Cooperative Two-Way Selective Relaying in Spectrum-Sharing Systems with Distributed Beamforming*

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***Abstract*—We consider in this paper distributed beamforming for two-way cognitive radio networks in an effort to improve the spectrum efficiency and enhance the performance of the cognitive (secondary) system. In particular, we consider a spectrum sharing system where a set of decode-and-forward (DF) relays are employed to help a pair of secondary transceivers in the presence of multiple licensed (primary) users. Among the available relays, only those that receive the signals reliably participate in the beamforming process, where the optimal beamformer weights are obtained via a linear optimization method. We derive closed-form expression for the probability distribution function (PDF) of the total end-to-end signal-to-noise ratio (SNR) at the secondary transceiver. We also derive closed-form expressions for the outage and error probabilities over independent and identically distributed (i.i.d.) Rayleigh fading channels. Numerical results show the effect of beamforming in enhancing the secondary system performance in addition to mitigating the interference to the primary users.**

I. Introduction

The notion of spectrum sharing is a promising solution for enhancing the utilization of the spectrum efficiency of wireless systems. To this end, spectrum sharing systems allow secondary users (SUs) to get access to the primary spectrum while adhering to the interference limitations to the latter [1], [2]. On the other side, employing the concept of cooperative relaying in spectrum-sharing systems has received considerable interest due to its vitality to ensure reliable transmission for the secondary systems [3]- [6]. In the literature, while cooperative one-way relaying in cognitive radio networks (CRNs) and non-CRNs are studied to a great extent, two-way relaying in spectrum sharing environments are rarely considered [7]- [12]. Authors in [10] derive the outage probability performance for a cooperative two-way decode-and-forward (DF) relaying system where a PU helps two secondary transceivers to communicate with each other. However, in [10], the interference from the SUs inflicted on the PUs is only limited by imposing constraints on the transmitted powers.

Beamforming is an alternative technique to alleviate the inflicted interference in spectrum-sharing systems, which has recently received great interest [11]– [13](and references therein). For instance, in [11], an iterative alternating optimization-based algorithm has been developed to obtain the optimal beamforming weights in order to maximize the worst signal to interference noise ratio. In [12], convex optimization tools were used to find the sub-optimal beamformers in relay assisted CRNs. However, these algorithms suffer from high computational complexity. In contrast, Zero forcing beamforming (ZFB) is a simpler sub-optimal approach that can be practically used. A ZFB approach is normally applied to improve the secondary system performance in a dual-hop underly CR scenario [3], [5]. However, all these works consider one-way relaying. We remark, however, that the authors in [13] optimize the beamforming coefficients in a cognitive two-way relaying system using iterative semidefinite programming and bisection search methods.

Motivated by the great potential of combining two-way relaying and beamforming, we tackle in our work these obstacles by using a collaborative distributed ZFB in two-way DF relaying in a spectrum sharing environment. In particular, we consider a CRN comprising two secondary sources communicating with each other in three consecutive time slots, a number of secondary DF relays and a number of PUs. The relays that can correctly decode both of the received signals (from the two sources) or at least one of them are used for relaying in the third time-slot. That is, the selected relays employ distributed ZFB to null the inflicted interference to the PUs in addition to improving the performance of the secondary system. Besides that, the interference from the secondary sources is limited by imposing peak power constraints on the interference received at the PUs. To analyze the performance, we derive the probability distribution function (PDF) of the end-to- end total received signal-to-noise ratio (SNR). Exploiting these statistics, closed-form expressions for the outage and error probabilities are evaluated. We show that the proposed distributed ZFB approach has the potential of improving the secondary system performance and limiting the interference in a simple practical manner compared to other iterative complex approaches [13].

The rest of this paper is organized as follows. Section

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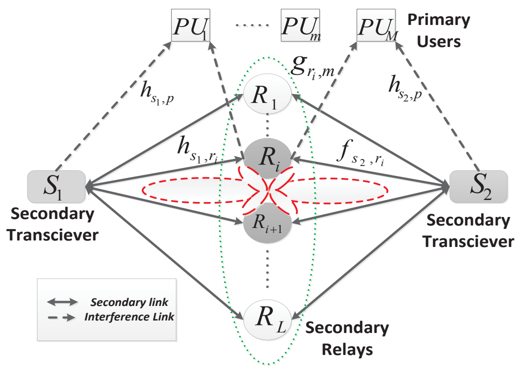

Fig. 1: Spectrum-sharing system with two-way DF relaying.

II describes the system model. The performance analysis is analyzed in Section III while Section IV describes the numerical results. Finally, Section V concludes the paper. Throughout this paper, the Frobenius norm of the vectors are denoted by $||.||$, The Transpose and the Conjugate Transpose operations are denoted by $(.)^T$ and $(.)^\dagger$, respectively. $|x|$ means the magnitude of a complex number x. $\mathcal{CN} \sim (0,1)$ refers to a complex Gaussian normal random variable with zero-mean and unit variance.

