Exact Error Analysis of Dual-Hop Fixed-Gain AF Relaying over Arbitrary Nakagami-m Fading

Imène Trigui, Sofiène Affes, and Alex Stéphenne
INRS-EMT, 800, de la Gauchetière Ouest, Bureau 6900, Montréal, H5A 1K6, Qc, Canada.

***Abstract*—Closed-form error analysis of dual-hop fixed-gain amplify-and-forward (AF) relaying systems for transmissions over Nakagami fading channels has, so far, been tractable only with integer fading parameters. For the general Nakagami-m fading with arbitrary m values, the exact closed-form error analysis is more challenging. This paper goes toward solving this problem by deriving a unified error analysis framework that embraces several general modulation schemes. The obtained Error Probability (EP) expressions involve common functions which can be efficiently evaluated using standard numerical softwares. This work represents a significant improvement on previous contributions, not only by unifying the error probability expressions pertaining to dual-hop fixed gain relaying over integer Nakagami-m fading, but also by extending them to the general arbitrary Nakagami-m fading. Simulation results sustaining our analysis are provided, and the impacts of various parameters on the overall AF relaying system performance are investigated.**

I. Introduction

Amplify and forward relay transmission is promising for next-generation wireless systems, owing to its ability to improve the transmission reliability and spectral efficiency while keeping a low-complexity and straightforward implementation [1]. Roughly speaking, in this communication paradigm, an intermediate node, called relay, is used to help the source forward signals to the destination if the direct channel from the source to the destination is in deep fading. The relay simply amplifies the received signals prior to transmission without implementing any decoding operation. Therefore, it is transparent to the modulation and coding schemes of the source. The amplification gain at the relay aims to invert the first hop channel subject to the output relay power constraint when the first-hop fading amplitude is low. AF relays can be classified into two categories, namely, channel state information CSI-assisted relays and blind relays. In the first case, the relay uses the channel information of the preceding hop to control the relay gain. In contrast, systems with blind relays use amplifiers with fixed gains resulting in a signal with variable power at the relay output. Systems with CSI-assisted relays are expected to perform better than systems equipped with blind relays, even though the latters are more attractive due to their ease of deployment.

Due to their practical merits, dual-hop AF relaying systems has garnered a lot of research endeavor especially over Rayleigh fading channels [2]-[4]. However, for the well-known Nakagami-m fading, despite many valuable contributions [5]-[13], exact and general error analysis of dual-hop relaying over this channel with arbitrary m values remains an open problem. So far, to obtain tractable error probabilities expressions, the authors in [5]-[10] assumed integer values of the Nakagami-m fading parameter. Nevertheless, in practical scenarios, namely the micro-cellular environment with strong specular components [11], the m parameters often adopt non-integer values, which limits the scope of [5]-[10]. For the scenario with arbitrary m values, closed-form expressions for the EP have been recently obtained in [12] but for fully CSI-assisted relays. For semi-blind relays, the moment generation function (MGF)-based approach is applied to analyze the average SEP in [13], where the obtained formulas involved the generalized Meijer's G-function. Due to its high complexity, the generalized Meijer's G-function is hard to be further processed and, therefore, the average EP is obtained by numerical integration and approximation. Whilst such an integration is numerically feasible, an analytical expression does not yet exist in closed form. Approximations have also been proposed in [8] in order to derive error bounds of the AF two-hop system in general Nakagami-m fading.

In spite of their significant contributions, none of the above papers investigated exact closed-form error analysis of the dual-hop fixed-gain relaying system under arbitrary Nakagami-m fading. The aim of this paper is to go one step towards filling this gap in the literature by deriving generic closed-form expressions for the error probabilities applicable for several modulation schemes.

