Multi-Antenna Cognitive AF Relay Systems with Multiple Primary Receivers

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***Abstract*—This paper investigates spectrum-sharing cognitive amplify-and-forward (AF) relay networks employing the maximum ratio transmission/maximum ratio combining (MRT/MRC) scheme at the multiple-antenna source-destination pair. It derives closed-from expressions for the ergodic capacity as well as for the symbol error rate (SER) lower bound and its asymptotic value when considering Nakagami-m fading and interference constraints on N primary receivers.**

***Index Terms*—Amplify-and-forward, two-hop relaying, MIMO, cognitive radio.**

I. Introduction

DUE to its strong potential in increasing transmission coverage and link reliability, relaying has garnered a wide interest from the wireless communication community [1]- [2]. Knowing that multiple antennas provide enormous performance gains in wireless systems, the idea of multiple-input multiple-output (MIMO) relaying is being investigated for emergent wireless system standards [2]. Nevertheless, terminals in such standards, will, inevitably, face a complex co-channel interference environment due to the highly aggressive frequency reuse. In this respect, cognitive spectrum sharing has arisen as a promising technique to combat spectrum scarcity in wireless relay networks. A common approach to cognitive spectrum sharing is the underlay model where the transmit power at the secondary users must be managed under a peak interference temperature to guarantee reliable communication between the primary users [3].

Aiming at understanding the performance limit of cognitive relay networks, significant contributions investigating such systems in various practical scenarios have appeared. As far as the analysis of single-antenna systems is concerned, some insightful results can be found in [4]-[8], where outage probability (OP) and symbol error rate expressions where derived for decode-and-forward (DF) and amplify-and-forward (AF) relaying in Nakagami-m fading. Recently, [9] and [10] investigated the OP of cognitive spectrum sharing from the viewpoint of multiple-input multiple-output (MIMO) in the primary and/or the secondary networks. Although, due to the prominence of multiple antennas in future cognitive networks, the findings in [9] and [10] are instructional, closed-form expressions were obtained therein only for integer Nakagami-m fading, thereby reducing their scope.

Here, we examine cognitive spectrum-sharing relay networks with multiple antennas at the secondary source-destination pair and provide new results for the ergodic capacity and error rate analysis in the presence of multiple primary users. It is noteworthy that such analysis has not thus far been addressed in Nakagami-m fading.

II. System and Channel Models

Consider a two-hop spectrum-sharing relay network consisting of one secondary user (SU) source S, one SU destination D, and N primary user (PU) receivers $PU_l (l = 1, \ldots, N)$. The SU source and destination are equipped with N_s and N_d antennas, respectively, communicate through a single-antenna SU relay R. Multiple-antenna source-destination pair and single-antenna relay systems are relevant for multipoint-to multipoint communications and cooperative virtual MIMO systems.

