mathbf{g}_0^H\mathbf{E}^{-1}\mathbf{g}_0\right)}} \\ &= \frac{e^{j\frac{2\pi}{\lambda}r_k\cos(\phi_0 - \psi_k)} - \sum_{i=1}^{L} e^{j\frac{2\pi}{\lambda}r_k\cos(\phi_i - \psi_k)}[\mathbf{E}^{-1}\mathbf{g}_0]_i}{\sqrt{K\left(1 - \mathbf{g}_0^H\mathbf{E}^{-1}\mathbf{g}_0\right)}} \end{aligned} \tag{16}$$

and, therefore, the null-steering beamformer can be implemented only if the k-th node ($k = 1, \dots, K$) is able to compute all $e^{j\frac{2\pi}{\lambda}r_k\cos(\phi_i - \psi_k)}$ for $i = 0, \dots, L$ as well as all entries of the $\mathbf{E}$ and $\mathbf{g}_0$. From $\mathbf{A1}$ and $\mathbf{B1}'$ it follows that the k-th node can use its available knowledge to derive $e^{j\frac{2\pi}{\lambda}r_k\cos(\phi_i - \psi_k)}$ for $i = 0, \dots, L$. However, this knowledge is not sufficient to compute $\mathbf{E}$ and $\mathbf{g}_0$ as it turns out that $\mathbf{E}$ and $\mathbf{g}_0$ depend on the locations of all DBF nodes [5]. Therefore, $\mathbf{w}_{\text{ns}}$ is not implementable in the distributed network of our concern. An approach to get around this problem is to approximate $\mathbf{E}$ and $\mathbf{g}_0$ with other quantities that are P1) good

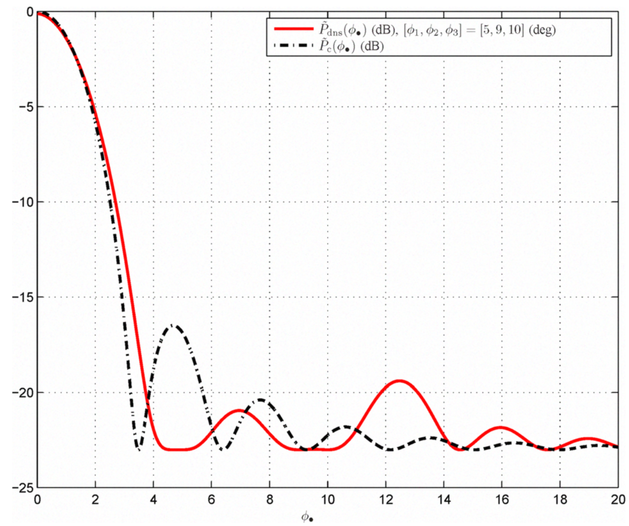

Fig. 3. $\tilde{P}_{\text{dns}}(\phi_\bullet)$ and $\tilde{P}_c(\phi_\bullet)$ versus $\phi_\bullet$. Three unintended receivers at $[\phi_1, \phi_2, \phi_3] = [5, 9, 10]$ (deg) are considered.

approximations of their original counterparts; and P2) depend only on the parameters *commonly* known at every node, that is, $\phi_l,\ l = 0, 1, \dots, L$.

As the total transmission power from the whole network is fixed and each DBF node has a limited nonrenewable energy resource, it is sensible to use a large number of DBF nodes in return to a low transmission power from each individual node. When K is large enough, $\bar{\mathbf{E}} \triangleq \lim_{K\to\infty}\mathbf{E}$ and $\bar{\mathbf{g}}_0 \triangleq \lim_{K\to\infty}\mathbf{g}_0$ are accurate approximations of $\mathbf{E}$ and $\mathbf{g}_0$, respectively. It can also be shown that when the DBF nodes are randomly selected from $D(O, R)$, we have [5]

$$[\bar{\mathbf{E}}]_{mn} = \begin{cases} \frac{2}{R\beta(\phi_n - \phi_m)} J_1\left(R\beta(\phi_n - \phi_m)\right) & m \ne n \\ 1 & m = n \end{cases} \tag{17}$$

$$[\bar{\mathbf{g}}_0]_m = \frac{2}{R\beta(\phi_m)} J_1\left(R\beta(\phi_m)\right). \tag{18}$$