The rest of this paper is organized as follows. First, Section II briefly introduces the system model. Next, Section III presents the end-to-end error performance over arbitrary Nakagami-m fading in dual hop AF relaying systems along with the newly derived EP closed-from expressions. Numerical and simulation results are presented in Section IV, and a conclusion summarizing the contributions of this work is provided in Section V

II. System and Channel Models

We consider a dual-hop amplify and forward transmission system consisting of one source (S), one destination (D) and a relay (R). Both the first hop (source-to-relay) and the second hop (relay-to-destination) experience independent but not necessarily identically distributed Nakagami-m fading with fading shape factors $m_1, m_2 \geq 0.5$, respectively. The source has no direct link with the destination due to the

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unsatisfactory quality of the channel, and the transmission is performed only through the relay (R). Let γ_1 and γ_2 be the instantaneous SNRs pertaining to the the first- and second-hop transmissions, respectively. Skipping the mathematical details about the signal transmission, the instantaneous end-to-end SNR can be expressed as [2]

$$\gamma = \frac{\gamma_1\gamma_2}{\gamma_2 + C}, \tag{1}$$

where C is a constant describing the relay gain. Under Nakagami-m fading, it can be easily shown that the probability density function (PDF) of γ_i, $(i = 1, 2)$ is given by

$$f_{\gamma_i}(\gamma_i) = \frac{m_i^{m_i}}{\Gamma(m_i)\bar{\gamma_i}^{m_i}}\gamma_i^{m_i-1}\exp\left(-\frac{m_i\gamma_i}{\bar{\gamma_i}}\right), \tag{2}$$

whereby $\Gamma(\cdot)$ is the Gamma function and $\bar{\gamma_i} = \mathrm{E}(\gamma_i)$, with $\mathrm{E}(\cdot)$ denoting expectation. Following the procedure adopted in [2], the constant C can be written as

$$C = \frac{\bar{\gamma_1}}{m_1^{m_1}\Gamma(1-m_1,\frac{m_1}{\bar{\gamma_1}})}e^{-\frac{m_1}{\bar{\gamma_1}}}, \tag{3}$$

where $\Gamma(\cdot,\cdot)$ is the complementary incomplete gamma function [1, Eq. (8.350.2)].

III. END-TO-END ERROR PROBABILITY ANALYSIS

Here, we present our main results on the error probability analysis of dual-hop fixed gain relay systems. Different modulations schemes are therefore considered including binary and M-ary modulations.

A. Binary Modulations

Theorem 1 : The bit error probability (BEP) of coherent, differentially coherent and non-coherent detection of binary signals transmitted over the dual-hop fixed gain system can be written as follows

$$\begin{aligned}P_b = &\frac{1}{2} - \frac{\left(\frac{a\bar{\gamma}_1\bar{\gamma}_2}{Cm_1m_2}\right)^b}{2\Gamma(b)\Gamma(m_1)\Gamma(m_2)}\times\\ &\left(\mathrm{G}_{3,2}^{1,3}\left(\frac{a\bar{\gamma}_1\bar{\gamma}_2}{Cm_1m_2}\middle|\begin{matrix}1-b-m_1,1-m_2-b,1-b\\0,-b\end{matrix}\right)\right.\\ &\left.-\mathrm{G}_{2,1,0,2}^{2,1,1,1,1}\left(\begin{matrix}\frac{a\bar{\gamma}_1\bar{\gamma}_2}{Cm_1m_2}\\ \frac{\bar{\gamma}_2}{Cm_2}\end{matrix}\middle|\begin{matrix}1-m_2-b,1-b\\b+m_1,0\\\frac{m_2}{2}+b\\0,-b,1,0\end{matrix}\right)\right),\end{aligned} \tag{4}$$

where $G_{a,b}^{c,d}(\cdot)$ is the Meijer's G-function [15, Eq. (9.301)] and $G[\cdot,\cdot]$ denotes the generalized Meijer's G-function of two variables[1] [18].

Proof: : A generic expression for the BEP of binary signals is given by

$$P_b = \mathrm{E}\left\{\frac{\Gamma(b, a\gamma)}{2\Gamma(b)}\right\}. \tag{5}$$

[1] Although the bivariate G-function in (4) cannot be directly calculated by popular mathematical softwares such as Matlab or Mathematica, it can be easily evaluated by the algorithm recently developed in [17, Table II], which is based on the double Mellin-Barnes type integrals.