II. System and Channel Models

We consider a two-way relaying system that is composed of two secondary transceivers S_j, $j = 1, 2$ and a set of L DF secondary relays denoted by R_i for $i = 1, .., L$ coexisting in the same spectrum band with M primary receivers (PUs) as shown in Fig. 1. All nodes are equipped with one antenna.The two sources wish to communicate with each other in a half-duplex way. There is no direct link between the sources and thus they only can exchange messages via relay nodes using time division broadcast (TDBC) protocol during three time slots $(\text{TS}_i), i = 1, 2, 3$ [9]. The SUs are allowed to share the same frequency spectrum with the PUs as long as the interference to the PUs is limited to a predefined threshold. Both systems transmit simultaneously in an underlay manner. In TS_1, based on the interference channel state information (CSI) from S_1 to the m^{th} PU, which suffers the most interference caused by S_1, S_1 adjusts its transmit power under a predefined threshold Q_1 and broadcasts its message to the set of relays. Similarly, in TS_2, S_2 transmits its message to all relays under a tolerable threshold Q_2. In TS_3, ZFB is applied to null the interference from the selected potential relays L_s (which decided to participate) to the PUs so that the relays are always able to transmit without interfering with the PUs. Two ZFB weight vectors, namely $\mathbf{w_{zf_1}}$ and $\mathbf{w_{zf_2}}$ are optimized so as to maximize the received SNRs at S_1 and S_2 respectively, while nulling the inflicted interference to the existing PUs.
All channel coefficients are assumed to be independent Rayleigh flat fading and quasi-static, so that the channel gains remain unchanged during the transmission period, i.e., for example $|h_{s_j,r_i}|^2 = |h_{r_i,s_j}|^2$. Let h_{s_1,r_i}, f_{s_2,r_i} denote the channel coefficient from the sources S_1 and S_2 to the i^{th} relay respectively, which are modeled as a zero mean, circularly symmetric complex Gaussian (CSCG) random variable with variance λ_{s_1,r_i},λ_{s_2,r_i}. Denote $h_{s_1,p}$ and $h_{s_2,p}$ as the interference channel coefficients from S_1 and S_2 to the m^{th} PU, respectively, and their respective channel power gains are $|h_{s_1,p}|^2$ and $|h_{s_2,p}|^2$, which are exponentially distributed with parameter $\lambda_{s_1,p}$ and $\lambda_{s_2,p}$. It is assumed that S_1 and S_2 have perfect knowledge of their interference channel power gains which can be acquired through a spectrum-band manager that mediates between the primary and secondary users [4]. To be able to implement beamforming, local CSI is required at every node i.e. S_1 and S_2 need to obtain the CSI of all $(S_1, S_2 - R_i)$ channels. Each relay needs to obtain the CSI of its own $(R_i - S_1, S_2)$ channels. The interference from the primary transmitter is neglected and can be represented in terms of noise when its message is generated by random Gaussian codebooks [4]. As mentioned before, the communication process occurs over three time-slots. In TS_1, S_1 broadcasts its data to all L relays, then the received signal at the i^{th} relay is given as

$$y^1_{r_i} = \sqrt{P_1} h_{s_1,r_i} x_{s_1} + n_1, \tag{1}$$

where P_1 is S_1 transmit power, x_{s_1} is the information symbol of S_1 with $\text{E}[|x_{s_1}|^2] = 1$ and n_1 denotes the zero-mean CSCG noise at the i^{th} relay with variance σ^2 in TS_1. In TS_2, similarly, S_2 also broadcasts its data to all L relays, and the received signal at the i^{th} relay is

$$y^2_{r_i} = \sqrt{P_2} f_{s_2,r_i} x_{s_2} + n_2, \tag{2}$$

where P_2 is the S_2 transmit power, x_{s_2} is the S_2 symbol with $\text{E}[|x_{s_2}|^2] = 1$ and n_2 denotes the zero-mean CSCG noise at the i^{th} relay with variance σ^2 in TS_2. In TS_3, the decoding set $\mathcal{C}$ which consists of the relays that can correctly decode x_{s_1} and/or x_{s_2} by using cyclic redundancy codes transmits to both sources simultaneously by using beamforming. The relay combines linearly the received signals from TS_1 and TS_2. If R_i decodes both x_{s_1} and x_{s_2}, it forwards the sum $(w_{i,2}x_{s_1} + w_{i,1}x_{s_2})$ to both of transceivers. If R_i correctly decodes x_{s_1} or x_{s_2}, it only broadcasts $(w_{i,2}x_{s_1})$ or $(w_{i,1}x_{s_2})$. Otherwise, R_i keeps silent and does not transmit any signal. Then the received signal at S_2 in a vector form

$$\begin{aligned} y^3_{S_2} &= \sum_{i=1}^{N_1} \sqrt{P_r} f_{r_i,s_2} (w_{i,2}x_{s_1} + w_{i,1}x_{s_2}) \\ &+ \sum_{i=N_1+1}^{N_1+N_2} \sqrt{P_r} f_{r_i,s_2} w_{i,2} x_{s_1} \\ &+ \sum_{i=N_1+N_2}^{N_1+N_2+N_3} \sqrt{P_r} f_{r_i,s_2} w_{i,1} x_{s_2} + n_{s_2}, \end{aligned} \tag{3}$$