Eqs. (17) and (18) establish the fact that $\bar{\mathbf{E}}$ and $\bar{\mathbf{g}}_0$ satisfy P2. Therefore, we propose to use

$$\bar{\mathbf{w}}_{\text{ns}} = \frac{\mathbf{a}_0 - \mathbf{A}\bar{\mathbf{E}}^{-1}\bar{\mathbf{g}}_0}{\sqrt{K\left(1 - \bar{\mathbf{g}}_0^H\bar{\mathbf{E}}^{-1}\bar{\mathbf{g}}_0\right)}}. \tag{19}$$

in lieu of $\mathbf{w}_{\text{ns}}$. Note that $\bar{\mathbf{w}}_{\text{ns}}$ accurately approximates the null-steering beamformer $\mathbf{w}_{\text{ns}}$ for a large K, and, in addition, can be implemented by DBF nodes in a distributed fashion. When $\bar{\mathbf{w}}_{\text{ns}}$ is used, the average beampattern is given by [5]

$$\tilde{P}_{\text{dns}}(\phi_\bullet) = \frac{1}{K} + \left(1 - \frac{1}{K}\right) \cdot \frac{\left(2\frac{J_1(R\beta(\phi_\bullet))}{R\beta(\phi_\bullet)} - \bar{\mathbf{g}}_0^T\bar{\mathbf{E}}^{-1}\bar{\mathbf{g}}_{\phi_\bullet}\right)^2}{1 - \bar{\mathbf{g}}_0^T\bar{\mathbf{E}}^{-1}\bar{\mathbf{g}}_0} \tag{20}$$

where $\bar{\mathbf{g}}_{\phi_\bullet}$ is an $L \times 1$ vector with

$$[\bar{\mathbf{g}}_{\phi_\bullet}]_l \triangleq \begin{cases} 2\frac{J_1(R\beta(\phi_\bullet - \phi_l))}{R\beta(\phi_\bullet - \phi_l)} & \phi_\bullet \ne \phi_l \\ 1 & \phi_\bullet = \phi_l \end{cases}. \tag{21}$$

It can be readily shown from (20) that

$$\tilde{P}_{\text{dns}}(\phi_l) = \frac{1}{K} \tag{22}$$

for $l = 1, \ldots, L$. Eq. (22) verifies that $\phi_l,\ l = 1, \ldots, L$ are in fact the minimum points of the average beampattern $\tilde{P}_{\text{dns}}(\phi_\bullet)$. The latter equation also shows that the price of using $\bar{\mathbf{w}}_{\text{ns}}$ in lieu of $\mathbf{w}_{\text{ns}}$ is to elevate the minimum levels at $\phi_l,\ l = 1, \ldots, L$ from zero to $1/K$. Of course, as K grows large, $\bar{\mathbf{w}}_{\text{ns}}$ converges to $\mathbf{w}_{\text{ns}}$ and $1/K$ diminishes to zero. It is also straightforward to show from (20) that

$$\tilde{P}_{\text{dns}}(0) = 1 - \bar{\mathbf{g}}_0^T \bar{\mathbf{E}}^{-1} \bar{\mathbf{g}}_0 + \frac{1}{K} \bar{\mathbf{g}}_0^T \bar{\mathbf{E}}^{-1} \bar{\mathbf{g}}_0. \tag{23}$$

Under mild conditions we have $\bar{\mathbf{g}}_0^T \bar{\mathbf{E}}^{-1} \bar{\mathbf{g}}_0 \approx 0$ [5], and, therefore, $\tilde{P}_{\text{dns}}(0)$ is approximately equal to the maximal possible value of 1.

Fig. 3 plots $\tilde{P}_{\text{c}}(\phi_\bullet)$ and $\tilde{P}_{\text{dns}}(\phi_\bullet)$ versus $\phi_\bullet$ for $[\phi_1, \phi_2, \phi_3] = [5, 9, 10]$ (deg). $K = 200$ and $R/\lambda = 10$ are assumed. As can be observed from Fig. 3, three nulls of $\tilde{P}_{\text{dns}}(\phi_\bullet)$ are exactly located at ϕ_1, ϕ_2, and ϕ_3, and, moreover, we have that $\tilde{P}_{\text{dns}}(0) \approx 1$.