In (5), γ is the end-to-end SNR, and a and b depend on the particular form of modulation and detection. Using the McLauren series of $\Gamma(\cdot,\cdot)$ given in [15, Eq. (8.354.2)], one obtains

$$\begin{aligned}P_b &= \frac{1}{2} - \sum_{n=0}^{\infty}\frac{(-1)^n a^{b+n}}{2\Gamma(b)n!(b+n)}\mathrm{E}\left\{\gamma^{b+n}\right\}\\ &= \frac{1}{2} - \sum_{n=0}^{\infty}\frac{(-1)^n a^{b+n}\mathrm{E}\left\{\gamma_1^{b+n}\right\}}{2\Gamma(b)n!(b+n)}\mathrm{E}\left\{\left(\frac{\gamma_2}{\gamma_2+C}\right)^{b+n}\right\}.\end{aligned} \tag{6}$$

Under the assumption of Nakagami-m fading channels, it can easily be shown that

$$\mathrm{E}\left\{\gamma_1^{b+n}\right\} = \frac{\Gamma(m_1+b+n)}{\Gamma(m_1)}\left(\frac{\bar{\gamma_1}}{m_1}\right)^{b+n}, \tag{7}$$

and

$$\mathrm{E}\left\{\left(\frac{\gamma_2}{\gamma_2+C}\right)^{b+n}\right\} = \frac{\int_0^\infty s^{b+n-1}e^{-s}M_{\frac{1}{\gamma_2}}(Cs)ds}{(b+n-1)!}, \tag{8}$$

whereby $M_{\frac{1}{\gamma_2}}(\cdot)$ stands for the MGF of $1/\gamma_2$ given by

$$M_{\gamma_2^{-1}}(s) = 2\left(\frac{\left(\frac{m_2}{\bar{\gamma_2}}\right)^{\frac{m_2}{2}}}{\Gamma(m_2)}\right)s^{\frac{m_2}{2}}K_{m_2}\left(2\sqrt{\frac{sm_2}{\bar{\gamma_2}}}\right), \tag{9}$$

where $K_\lambda(\cdot)$ is the modified bessel function of the second kind and order λ [15, Eq. (8.485)]. Upon substitution of (7) and (8) into (6), one obtains

$$\begin{aligned}P_b &= \frac{1}{2} - \frac{\left(\frac{Cm_2}{\bar{\gamma_2}}\right)^{\frac{m_2}{2}}\left(\frac{a\bar{\gamma_1}}{m_1}\right)^b\Gamma(b+m_1)}{\Gamma(b)\Gamma(m_1)\Gamma(m_2)\Gamma(1+b)}\int_0^\infty s^{\frac{m_2}{2}+b-1}\\ &\times e^{-s}{}_1\mathrm{F}_1\left(b+m_1,1+b,\frac{-a\bar{\gamma_1}s}{m_1}\right)K_{m_2}\left(2\sqrt{\frac{Cm_2s}{\bar{\gamma_2}}}\right)ds,\end{aligned} \tag{10}$$

where ${}_1\mathrm{F}_1(a;b;z)$ is the confluent Hypergeometric function [15, Eq. (9.210)]. By recognizing that the three integrands involved in (10) can be expressed in terms of the Meijer's G-function [15, Eqs. (9.304)] as shown in (11) and using the Mellin-Barnes integration theorems in [15, Eq. (7.811.1)] and [18, Eq. (3.1)], a closed-form expression of the BEP is obtained according to (4). It is worth mentioning that (4) greatly simplifies the error probability evaluation, in contrast to the recent results in [13, Eq. (23)] where the intractability arises in seeking a closed-form solution to the finite-range integration of the MGF expression of the end-to-end SNR which involves the double Meijer's G-function.