where P_r is total power available at the relays, and without loss of generality, N_1 is the number of relays that decode both of signals, N_2 is the number of relays that decode only x_{s_1}, and N_2 is the number of relays that decode only x_{s_2}, so that the cardinality of $\mathcal{C}$, denoted as $|\mathcal{C}|$, equals $|\mathcal{C}| = L_s = N_1 + N_2 + N_3$, and n_{s_2} denotes the zero-mean CSCG noise at S_2 with variance σ^2. Removing the self-interference terms, S_2 obtains a desired signal as follows

$$\begin{aligned} y_{S_2}^3 &= \sum_{i=1}^{N_1} \sqrt{P_r} f_{r_i,s_2} w_{i,2} x_{s_1} \\ &+ \sum_{i=N_1+1}^{N_1+N_2} \sqrt{P_r} f_{r_i,s_2} w_{i,2} x_{s_1} + n_{s_2}. \end{aligned} \tag{4}$$

Therefore, the corresponding total received SNR at S_2 given $\mathcal{C}$, denoted , $\gamma_{tot_2|\mathcal{C}}$ is given by

$$\gamma_{tot_2|\mathcal{C}} = \frac{P_r |\mathbf{f}^\dagger \mathbf{w_{zf_2}}|^2}{\sigma^2}. \tag{5}$$

Similarly, the corresponding total received SNR at S_1 given $\mathcal{C}$, denoted , $\gamma_{tot_1|\mathcal{C}}$ is given by

$$\gamma_{tot_1|\mathcal{C}} = \frac{P_r |\mathbf{h}^\dagger \mathbf{w_{zf_1}}|^2}{\sigma^2}. \tag{6}$$

where $\mathbf{w_{zf_1}}^T = [w_{11}, w_{12}, ...w_{1(N_1+N_2)}, ..., w_{1L_s}]$ used to direct the signal to S_1 and $\mathbf{w_{zf_2}}^T = [w_{21}, w_{22}, ...w_{2(N_1+N_3)}, ..., w_{2L_s}]$ used to direct the signal to S_2. $\mathbf{h}^T = [h_{r_1,s_1}, ..., h_{r_{N_1},s_1}, \mathbf{0}_{1\times(N_2)}, h_{r_{(N_1+N_2)},s_1}, ..h_{r_{L_s},s_1}]$, and $\mathbf{f}^T = [f_{r_1,s_2}, ..., f_{r_{N_1+N_2},s_2}, \mathbf{0}_{1\times(L_s-(N_1+N_2))}]$. As the analysis at both transceivers are similar, hereafter, we only consider S_2.

A. Mathematical Model and Size of $\mathcal{C}$

In the underlay approach of this model, the secondary source can utilize the PU's spectrum as long as the interference it generates at the PUs remains below the interference threshold Q_j. For that reason, P_j is constrained as $P_j \leq \min\left\{\frac{Q_j}{|h_{s_j,p}|^2}, P_{s_j}\right\}$ where P_{s_j} is the maximum transmission power of S_j, $\forall j = 1, 2$ [5].

So the received SNR γ_{s_2,r_i} at the i^{th} relay is given as

$$\gamma_{s_2,r_i} = \begin{cases} \frac{P_{s_2}|f_{s_2,r_i}|^2}{\sigma^2}, & P_{s_2} < \frac{Q_2}{|h_{s_2,p}|^2} \\ \frac{Q_2|f_{s_2,r_i}|^2}{\sigma^2|h_{s_2,p}|^2}, & P_{s_2} \geq \frac{Q_2}{|h_{s_2,p}|^2} \end{cases}, \tag{7}$$

where σ^2 is the noise variance at each relay. Firstly, we find the CDF of γ_{s,r_i} as

$$\begin{aligned} \Pr(\gamma_{s_2,r_i} < \gamma) &= \Pr(P_{s_2}|f_{s_2,r_i}|^2 < \sigma^2\gamma, P_{s_2} < \frac{Q_2}{|f_{s_2,p}|^2}) \\ &+ \Pr(\frac{Q_2|f_{s_2,r_i}|^2}{|h_{s_2,p}|^2} < \sigma^2\gamma, P_{s_2} \geq \frac{Q_2}{|h_{s_2,p}|^2}). \end{aligned}$$

Performing the integration, the CDF of γ_{s_2,r_i} is given as [5]

$$F_{\gamma_{s_2,r_i}}(\gamma) = 1 - e^{\frac{-\lambda_{s_2,r_i}\gamma}{\gamma_s}} + \frac{\lambda_{s_2,r_i}\gamma e^{-\frac{\lambda_{s_2,r_i}\gamma + \frac{\lambda_{s_2,p}Q_2}{\sigma^2}}{\gamma_s}}}{\lambda_{s_2,r_i}\gamma + \lambda_{s_2,p}Q_2}, \tag{8}$$

where $\gamma_s = \frac{P_{s_2}}{\sigma^2}$. By differentiating $F_{\gamma_{s_2,r_i}}(\gamma)$ with respect to γ, we get the PDF of γ_{s_2,r_i} as