Let[1] the $N_v \times 1$ vectors $\mathbf{h}_i,\ (v, i) \in \{(s, 1), (d, 2)\}$ denote the channels for the source-relay and the relay-destination links, respectively, with entries following independent identically distributed (i.i.d) Nakagami-m random variables (RVs) with parameters $(m_i, \lambda_i),\ i = \{1, 2\}$. Let also x denote the source symbol satisfying $\mathrm{E}\{xx^*\} = P_s$, and $\mathbf{n}_u,\ u \in \{r, d\}$ denote the $N_v \times 1$ AWGN at the relay and destination nodes, respectively, with $\mathrm{E}\{\mathbf{n}_u \mathbf{n}_u^*\} = N_0 \mathbf{I}$, where $\mathbf{I}$ is the identity matrix. Then the received signals at both the relay and the destination are given by $y_r = \mathbf{h}_1^\dagger \mathbf{w}_1 x + n_r$, and $y_d = \widetilde{\mathbf{w}}_2^\dagger \left[\mathbf{h}_2 y_r + \mathbf{n}_d\right]$, where, for MRT/MRC, $\mathbf{w}_1$ is set to match the first hop, i.e., $\mathbf{w}_1 = \mathbf{h}_1 / \|\mathbf{h}_1\|$ and $\widetilde{\mathbf{w}}_2 = w \mathbf{w}_2$ where w is the power constraint factor and $\mathbf{w}_2$ is set to match the second hop, i.e., $\mathbf{w}_2 = \mathbf{h}_2 / \|\mathbf{h}_2\|$. The relay mode is non-regenerative with a variable gain in which the amplification factor is determined by the instantaneous channel statistics of the source-relay link. Hence w^2 can be computed as $w^2 = P_r / (P_s \mathbf{h}_1^\dagger \mathbf{h}_1)$, where $P_r = \mathrm{E}\{\|\widetilde{\mathbf{w}}_2 y_r\|^2\}$. In CRNs, the interference from the SU should be strictly constrained below a maximum tolerable interference level I_p at the PU receiver. Let the $N_S \times 1$ vector $\mathbf{g}_{1j}$ denote the channel form the SU source to the jth PU with coefficients $g_{1jl},\ l = 1, \ldots, N_s$, and $g_{2j},\ j = 1, \ldots, N$ denote the channel coefficient form the relay to the jth PU, all following i.i.d Nakagami-m RVs with parameters $(m_{I_i}, \lambda_{I_i}),\ i = \{1, 2\}$. Then, by considering MRC at the PU receivers, the SUs should adaptively adjust their transmit powers[2] as $P_s \leq I_p / |\mathbf{g}_{1j*}|^2$ and $P_r \leq I_p / |g_{2j*}|^2$, where $|\mathbf{g}_{1j*}| = \max_{j=1,\ldots,N}\{|\mathbf{g}_{1j}|\}$ and $|g_{2j^*}| = \max_{j=1,\ldots,N} |g_{2j}|$.

[1] Bold lower case letters denote vectors and lower case letters denote scalars. $\mathrm{E}\{x\}$ stands for the expectation of the random variable x, $*$ denotes the conjugate operator and, $\dagger$ denotes the conjugate transpose operator.

[2] When S and R are not power-limited terminals, the transmit power constraint depends on interference only.

Therefore, the end-to-end SNR of the SU $S \to R \to D$ link can be expressed as

$$\gamma = \frac{\gamma_r \gamma_d}{\gamma_r + \gamma_d}, \quad (1)$$

where $\gamma_r = \frac{\bar{\gamma}|\mathbf{h}_1|^2}{|\mathbf{g}_{1j^*}|^2}$, $\gamma_d = \frac{\bar{\gamma}|\mathbf{h}_2|^2}{|g_{2j^*}|^2}$, and $\bar{\gamma} = \frac{I_p}{N_0}$.

III. Ergodic Capacity

The ergodic capacity is an important performance metric since it quantifies the maximum achievable transmission rate under which errors are recoverable.

Lemma 1: Let $\mathrm{F_A}$ be the Lauricella hypergeometric function of the first kind [11], then the ergodic capacity of CRNs employing AF relaying over Nakagami-m fading is given as in (2) at the top of the next page.

Proof: Resorting to the moment generating function (MGF)-based approach proposed in [12], the ergodic capacity can be computed as

$$\begin{aligned} C &= \frac{1}{2}\mathrm{E}\left[\log_2\left(1+\gamma\right)\right] \\ &= \frac{1}{2\ln(2)}\int_0^\infty \frac{1-e^{-s}}{s} M_{\gamma^{-1}} ds, \end{aligned} \quad (3)$$

where $M_{\gamma^{-1}}(s) = M^{-1}_{\gamma_r}(s) M^{-1}_{\gamma_d}(s)$ is the MGF of the end-to-end SNR. In its turn, the per-hop SNR MGF $M_{\gamma_X^{-1}}(s), X \in \{r, d\}$ is derived as