Even though, in general, (4) can be afforded easily, for some applications and system optimizations one may be interested in having a very simple framework.

lemma 1: This framework can be obtained when m_2 is constrained to be a non-integer fading parameter. In this case, the bit error probability can be written according to

$$P_b = \frac{1}{2} - \frac{\left(\frac{Cm_2}{\bar{\gamma_2}}\right)^{\frac{m_2}{2}}\left(\frac{a\bar{\gamma_1}}{m_1}\right)^b\Gamma(b+m_1)}{\Gamma(b)\Gamma(m_1)\Gamma(m_2)\Gamma(1+b)}\Upsilon, \tag{12}$$

$$P_b = \frac{1}{2} - \frac{\left(\frac{Cm_2}{\bar{\gamma}_2}\right)^{\frac{m_2}{2}}\left(\frac{a\bar{\gamma}_1}{m_1}\right)^b}{2\Gamma(b)\Gamma(m_1)\Gamma(m_2)} \int_0^\infty s^{\frac{m_2}{2}+b-1} \mathrm{G}_{1,2}^{1,1}\left(\frac{a\bar{\gamma}_1}{m_1}s \,\middle|\, \begin{matrix} 1-b-m_1 \\ 0,-b \end{matrix}\right) \mathrm{G}_{0,2}^{2,0}\left(\frac{Cm_2}{\bar{\gamma}_2}s \,\middle|\, \begin{matrix} - \\ \frac{m_2}{2},-\frac{m_2}{2} \end{matrix}\right) ds - \int_0^\infty s^{\frac{m_2}{2}+b-1} \mathrm{G}_{1,2}^{1,1}\left(s \,\middle|\, \begin{matrix} 1 \\ 1,0 \end{matrix}\right) \mathrm{G}_{1,2}^{1,1}\left(\frac{a\bar{\gamma}_1}{m_1}s \,\middle|\, \begin{matrix} 1-b-m_1 \\ 0,-b \end{matrix}\right) \mathrm{G}_{0,2}^{2,0}\left(\frac{Cm_2}{\bar{\gamma}_2}s \,\middle|\, \begin{matrix} - \\ \frac{m_2}{2},-\frac{m_2}{2} \end{matrix}\right) ds, \quad (11)$$

$$P_b = \frac{1}{2} - \left(\frac{a\bar{\gamma}_1}{m_1}\right)^b \frac{\Gamma(m_1+b)}{2\Gamma(m_1)\Gamma(1+b)\Gamma(b)} \sum_{n=0}^\infty \left(\frac{Cm_2}{\bar{\gamma}_2}\right)^n \left(\frac{(b)_n}{(1-m_2)_n n!}\, {}_2\mathrm{F}_1\left(b+n, b+m_1; 1+b; -\frac{a\bar{\gamma}_1}{m_1}\right) + \frac{Cm_2}{\bar{\gamma}_2}\frac{\Gamma(-m_2)(m_2+b)_n}{\Gamma(m_2)(1+m_2)_n n!}\, {}_2\mathrm{F}_1\left(m_2+b+n, b+m_1; 1+b; -\frac{a\bar{\gamma}_1}{m_1}\right)\right). \quad (13)$$

where Υ is given (26). After performing the necessary substitutions in (26), the BEP expression can be derived as shown in (13)[2]. Note that even though the BEP expression in (4) seems simpler than (13), (13) involves common simple functions and thus has the advantage of being directly computable using mathematical software packages.

B. M-Ary Modulations

An arbitrary rectangular $M_I \times M_J$ QAM signal constellation is assumed to be formed by drawing the in-phase and quadrature components from two independent M-ary pulse amplitude modulation (PAM) schemes, M_I-ary PAM and M_J-ary PAM, respectively. The symbol error probability of the ensuing M-ary rectangular QAM ($M = M_I M_J$) is

$$\begin{aligned} P_e = {} & 2(1-\tfrac{1}{M_I})\mathrm{E}\left(Q(A\sqrt{\gamma})\right) + 2(1-\tfrac{1}{M_J})\mathrm{E}\left(Q(B\sqrt{\gamma})\right) \\ & -4(1-\tfrac{1}{M_I})(1-\tfrac{1}{M_J})\mathrm{E}\left(Q(A\sqrt{\gamma})Q(B\sqrt{\gamma})\right), \end{aligned} \quad (14)$$

where $A = \sqrt{6/[(M_I^2-1)+(M_J^2-1)\zeta]}$ and $B = \sqrt{\zeta}A$ where ζ denotes the squared quadrature to in-phase distance ratio. It is seen that the evaluation of (14) involves the evaluations of two expectation forms, namely, the expectation of the Gaussian-Q function and the expectation of the product of two Gaussian-Q functions with different arguments.