$$\begin{aligned} f_{\gamma_{s_2,r_i}}(\gamma) &= \frac{\lambda_{s_2,r_i}}{\gamma_s} e^{\frac{-\lambda_{s_2,r_i}\gamma}{\gamma_s}} - \frac{\lambda_{s_2,r_i}}{\gamma_s} e^{-\frac{\lambda_{s_2,r_i}\gamma + \lambda_{s_2,p}Q_2}{\gamma_s}} \\ &\times \left(\frac{\lambda_{s_2,p}Q_2(\lambda_{s_2,r_i}\gamma + \lambda_{s_2,p}Q_2 + \gamma_s)}{(\lambda_{s_2,r_i}\gamma + \lambda_{s_2,p}Q_2)^2} - 1\right) \end{aligned} \tag{9}$$

As stated before, we define $\mathcal{C}$ to be the set of relays which can correctly decode both of signals or at least one of them, hence, the probability $\Pr[|\mathcal{C}| = L_s]$ becomes

$$\Pr[|\mathcal{C}| = L_s] = \binom{L}{L_s} P_{\text{off}}^{L-L_s}(1 - P_{\text{off}})^{L_s}, \tag{10}$$

where P_{off} denotes the probability that the relay does not decode correctly any signal and keeps silent in the third time-slot. Let P_{e_1} and P_{e_2} be the error probability at any R_i for a signal transmitted from S_1 and S_2, respectively, then P_{off} is computed as

$$P_{\text{off}} = P_{e_1} P_{e_2}. \tag{11}$$

The probability that the i^{th} relay decodes incorrectly the signal transmitted from S_2 can be computed as [14]

$$\begin{aligned} P_{e_2} &= \int_0^\infty a\mathcal{Q}(\sqrt{b\gamma}) f_{\gamma_{s_2,r_i}}(\gamma) d\gamma \\ &= \int_0^\infty a\mathcal{Q}(\sqrt{b\gamma}) \left\{ \frac{\lambda_{s_2,r_i}}{\gamma_s}(1 - e^{-\frac{\lambda_{s_2,p}Q_2}{\gamma_s}}) e^{\frac{-\lambda_{s_2,r_i}\gamma}{\gamma_s}} \right. \\ &+ e^{-\frac{\lambda_{s_2,r_i}\gamma + \lambda_{s_2,p}Q_2}{\gamma_s}} \lambda_{s,r_i}\lambda_{s,p}Q_2 \\ &\times \left(\frac{1}{\gamma_s(\lambda_{s_3,r_i}\gamma + \lambda_{s_2,p}Q_2)} \right. \\ &+ \left. \left. \frac{1}{(\lambda_{s_2,r_i}\gamma + \lambda_{s_2,p}Q_2)^2} \right) \right\} d\gamma, \end{aligned} \tag{12}$$

where the constants (a, b) are chosen according to the type of modulation and $\mathcal{Q}$ is the Q-function defined as $\frac{1}{\sqrt{2\pi}} \int_u^\infty e^{-x^2/2} dx$. After distributing the integral, utilizing the closed-form expression derived in [15, Appendix 5A] to slove the first integral of (12) and also making use of [14, Equ. 11] to evalute the second integral of (12), P_{e_2} results in

$$\begin{aligned} P_{e_2} &= a\left(\frac{1}{2} - \frac{e^{-\frac{\lambda_{s_2,p}Q_2}{\gamma_s}}}{2}\right)\left[1 - \sqrt{\frac{b}{b + 2\frac{\lambda_{s_2,r_i}}{\gamma_s}}}\right] \\ &+ \frac{ae^{-\frac{\lambda_{s_2,p}Q_2}{\gamma_s}}}{2}\left(1 - \frac{\sqrt{1-\mu}}{\mu}\Psi\left(1; \frac{3}{2}; \frac{\lambda_{s_2,p}Q_2}{\mu\gamma_s}\right)\right), \end{aligned} \tag{13}$$

where $\mu = \frac{\lambda_{s_2,r_i}}{b\gamma_s + 1}$ and $\Psi(x; y; z)$ is the confluent hypergeometric function of the second kind defined as [16, 9.211.4]. Similarly, P_{e_1} is computed with notations interchangeable, i.e. λ_{s_1,r_i} replaces λ_{s_2,r_i} and Q_1 replaces Q_2.

B. ZFB Weights Design

Our objective is to maximize the received SNRs at the two transceivers in order to enhance the performance of the secondary system while limiting the interference reflected on

PUs. Due to its simplicity and low complexity, ZFB approach is applied as an alternative for the optimal scheme. To be able to apply ZFB, the general assumption that the number of relays must be greater than the number of primary receivers is considered, hence, $L_s > M$.