$$M_{\gamma_X^{-1}}(s) = \int_0^\infty \int_0^\infty e^{-s\frac{\bar{\gamma} z}{y}} f_{|\mathbf{h}_i|^2}(y) f_{|\mathbf{g}_{ij^*}|^2}(z) dy dz, \quad (4)$$

where $f_{|\mathbf{h}_i|^2}$ and $f_{|\mathbf{g}_{ij^*}|^2}$ denote the probability density functions (PDFs) of $|\mathbf{h}_i|^2$ and $|\mathbf{g}_{ij^*}|^2$ and are, respectively, given by

$$f_{|\mathbf{h}_i|^2}(x) = \frac{\left(\frac{m_i}{\lambda_i}\right)^{N_u m_i}}{\Gamma(N_u m_i)} x^{N_u m_i - 1} e^{-\frac{m_i}{\lambda_i}x}, \quad (5)$$

with $(u, i) = \{(s, 1), (d, 2)\}$, and

$$f_{|\mathbf{g}_{ij^*}|}(x) = \frac{N\left(\frac{m_{I_i}}{\lambda_{I_i}}\right)^{N_v m_{I_i}} x^{N_v m_{I_i}-1} e^{-\frac{m_{I_i}}{\lambda_{I_i}}x}}{\Gamma(N_v m_{I_i})} \left(1 - \frac{\Gamma(N_v m_{I_i}, \frac{m_{I_i}}{\lambda_{I_i}}x)}{\Gamma(N_v m_{I_i})}\right)^{N-1}, \quad (6)$$

with $(v, i) = \{(s, 1), (r, 2)\}$ and $N_r = 1$. By performing the necessary substitutions in (4) along with [13, Eqs. (3.351.3) and (9.211.4)], we obtain

$$M_{\gamma_X^{-1}}(s) = \frac{N\left(\frac{m_{I_i}}{\lambda_{I_i}}\right)^{N_v m_{I_i}}}{\Gamma(N_u m_i)\Gamma(N_v m_{I_i})} \sum_{n=0}^{N-1}\binom{N-1}{n}(-1)^n \sum_{\Omega(n, N_u m_i)} \tau_\Omega^n \Psi\left(\delta_n + N_v m_{I_i}, 1 - N_u m_i, \frac{m_i \lambda_{I_i}}{m_{I_i}\lambda_i (n+1)\bar{\gamma}} s\right), \quad (7)$$

where $X \in \{r, d\}$, $(i, u, v) = \{(1, s, s), (2, r, d)\}$ with $N_r = 1$, $\Psi(a; b; z)$ denotes the Triconomi confluent hypergeometric function [13, Eq. (9.211.1)] and τ_Ω^n is given by

$$\tau_\Omega^n = \frac{n!\Gamma(\delta_n + N_v m_{I_i})\Gamma(\delta_n + N_v m_{I_i} + N_u m_i)}{\left(\frac{m_{I_i}}{\lambda_{I_i}}(n+1)\right)^{\delta_n + N_v m_{I_i}} \prod_{k=1}^{N_v m_{I_i}} n_k! \prod_{p=0}^{N_v m_{I_i}-1}\left(\frac{\left(\frac{m_{I_i}}{\lambda_{I_i}}\right)^p}{p!}\right)^{-n_{p+1}}}. \quad (8)$$

Subsequently, the ergodic capacity is derived by replacing (7) into (3) as

$$C = \frac{N^2\left(\frac{m_{I_1}}{\lambda_{I_1}}\right)^{m_{I_1}}\left(\frac{m_{I_2}}{\lambda_{I_2}}\right)^{m_{I_2}}}{2\ln(2)\Gamma(N_s m_1)\Gamma(N_s m_{I_1})\Gamma(N_d m_2)\Gamma(m_{I_2})} \times \sum_{n,p=0}^{N-1}\binom{N-1}{n}\binom{N-1}{p}(-1)^{n+p}\widetilde{\sum}\tau_\Omega^n \tau_\Omega^p I_{n,p}, \quad (9)$$