Theorem 2: For the system under study, the expectation of the gaussian Q function can be written according to

$$\mathrm{E}(Q(A\sqrt{\gamma})) = \frac{1}{2} - \frac{A\sqrt{\frac{a\bar{\gamma}_1\bar{\gamma}_2}{Cm_1m_2}}}{2\sqrt{2\pi}\Gamma(m_1)\Gamma(m_2)} \times \left(\mathrm{G}_{3,2}^{1,3}\left(\frac{A^2\bar{\gamma}_1\bar{\gamma}_2}{2Cm_1m_2} \,\middle|\, \begin{matrix} \frac{1}{2}-m_1, \frac{1}{2}-m_2, \frac{1}{2} \\ 0, -\frac{1}{2} \end{matrix}\right) - \mathrm{G}_{2,1,0,2}^{2,1,1,1,1}\left(\begin{matrix} \frac{A^2\bar{\gamma}_1\bar{\gamma}_2}{2Cm_1m_2} \\ \frac{\bar{\gamma}_2}{Cm_2} \end{matrix} \,\middle|\, \begin{matrix} \frac{1}{2}-m_2, \frac{1}{2} \\ \frac{1}{2}+m_1, 0 \\ \frac{m_2}{2}+\frac{1}{2} \\ 0, -\frac{1}{2}, 1, 0 \end{matrix}\right)\right). \quad (15)$$

Proof: : The result immediately follows by using the relation

$$\mathrm{E}(Q(A\sqrt{\gamma})) = \frac{\Gamma(\frac{1}{2}, \frac{A^2\gamma}{2})}{2\Gamma(\frac{1}{2})}. \quad (16)$$

Accordingly, (15) follows from the development in the preceding section. By substituting A with B in (15), we obtain the closed-from solution for the second expectation with argument B in (14). Nevertheless, there are some challenges for the evaluation of the expectation of the product of two Gaussian Q-functions with different arguments, a process which involves the integration of the product of two Gaussian Q-functions with different arguments. In [19], the authors sidestepped this hurdle by introducing a simple and accurate exponential approximation of the product of two Gaussian Q-functions with different arguments given by

$$Q(A\sqrt{\gamma})Q(B\sqrt{\gamma}) \simeq \sum_{i,j=1}^{2} c_i c_j e^{-(A^2 b_i + B^2 b_j)\gamma}, \quad (17)$$

where $\{c_i\} = \{1/12, 1/4\}$ and $\{b_i\} = \{1/2, 2/3\}$. Based on the above approximation and proceeding by using the McLauren series of $e^{(\cdot)}$ given in [15, Eq. (1.211.1)], it can be easily shown, using [15, Eq. (8.402)], that the expectation of the product of two Gaussian Q-functions with different arguments can be expressed as

$$\mathrm{E}\left(Q(A\sqrt{\gamma})Q(B\sqrt{\gamma})\right) \simeq -\frac{\left(\frac{Cm_2}{\bar{\gamma}_2}\right)^{m_2}\bar{\gamma}_1}{\Gamma(m_2)} \sum_{u,v=1}^{2} c_u c_v d_{uv} \int_0^\infty s^{m_2} e^{-s}\, {}_1\mathrm{F}_1\left(m_1+1, 2, -\frac{d_{uv}\bar{\gamma}_1}{m_1}s\right) K_{m_2}\left(2\sqrt{\frac{Cm_2 s}{\bar{\gamma}_2}}\right) ds \quad (18)$$

where $d_{ij} = A^2 b_i + B^2 b_j$. By following the same rationale employed in (4), we obtain