Let $\mathbf{G}_{\mathbf{rp}}^T = [\mathbf{g}_{\mathbf{r},\mathbf{p_1}}, ..., \mathbf{g}_{\mathbf{r},\mathbf{p_M}}]$ where $\mathbf{g}_{\mathbf{r},\mathbf{p_m}} = [g_{r_1,p_m}, ..., g_{r_{L_s},p_m}]$ be the channel vectors between the relays and S_1, S_2 and between the relays and the M of PUs, respectively. According to the ZFB principles, the transmit weight vectors $\mathbf{w_{zf_1}}$, $\mathbf{w_{zf_2}}$ are chosen to lie in the orthogonal space of $\mathbf{G}_{\mathbf{rp}}^\dagger$ such that $|\mathbf{g}_{\mathbf{r},\mathbf{p_i}}^\dagger \mathbf{w_{zf_1}}| = 0$, $|\mathbf{g}_{\mathbf{r},\mathbf{p_i}}^\dagger \mathbf{w_{zf_2}}| = 0$, $\forall i = 1, .., M$ and $|\mathbf{h}^\dagger \mathbf{w_{zf_1}}|$, $|\mathbf{f}^\dagger \mathbf{w_{zf_2}}|$ are maximized. So the problem formulation for finding the optimal weight vectors is divided into two parts as follows.

$$\begin{aligned} \max_{\mathbf{w_{zf_1}}} \quad & |\mathbf{h}^\dagger \mathbf{w_{zf_1}}| \\ \text{s.t.:} \quad & |\mathbf{g}_{\mathbf{r},\mathbf{p_i}}^\dagger \mathbf{w_{zf_1}}| = 0, \quad \forall i = 1, .., M \\ & \|\mathbf{w_{zf_1}}\| = 1. \end{aligned} \tag{14}$$

$$\begin{aligned} \max_{\mathbf{w_{zf_2}}} \quad & |\mathbf{f}^\dagger \mathbf{w_{zf_2}}| \\ \text{s.t.:} \quad & |\mathbf{g}_{\mathbf{r},\mathbf{p_i}}^\dagger \mathbf{w_{zf_2}}| = 0, \quad \forall i = 1, .., M \\ & \|\mathbf{w_{zf_2}}\| = 1. \end{aligned} \tag{15}$$

To obtain the optimal weights, we consider the following Lemma from projection matrix theory [17].

Lemma 1: Let $\mathbf{G}$ be an $n \times k$ matrix with full column rank k, $k < n$, then the nonzero matrix $\mathbf{G}(\mathbf{G^H G})^{-1}\mathbf{G^H}$ is an idempotent symmetric matrix and its orthogonal projection matrix is $\mathbf{I} - \mathbf{G}(\mathbf{G^H G})^{-1}\mathbf{G^H}$ with rank $(n-k)$ [17, Theorems 4.21, 4.22].

By applying a standard Lagrangian multiplier method, the weight vectors that satisfy the above optimization methods are given as

$$\mathbf{w_{zf_1}} = \frac{\mathbf{\Xi}^\perp \mathbf{h}}{||\mathbf{\Xi}^\perp \mathbf{h}||}, \tag{16}$$

and

$$\mathbf{w_{zf_2}} = \frac{\mathbf{\Xi}^\perp \mathbf{f}}{||\mathbf{\Xi}^\perp \mathbf{f}||}, \tag{17}$$

where $\mathbf{\Xi}^\perp = \left(\mathbf{I} - \mathbf{G_{rp}}(\mathbf{G}_{\mathbf{rp}}^\dagger \mathbf{G_{rp}})^{-1}\mathbf{G}_{\mathbf{rp}}^\dagger\right)$ is the projection idempotent matrix with rank $(L_s - M)$.

Now, after finding the ZFB vectors, we substitute them into equations (5) and (6) to get

$$\gamma_{tot_1|\mathcal{C}} = \frac{P_r ||\mathbf{\Xi}^\perp \mathbf{h}||^2}{\sigma^2}, \tag{18}$$

$$\gamma_{tot_2|\mathcal{C}} = \frac{P_r ||\mathbf{\Xi}^\perp \mathbf{f}||^2}{\sigma^2}, \tag{19}$$

where we exploit the idempotent matrix property $(\mathbf{\Xi}^\perp)^2 = \mathbf{\Xi}^\perp$. To proceed, we need the following Lemma to find the CDF of $\gamma_{tot_2|\mathcal{C}}$.

Lemma 1:(CDF of $\gamma_{tot_2|\mathcal{C}}$): Let each entry of $\mathbf{f}$ be i.i.d. $\mathcal{CN} \sim (0, 1)$, then $\left\|\mathbf{\Xi}^\perp \mathbf{f}\right\|^2$ is a chi squared random variable with $2(L_s - M)$ degrees of freedom [18, theorem 2 Ch.1] then the CDF of $\gamma_{tot_2|\mathcal{C}}$

$$F_{\gamma_{tot_2|\mathcal{C}}}(\gamma) = 1 - \frac{\Gamma\left(L_s - M, \frac{\gamma}{\gamma_r}\right)}{\Gamma(L_s - M)}, \ \gamma \geq 0, \tag{20}$$

where $\gamma_r = \frac{P_r}{\sigma^2}$. Accordingly, the PDF of $f_{\gamma_{tot_2|\mathcal{C}}}(\gamma)$ is given as

$$f_{\gamma_{tot_2|\mathcal{C}}}(\gamma) = \frac{(\gamma)^{L_s - M - 1} e^{-\frac{\gamma}{\gamma_r}}}{\Gamma(L_s - M)(\gamma_r)^{L_s - M}}, \ \gamma \geq 0, \ L_s > M. \tag{21}$$