where

$$I_{n,p} = \int_0^\infty \frac{1-e^{-s}}{s}\Psi\left(\delta_n + N_s m_{I_1}, 1 - N_s m_1, \frac{m_1\lambda_{I_1}}{m_{I_1}\lambda_1(n+1)\bar{\gamma}} s\right) \Psi\left(\delta_p + m_{I_2}, 1 - N_d m_2, \frac{m_2 \lambda_{I_2}}{m_{I_2}\lambda_2(p+1)\bar{\gamma}} s\right) ds. \quad (10)$$

To resolve (10), we invoke the expansion formulas of Ψ in terms of the confluent hypergeometric function ${}_1\mathrm{F}_1$ in [13, Eq. (9.210.1)] and the fact that $(1-e^{-s})/s = e^{-s}{}_1\mathrm{F}_1(1; 2; s)$. Subsequently, we can obtain the following expression of $I_{n,p}$

$$\begin{aligned} I_{n,p} = & \frac{\Gamma(N_s m_1)\Gamma(N_d m_2)}{\Gamma(N_s m_1 + \delta_n + N_s m_{I_1})\Gamma(N_d m_2 + \delta_p + m_{I_2})}\int_0^\infty e^{-s} \\ & {}_1\mathrm{F}_1(1; 2; s)\left({}_1\mathrm{F}_1\left(\delta_n + N_s m_{I_1}; 1 - N_s m_1; \frac{m_1\lambda_{I_1}}{m_{I_1}\lambda_1(n+1)\bar{\gamma}} s\right)\right. \\ & \left. + \rho_n {}_1\mathrm{F}_1\left(N_s m_1 + \delta_n + N_s m_{I_1}; 1 + N_s m_1; \frac{m_1\lambda_{I_1}}{m_{I_1}\lambda_1(n+1)\bar{\gamma}} s\right)\right) \\ & \times\left({}_1\mathrm{F}_1\left(\delta_p + m_{I_2}; 1 - N_d m_2; \frac{m_2\lambda_{I_2}}{m_{I_2}\lambda_2(p+1)\bar{\gamma}} s\right) + \rho_p\right. \\ & \left. {}_1\mathrm{F}_1\left(N_d m_2 + \delta_p, 1 + m_{I_2}; 1 + N_d m_2; \frac{m_2\lambda_{I_2}}{m_{I_2}\lambda_2(p+1)\bar{\gamma}} s\right)\right) ds, \end{aligned} \quad (11)$$

where $\rho_k = \frac{\Gamma(-m_i)\left(\frac{m_i\lambda_{I_i}}{m_{I_i}\lambda_i(k+1)\bar{\gamma}}\right)^{m_i}}{\mathrm{B}(\delta_k + N_v m_{I_i}, m_i)}$ with $(k, i, v) \in \{(n, 1, s), (p, 2, r)\}$. Now from (9) and (11), the desired result can be obtained by appealing to [11]

$$\mathrm{F_A^{(r)}}\left(a; b_1, \ldots, b_r; c_1, \ldots, c_r; \frac{x_1}{\nu}, \ldots, \frac{x_r}{\nu}\right) = \frac{\nu^a}{\Gamma(a)} \int_0^\infty e^{-\nu t} t^{a-1}\left(\prod_{k=1}^r {}_1F_1(b_k; c_k, x_k t)\right) dt; \text{ where } \mathfrak{Re}(a) > 0. \quad (12)$$