$$\mathrm{E}\left(Q(A\sqrt{\gamma})Q(B\sqrt{\gamma})\right) \simeq -\frac{\left(\frac{Cm_2}{\bar{\gamma}_2}\right)^{m_2}\frac{\bar{\gamma}_1}{m_1}}{2\Gamma(m_2)\Gamma(m_1)} \sum_{u,v=1}^{2} c_u c_v d_{uv} \times \left(\mathrm{G}_{3,2}^{1,3}\left(\frac{d_{uv}\bar{\gamma}_1\bar{\gamma}_2}{Cm_1m_2} \,\middle|\, \begin{matrix} -m_1, -m_2, 0 \\ 0, -1 \end{matrix}\right) - \mathrm{G}_{2,1,0,2}^{2,1,1,1,1}\left(\begin{matrix} \frac{d_{uv}\bar{\gamma}_1\bar{\gamma}_2}{Cm_1m_2} \\ \frac{\bar{\gamma}_2}{Cm_2} \end{matrix} \,\middle|\, \begin{matrix} -m_2, 0 \\ 1+m_1, 0 \\ \frac{m_2}{2}+1 \\ 0, -1, 1, 0 \end{matrix}\right)\right). \quad (19)$$

[2] Note that (13) is only valid for real-valued non-integer m_2. Nevertheless, practically, very similar results can be obtained at m_2 and $m_2 + \epsilon$ for sufficiently small ϵ values.

It is observed that employing (17) allows to evaluate (18) in closed-form. Moreover, owing to the structure of (17), the obtained formulas, i.e. (15) and (19), have similar structures. This fact facilitates further numerical calculations. Then, by properly substituting (15) and (19) into (14) yields the error probability expression for an arbitrary M-QAM modulation used along dual-hop AF transmission over Nakagami-m fading.

Similar to *lemma 1*, the error probability function of an arbitrary rectangular $M_I \times M_J$ QAM signal when $m_2 \in R\backslash N$ can be derived using (26). The final formulas are not reported for the sake of conciseness.

It is noteworthy that the developed EP framework constitutes a nontrivial and useful add-on of the framework proposed [9] and [10] dealing with the error probability of fixed gain relaying systems over Nakagami-m fading when m is an integer. Moreover, we would like to emphasize that, although not explicitly shown in this paper, there are several cases of interest in which truly closed-form results may be obtained by using the frameworks proposed in this paper. For example, there are several scenarios in which the generalized moments of the first hop SNR and the MGF of second-hop SNR can conveniently be expressed in terms of a Meijer's G-function. In these scenarios, closed-form results for the error probability of several modulation schemes might readily be computed by using the Mellin-Barnes theorem. This comment further corroborates the usefulness of the analytical framework proposed in this paper.

IV. NUMERICAL RESULTS AND DISCUSSIONS

In this section, numerical examples for the above analysis are presented and corroborated by simulation results. Fig.1 depicts the BEP as a function of the average SNR per hop with $\bar{\gamma_1} = \bar{\gamma_2}$. BPSK modulation is applied in the simulations, that is, $a = 1$ and $b = 0.5$ in (4) and (13) where we truncate the infinite series at $N = 6$. It is observed that the analytical results match perfectly with the simulation and numerical results. As expected, the BEP performance improves with increasing values of m_i. On the other hand, it is observed that the curve with $(m_1 = 1.3, m_2 = 1.7)$ has the same slope as the curve with $(m_1 = 1.3, m_2 = 2.5)$, in contrast to the curve with $(m_1 = 2, m_2 = 1.7)$ who has a steeper slope. This implies that the weakest hop dominates the bit/symbol error rate performance, and that better performance is attained when severe fading conditions characterize the R-D link. This is because the gain applied to the signal depends on the statistics of the first hop only. In Fig.2 we highlight the effect of the truncation order N on the accuracy of the EP. For that purpose, we plot the performance of a QPSK modulation for different sets of Nakagami-m fading parameters. It is observed that the infinite series using (26) converges steadily and rapidly to the exact values, especially in the high SNR regions. Exhaustive tests have been carried out by the authors and, in all of them, $N = 6$ was sufficient to achieve the concordance.