V. Conclusions

We have considered two problems that hamper the applicability of beamfoming techniques in wireless sensor networks (WSNs): 1) The nodes disconnectivity problem when forming a beampattern with a narrow mainlobe and; 2) The nodes obliviousness to other nodes' locations when implementing a null-steering beamformer.

Assuming that nodes are uniformly distributed on a planar area, the first problem has been resolved by establishing the fact that if the participating nodes in the beamforming are randomly selected from a narrow ring, an average beampattern with a narrow mainlobe can be formed, and, further, the probability that the active nodes fall outside of the transmission range of one another is substantially reduced. The second problem has been successfully treated by proposing a null-steering beamformer that, in contrary to its existing counterparts, is applicable to WSNs wherein each node is unaware of other nodes' locations. The average beampattern expression of the proposed beamformer has been derived and it has been shown that the average gain of this beamformer at the directions of unintended receivers is inversely proportional to the number of nodes.

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References

[1] H. Ochiai, P. Mitran, H. V. Poor, and V. Tarokh, "Collaborative beamforming for distributed wireless ad hoc sensor networks," *IEEE Trans. Signal Process.,* vol. 53, pp. 4110-4124, Nov. 2005.

[2] R. Mudumbai, G. Barriac, U. Madhow, "On the feasibilty of distributed beamforming in wireless sensor networks," *IEEE Trans. Wirless Commun.,* vo. 6, pp. 1754-1763, May 2007.

[3] M. F. A. Ahmed and S. A. Vorobyov, "Collaborative beamforming for wireless sensor networks with Gaussian distributed sensor nodes," *IEEE Trans. Wireless Commun.,* pp. 638-643, Feb. 2009.

[4] K. Zarifi, A. Ghrayeb, and S. Affes, "Distributed beamforming for wireless sensor networks with improved graph connectivity and energy efficiency," *IEEE Trans. Signal Process.,* to appear.

[5] K. Zarifi, S. Affes, and A. Ghrayeb, "Collaborative null-steering beamforming for uniformly distributed wireless sensor networks," *IEEE Trans. Signal Process.,* to appear.

[6] B. Friedlander and B. Porat, "Performance analysis of a null-steering algorithm based on direction-of-arrival estimation," *IEEE Trans. Signal Process.,* vol. 37, pp. 461-466, Apr. 1989.

[7] L. C. Godara, "Application of antenna arrays to mobile communications, Part II: Beam-forming and direction-of-arrival considerations," *Proc. IEEE,* vol. 85, pp. 1195-1245, Aug. 1997.

[8] A. Paulraj and T. Kailath, "Eigenstructure methods for direction of arrival estimation in the presence of unknown noise fields," *IEEE Trans. Acoust. Speech Signal Process.,* vol. 34, pp. 13-20, Feb. 1986.

[9] S. A. Vorobyov, A. B. Gershman, and Z.-Q. Luo, "Robust adaptive beamforming using worst-case performance optimization: A solution to the signal mismatch problem," *IEEE Trans. Signal Process.,* vol. 51, pp. 313-324, Feb. 2003.

[10] D. R. Brown III and H. V. Poor, "Time-slotted round-trip carrier synchronization for distributed beamforming," *IEEE Trans. Signal Process.,* vol. 56, pp. 5630-5643, Nov. 2008.

[11] S. Boyd, A. Ghosh, B. Prabhakar, and D. Shah, "Randomized gossip algorithms," *IEEE Trans. Inf. Theory,* vo. 52, pp. 2508-2530, June 2006.

[12] R. Karp, C. Schindelhauser, S. Shenker, and B. Vöking, "Randomized rumor spreading," *in Proc. Symp. Foundations of Computer Science,* 2000, pp. 564-574.

[13] P. Gupta and P. R. Kumar, "Critical power for asymptotic connectivity in wireless networks," in *Stochastic Analysis, Control, Optimization and Applications: A Volume in Honor of W. H. Fleming,* W. M. McEneany, G. Yin, and Q. Zhang, Eds. Boston, MA: Birkhauser, 1998, pp. 547-566.

[14] C. Bettstetter and C. Hartmann, "Connectivity of wireless multihop networks in a shadow fading environment," in *ACM/Springer Wireless Networks,* vol. 11, pp. 571-579, Sep. 2005.