By using the theorem of total probability, the unconditional PDF of the total received SNR, denoted γ_{tot_2}, can be written as

$$\begin{aligned} f_{\gamma_{tot_2}}(\gamma) \quad = \quad & \sum_{L_s=0}^{M} \binom{L}{L_s} P_{\text{off}}^{L-L_s} (1 - P_{\text{off}})^{L_s} \delta(\gamma) \\ & + \sum_{L_s=M+1}^{L} \binom{L}{L_s} P_{\text{off}}^{L-L_s} (1 - P_{\text{off}})^{L_s} \\ & \times \left(\frac{(\gamma)^{L_s-M-1} e^{-\frac{\gamma}{\gamma_r}}}{\Gamma(L_s - M)(\gamma_r)^{L_s-M}} \right), \end{aligned} \tag{22}$$

where $\delta(.)$ is the dirc function that refers to received SNR when the relays are not active. $f_{\gamma_{tot_2}}(\gamma)$ is plotted in Fig. 2 for various values of L and M. It clarifies the effect of increasing the number of the selected relays on improving the total received SNR at S_2. As the number of relays increases, the curve shifts towards the right-side, which means that the probability to have a better received SNR for a certain channel condition becomes higher. On the opposite side, as M increases, the effect is reversed.

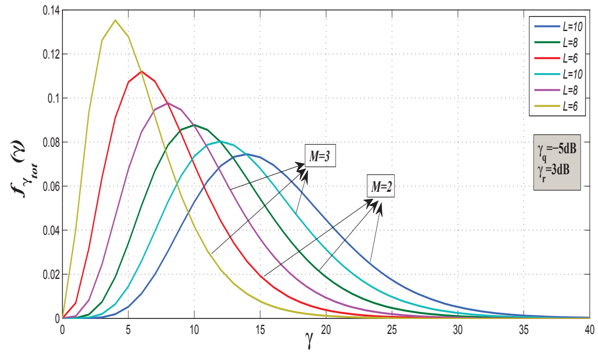

Fig. 2: PDF of the total received SNR at S_2, $f_{\gamma_{tot_2}}(\gamma)$.

III. Performance Analysis

A. Outage Probability Analysis

1. User Outage Probability

An outage event occurs when the total received SNR falls below a certain threshold γ_{th} and is expressed as $P_{out} = \Pr(\gamma_{tot_j} < \gamma_{th})$. There exist two exclusive outage events for the secondary system with distributed ZFB which are failing to apply ZFB when $L_s \leq M$, and failing to achieve the threshold when $L_s > M$. By using (20) and according to the total probability theorem, the user outage probability of S_2 can be written as

$$P_{out_2} = \sum_{L_s=0}^{M} \binom{L}{L_s} P_{\text{off}}^{L-L_s}(1-P_{\text{off}})^{L_s} + \sum_{L_s=M+1}^{L} \binom{L}{L_s} \times P_{\text{off}}^{L-L_s}(1-P_{\text{off}})^{L_s} \left\{ 1 - \frac{\Gamma\left(L_s - M, \frac{\gamma_{th}}{\gamma_r}\right)}{\Gamma(L_s - M)} \right\}. \tag{23}$$

For simpler and clearer expression, P_{out} can also be rewritten in terms of $1 - \Pr(\text{no outage})$ as

$$P_{out_2} = 1 - \sum_{L_s=M+1}^{L} \binom{L}{L_s} P_{\text{off}}^{L-L_s}(1-P_{\text{off}})^{L_s} \times \left(\frac{\Gamma\left(L_s - M, \frac{\gamma_{th}}{\gamma_r}\right)}{\Gamma(L_s - M)} \right). \tag{24}$$

2. System Outage Probability

Knowing that the two-way system is considered as a multiuser system, the system outage occurs when any user is in outage. Therefore, the two-way system is in outage when either of the received SNRs at S_1 and S_2 is smaller than a threshold, i.e.

$$P_{out}^{sys} = \Pr\left(\min(\gamma_{tot_1}, \gamma_{tot_2}) < \gamma_{th}\right). \tag{25}$$

Using the well-known result for independent events [19], the system outage is computed as

$$P_{out}^{sys} = P_{out_1} + P_{out_2} - P_{out_1} P_{out_2}, \tag{26}$$

where P_{out_1} and P_{out_2} are the user outage probabilities of S_1 and S_2, respectively, and can be calculated from (23).