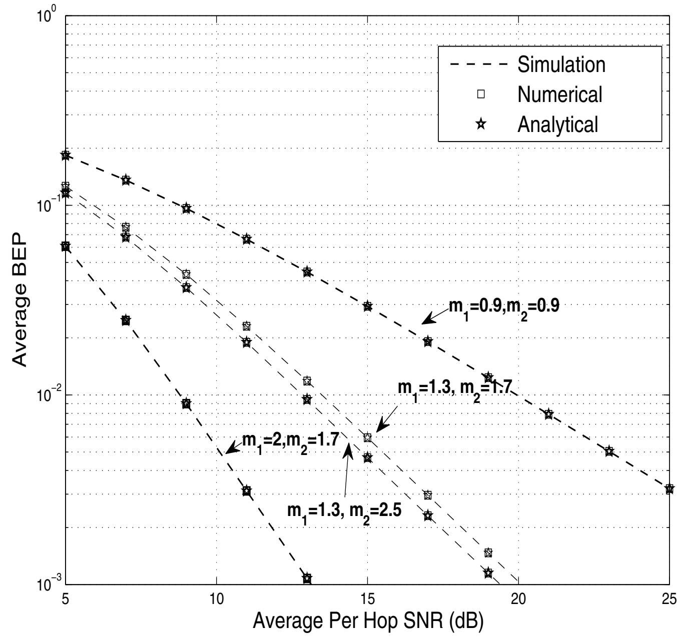

Fig. 1. Average BEP of the dual-hop fixed-gain AF relaying transmission

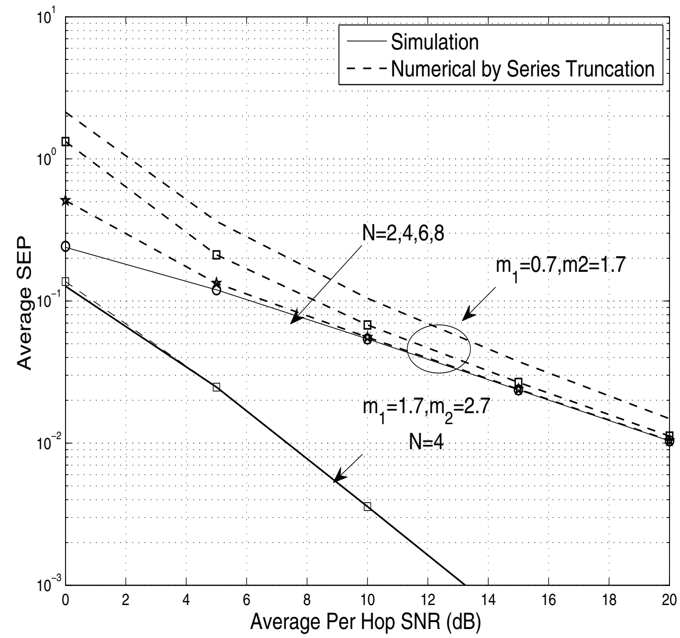

Fig. 2. Average SEP of dual-hop fixed-gain relaying transmission with QPSK modulation for different values of the truncation order N and different sets of the fading parameters m_i, $i = 1, 2$.

V. CONCLUDING REMARKS

In this paper, we have developed a comprehensive framework for the error probability analysis of dual-hop fixed-gain relying systems over general Nakagami-m channels with arbitrary fading parameters. The proposed framework can handle various modulation schemes and can be extended to various fading distributions, provided that the MGF of the second hop inverse SNR is known in closed form. All analytical results were demonstrated to offer efficient tools to accurately evaluate the AF system performance over arbitrary Nakagami fading channels.

VI. Appendix

Consider the integral of the form

$$\Upsilon = \int_0^\infty s^\nu e^{-s}{}_1\mathrm{F}_1\left(a;b;cx\right)K_\lambda\left(\sqrt{\beta x}\right)dx. \tag{20}$$

When λ takes non-integer values, the following equality holds

$$\begin{aligned}K_\lambda(z) &= 2^{-\lambda-1}\Gamma(-\lambda)z^\lambda \lim_{d\to\infty}{}_1\mathrm{F}_1(d;1+\lambda,\frac{z^2}{4d})+\\ &\quad 2^{\lambda-1}\Gamma(\lambda)z^{-\lambda}\lim_{d\to\infty}{}_1\mathrm{F}_1(d;1-\lambda,\frac{z^2}{4d}).\end{aligned} \tag{21}$$