IV. Symbol Error Rate

The SER is expressed in terms of the cumulative density function (CDF) of γ denoted by F_γ as [14]

$$P_e = \frac{a\sqrt{b}}{2\sqrt{\pi}}\int_0^\infty \frac{e^{-by}}{\sqrt{y}} F_\gamma(y) dy, \quad (13)$$

$$C = \frac{N^2}{2\ln(2)} \sum_{n,p=0}^{N-1} (-1)^{n+p} \binom{N-1}{n} \binom{N-1}{p} \widetilde{\sum} \frac{n!p!\Gamma(\delta_n)\Gamma(\delta_p)\left(\prod_{t=0}^{N_s m_{I_1}-1} \left(\frac{1}{t!}\right)^{n_t+1}\right)\left(\prod_{l=0}^{m_{I_2}-1} \left(\frac{1}{l!}\right)^{n_l+1}\right)}{\prod_{i=1}^{N_s m_{I_1}} n_i! \prod_{i=1}^{m_{I_2}} p_i! B(\delta_n, N_s m_{I_1}) B(\delta_p, m_{I_2})(n+1)^{\delta_n + N_s m_{I_1}} (p+1)^{\delta_p + m_{I_2}}}$$
$$\left(\mathrm{F_A}(\Xi_1, \Pi) + \rho_n \Gamma(1+N_s m_1) \mathrm{F_A}(\Xi_2, \Pi) + \rho_p \Gamma(1+N_d m_2) \mathrm{F_A}(\Xi_3, \Pi) + \rho_n \rho_p \Gamma(1+N_s m_1 + N_d m_2) \mathrm{F_A}(\Xi_4, \Pi) \right), \quad (2)$$

where $\widetilde{\sum} = \sum_{\Omega(n, N_s m_{I_1}), \Omega(p, m_{I_2})}$, $\Omega(n, m_i) = \{(n_1, ..., n_{m_i}) : n_k \geq 0; \sum_{k=1}^{m_i} n_k = n\}$, $\delta_n = \sum_{l=0}^{N_v m_{I_i}-1} l n_{l+1}$, $\Pi = \{1, \frac{m_1 \lambda_{I_1}}{m_{I_1} \lambda_1 (n+1)\bar{\gamma}}, \frac{m_2 \lambda_{I_2}}{m_{I_2} \lambda_2 (p+1)\bar{\gamma}})\}$, $\Xi_1 = \{1, 1, \delta_n + N_s m_{I_1}, \delta_p + m_{I_2}; 2, 1 - N_s m_1, 1 - N_d m_2\}$, $\Xi_2 = \{1 + N_s m_1; 1, \delta_n + N_s m_{I_1} + N_s m_1, \delta_p + m_{I_2}, 2, 1 + N_s m_1, 1 - N_d m_2\}$, $\Xi_3 = \{1 + N_d m_2; 1, \delta_p + m_{I_2} + N_d m_2, \delta_n + N_s m_{I_1}; 2, 1 + N_d m_2, 1 - N_s m_1\}$, $\Xi_4 = \{1 + N_s m_1 + N_d m_2; 1, \delta_n + N_s m_{I_1} + N_s m_1, \delta_p + m_{I_2} + N_d m_2; 2, 1 + N_s m_1, 1 + N_d m_2\}$.

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where $a, b > 0$ are modulation-specific constants. Unfortunately, for the MIMO CRN under study, F_γ is untractable in closed-form hampering the obtainment of (13). To simplify the analysis, an upper bound on γ is used as follows [6]

$$\gamma < \gamma_{up} = \min(\gamma_r, \gamma_d). \quad (14)$$

Lemma 2: Let Φ_2 be the confluent hypergeometric function of the second kind [11], then the average SER of CRNs employing AF relays over Nakagami-m is lower bounded as in (15), shown at the top of the next page[3].