B. Average Error Probability

We analyze the average error probability performance due to errors occurring at S_2 assuming that all participating relays have accurately decoded and regenerated the message. This probability could be evaluated by averaging the instantaneous error probability P_e over the PDF in (22). Since P_e depends on the modulation scheme, many expressions can be used. In this paper, we consider Binary Phase Shift Keying (BPSK) for which $P_e = \mathcal{Q}(\sqrt{2\gamma})$. After averaging this expression over the PDF in (22), $\bar{P}_e$ becomes [15, Eq.14-4-15]

$$\begin{aligned} \bar{P}_e &= 0.5 \sum_{L_s=0}^{M} \binom{L}{L_s} P_{\text{off}}^{L-L_s}(1-P_{\text{off}})^{L_s} \\ &+ \sum_{L_s=M+1}^{L} \binom{L}{L_s} P_{\text{off}}^{L-L_s}(1-P_{\text{off}})^{L_s} [\tfrac{1}{2}(1-\mu_o)]^{L_s-M} \\ &\times \sum_{k=0}^{L_s-M-1} \binom{L_s - M - 1 + k}{k} [\tfrac{1}{2}(1+\mu_o)]^k. \end{aligned} \tag{27}$$

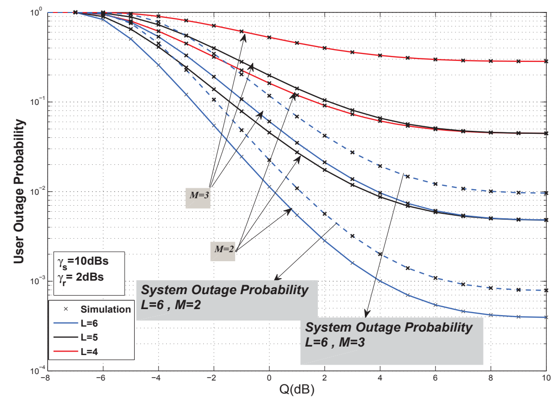

Fig. 3: Outage Probability vs. Q(dB) for different number of relays L=4,5,6 and M=2,3 PUs

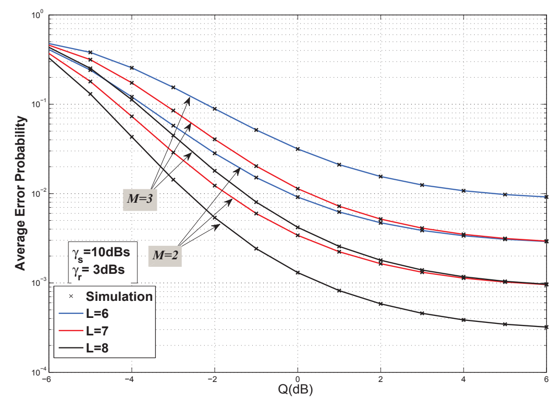

Fig. 4: Average error probability vs. Q(dB) for different number of relays L=6,7,8 and M=2,3 PUs

IV. Numerical Results and Discussions

In this section, we investigate the performance of some of the derived results through numerical evaluation and simulations. The two sources and the relays are located in straight line. The distance between the sources is normalized to one and the relays fall in the middle. We assume that $\lambda_{s_1,p} = \lambda_{s_2,p} = 0.5$, $\lambda_{s_1,r_i} = \lambda_{s_2,r_i} = 5$ and $Q_1 = Q_2 = Q$.

Fig. 3 shows the outage performance of SU S_2 versus the predefined threshold Q for different number of relays, $L = 4, 5, 6$ and different number of existing PUs $M = 2, 3$ at $\gamma_{th} = 1$, $\gamma_s = 10$ and $\gamma_r = 2$ dBs. As observed, as the values of Q become loose, the outage performance improves substantially. Moreover, by increasing the number of relays with ZFB approach applied, we observe significant improvement in the outage performance. It is attributed to the combined cooperative diversity and beamforming on enhancing the total received SNR at the receiver.

Fig. 4 illustrates the average error probability performance versus the tolerable interference threshold Q for different number of relays L=6, 7, 8, M=2, 3 and $\gamma_r = 3$ dBs at $\gamma_{th} = 1$dB. It is obvious that the average error probability performance

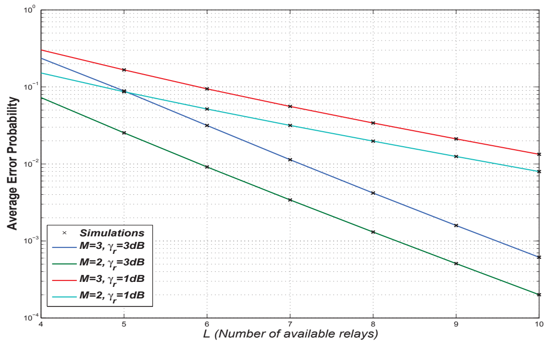

Fig. 5: Average error probability vs. L for different number of PUs M=2,3 and $\gamma_r = 1, 3$ dBs

improves substantially as the number of relays increases and Q becomes large. By beamforming and increasing the number of relays, the gain becomes higher. Clearly, in Figs. 2 and 3, the higher the number of existing PUs, the worse the outage and error probabilities. The figures also show that the performances saturate at high values of Q which is a result of the limitation on the maximum transmit power of the secondary transmitters.