Substituting (21) into (20), yields

$$\begin{aligned}\Upsilon &= \frac{1}{2}\lim_{d\to\infty}\Bigg(\left(\frac{\sqrt{\beta}}{2}\right)^\lambda\Gamma(-\lambda)\\ &\int_0^\infty s^{\nu+\frac{\lambda}{2}}e^{-s}{}_1\mathrm{F}_1\left(a,b,cx\right){}_1\mathrm{F}_1\left(d,1+\lambda,\frac{\beta x}{4d}\right)ds+\left(\frac{\sqrt{\beta}}{2}\right)^{-\lambda}\\ &\Gamma(\lambda)\int_0^\infty s^{\nu-\frac{\lambda}{2}}e^{-s}{}_1\mathrm{F}_1\left(a,b,cx\right){}_1\mathrm{F}_1\left(d,1-\lambda,\frac{\beta x}{4d}\right)ds\Bigg).\end{aligned} \tag{22}$$

Then, evaluating the integrals in the preceding expression, Υ can be rewritten according to

$$\begin{aligned}\Upsilon &= \frac{1}{2}\lim_{d\to\infty}\Bigg(\left(\frac{\sqrt{\beta}}{2}\right)^\lambda\Gamma(-\lambda)\Gamma(\nu+\frac{\lambda}{2}+1)\\ &\mathrm{F}^{(2)}\left(\nu+\frac{\lambda}{2}+1,a,d,b,1-\lambda,cx,\frac{\beta x}{4d}\right)+\left(\frac{\sqrt{\beta}}{2}\right)^{-\lambda}\Gamma(\lambda)\\ &\Gamma(\nu-\frac{\lambda}{2}+1)\mathrm{F}^{(2)}\left(\nu-\frac{\lambda}{2}+1,a,d,b,1+\lambda,cx,\frac{\beta x}{4d}\right)\Bigg),\end{aligned} \tag{23}$$

where $F^{(2)}(\cdot;\cdot;\cdot;\cdot)$ is the Appell hypergeometric function of the second kind [14, Eq. (6.4.1)]. As for the limit operation involved in (23), we recall the series expansion of $F^{(2)}(\cdot;\cdot;\cdot;\cdot)$ [14, Eq. (2.1.1)] given by

$$\begin{aligned}\mathrm{F}^{(2)}\left(u,a_1,a_1,b_1,b_2;x_1,x_2\right) &= \sum_{n=0}^\infty\frac{(u)_n(a_2)_n}{(b_2)_n}x_2^n\\ &\quad\times{}_2\mathrm{F}_1\left(u+n,a_1,b_1;x_1\right),\end{aligned} \tag{24}$$

whereby ${}_2\mathrm{F}_1(a,b;c;z)$ stands for the Gauss hypergeometric function [15, Eq. (9.100)] and $(a)_n$ is the Pochhammer symbol. Then, applying

$$\lim_{\lambda\to\infty}\left\{(\lambda)_n\left(\frac{z}{\lambda}\right)^n\right\}=z^n \qquad n\in N,|z|<\infty, \tag{25}$$

and after some manipulations, it follows that

$$\begin{aligned}\Upsilon &= \frac{1}{2}\Bigg(\left(\frac{\sqrt{\beta}}{2}\right)^\lambda\Gamma(-\lambda)\sum_{n=0}^\infty\frac{\Gamma(\nu+\frac{\lambda}{2}+1)}{(1-\lambda)_n}(\frac{\beta x}{4})^n\\ &\quad\times{}_2\mathrm{F}_1\left(\nu+\frac{\lambda}{2}+1+n,a;b;cx\right)\\ &\quad+\left(\frac{\sqrt{\beta}}{2}\right)^{-\lambda}\Gamma(\lambda)\sum_{n=0}^\infty\frac{\Gamma(\nu-\frac{\lambda}{2}+1)}{(1+\lambda)_n}(\frac{\beta x}{4})^n\\ &\quad\times{}_2\mathrm{F}_1\left(\nu-\frac{\lambda}{2}+1+n,a;b;cx\right)\Bigg).\end{aligned} \tag{26}$$

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