Fig. 5 shows the average error probability performance versus the number of available relays L for different number of existing PUs $M = 2, 3$ and two values of $\gamma_r = 1, 3$ dBs. It is clear that there is an opposite effect on the system performance between the increasing of L and the increasing of M at the same value of γ_r. As L increases, it positively improve the error probability performance while as M increases, it negatively affect the performance. Regarding γ_r value, it refers to the effect of the total power of relays on the received SNR at the secondary transceiver. The higher γ_r, the higher the received SNR. Its value can be optimized to improve the performance, however, in our system, it is fixed since its optimization problem is beyond our interest in this paper.

V. Conclusion

We proposed a selective DF relaying system model in a two-way CRN that limits the interference to the primary system by imposing peak interference power constraints on transceivers transmit powers and applying a distributed ZFB method in the relaying phase. We analyzed the performance of the secondary system by deriving the outage and error probabilities. Our numerical results showed the benefits of our proposed system. Results showed that the distributed ZFB method improves the performances by increasing the number of participating relays in addition to limiting the interference to PUs.

References

[1] S. Haykin, "Cognitive radio: brain-empowered wireless communications," *IEEE J. Select. Areas Commun.,* vol. 23, no. 9, pp. 201−220, 2005.

[2] A. Goldsmith, S. A. Jafar, I. Maric, and S. Srinivasa, "Breaking spectrum gridlock with cognitive radios: an information theoretic perspective," *Proc. IEEE,* vol. 97, no. 5, pp. 894−914, May 2009.

[3] A. Afana, V. Asghari, A. Ghrayeb and S. Affes, "Enhancing the performance of spectrum-sharing systems via collaborative distributed beamforming and AF relaying," *Proc. IEEE GlobeCom*, Anaheim, USA, Dec., 2012.

[4] V. Asghari, S. Affes and A. Ghrayeb, "Reactive relay selection in cooperative spectrum-sharing systems" *Proc. IEEE WCNC'2012,* France.

[5] A. Afana, V. Asghari, A. Ghrayeb and S. Affes, "Cooperative relaying in spectrum-sharing systems with beamforming and interference constraints," *Proc. IEEE SPAWC'12,* Cesme, Turkey, June 2012.

[6] V. Asghari and S. Aissa, "Performance of cooperative relaying in spectrum-sharing systems with amplify and forward relaying" *IEEE Trans. Wireless Comm.,* vol.11, no.4, pp.1295−1300, April 2012.

[7] X. Zeng, A. Ghrayeb and M. Hasna, "Joint optimal threshold-based relaying and ML detection in network-coded two-way relay," *IEEE Trans. on Commun.,* vol.60, no.9, pp.2657−2667, Sep. 2012.

[8] G. Al-Habian, A. Ghrayeb, M. Hasna, and A. Abu-Dayya, "Threshold-based relaying in coded cooperative networks," *IEEE Trans. on Vehicular Tech.,* vol. 60, no. 1, pp. 123−135, January 2011.

[9] Z. Yi and I. Kim, "ptimum beamforming in the broadcasting phase of bidirectional cooperative communication with multiple decode-and-forward relays," *IEEE Trans. on Wireless Commun.,* vol.8,no.12, pp.5806−5812, Dec. 2009.

[10] Q. Li, S. H. Ting, A. Pandharipande, and Y. Han, "Cognitive spectrum sharing with two-way relaying systems," *IEEE Trans. Veh. Technol.,* vol. 60, no. 3, pp. 1233−1240, Mar. 2011.

[11] K. Zarifi, A. Ghrayeb and S. Affes, "Jointly optimal source power control and relay matrix design in multipoint-to-multipoint cooperative communication networks," *IEEE Trans. on Signal Process.,* vol. 59, no. 9, pp. 4313−4330, Sept. 2011.

[12] K. Hamdi, K. Zarifi, K. Ben Letaief and A. Ghrayeb, "Beamforming in Relay-Assisted Cognitive Radio Systems: A Convex Optimization Approach," *Proc. IEEE ICC,* Kyoto, Japan, pp.1−5, June 2011.

[13] S.H. Safavi, M. Ardebilipour and S. Salari, "Relay beamforming in cognitive two-way networks with imperfect channel state information," *IEEE Wireless Commun. Lett.*, vol.1, no.4, pp.344−347, August 2012.

[14] S. Ikki, S. Aissa, "Impact of imperfect channel estimation and co-channel interference on regenerative cooperative networks," *IEEE Wireless Commun. Lett.*, accepted, Jun. 2012.

[15] J. Proakis, *Digital Communications*, 4th ed. McGraw-Hill, 2000.

[16] I. S. Gradshteyn, I. M. Ryzhik, A. Jeffrey, and D. Zwillinger, *Table of integrals, series and products*, 7th ed. Elsevier, 2007.

[17] A. Basilevsky,*Applied Matrix Algebra in the Statistical Sciences,* New York, NY: North-Holland, 1983.

[18] R. J. Pavur, "Quadratic forms involving the complex Gaussian," M.Sc. dissertation, Math. Dept., Texas Tech. Univ., Lubbock, TX, Aug. 1980.

[19] A. Papoulis and S. U. Pillai, *Probability, random variables, and stochastic processes*, 4th ed. Boston: McGraw-Hill, 2002.