Proof: The CDF of γ^{up} can be written as

$$F_{\gamma_{up}}(x) = F_{\gamma_r}(x) + F_{\gamma_d}(x) - F_{\gamma_r}(x) F_{\gamma_d}(x), \quad (17)$$

where from (5) and (6) and appealing to [13, Eq. (3.194.1)], we obtain F_{γ_X}, $X \in \{r, d\}$ as

$$F_{\gamma_X}(x) = \sum_{n=0}^{N-1} \sum_{\Omega(n, N_v m_{I_i})} \Theta_n \frac{\left(\frac{m_i \lambda_{I_i}}{m_{I_i} \lambda_i (n+1)\bar{\gamma}} x\right)^{N_u m_i}}{N_u m_i}$$
$${}_2\mathrm{F}_1\left(N_u m_i, N_u m_i + N_v m_{I_i}, N_u m_i + 1, \frac{-m_i \lambda_{I_i} x}{m_{I_i} \lambda_i (n+1)\bar{\gamma}}\right), \quad (18)$$

where $\Theta_n = \frac{N\binom{N-1}{n}(-1)^n \tau_\Omega^n \left(\frac{m_{I_i}}{\lambda_{I_i}}\right)^{N_v m_{I_i}}}{B(N_u m_i, N_v m_{I_i}) \Gamma(\delta_n + N_v m_{I_i} + N_u m_i)}$, and ${}_2\mathrm{F}_1$ is the Gauss hypergeometric function [13, Eq. (9.100)].

Substituting (17) and (18) into (13) and resorting to the key transformation

$${}_2F_1(a, b, b-n, z) = (1-z)^{-a-n} \sum_{k=0}^{n} \frac{(-n)_k (b-a-n)_k}{(b-n)_k} \left(\frac{z}{1+z}\right)^k, \quad (19)$$

the desired result is obtained after applying [13, Eq. (9.211.1)] and recognizing the fact that

$$\Phi_2(a; b_1, ..., b_K; z; x_1, ..., x_K, y) = \frac{1}{\Gamma(a)} \int_0^\infty \exp(-yt) t^{a-1} (1+t)^{a-z-1} \prod_{k=1}^{K} (1 + x_k t)^{-b_k} dt. \quad (20)$$

[3]Note that the obtainment of (15) inflicts the quantities $N_s m_{I_1}$ and m_{I_2} to be integer valued. However, this does not limit the scope of the paper since we already show in [12] that only interference power and number affect the system performance.

A. Asymptotic SER

Corollary 1: The asymptotic SER of multiple antenna CRNs with AF relaying in (15), derived as $\bar{\gamma} \to \infty$, is

$$P_e^{l\infty} = \frac{aN}{2\sqrt{\pi}} \Bigg[\sum_{n=0}^{N-1} \sum_{\Omega(n, N_s m_{I_1})} \frac{\widetilde{\Sigma}(1, s, s)\Gamma(\frac{1}{2} + N_s m_1)}{\left(\frac{b m_{I_1} \lambda_1 (n+1)}{2 m_1 \lambda_{I_1}}\right)^{N_s m_1}} \bar{\gamma}^{-N_s m_1}$$
$$+ \sum_{n=0}^{N-1} \sum_{\Omega(n, m_{I_2})} \frac{\widetilde{\Sigma}(2, r, d)_{N_r=1} \Gamma(\frac{1}{2} + N_d m_2)}{\left(\frac{b m_{I_2} \lambda_2 (n+1)}{2 m_2 \lambda_{I_2}}\right)^{N_d m_2}} \bar{\gamma}^{-N_d m_2}$$
$$- N \sum_{n,p=0}^{N-1} \widetilde{\sum} \frac{\widetilde{\Sigma}(1, s, s) \widetilde{\Sigma}(2, r, d)_{N_r=1}}{\left(\frac{b m_{I_1} \lambda_1 (n+1)}{m_1 \lambda_{I_1}}\right)^{N_s m_1} \left(\frac{b m_{I_2} \lambda_2 (p+1)}{m_2 \lambda_{I_2}}\right)^{N_d m_2}}$$
$$\times \Gamma(\frac{1}{2} + N_s m_1 + N_d m_2) \bar{\gamma}^{-N_s m_1 - N_d m_2} \Bigg], \quad (21)$$

where

$$\widetilde{\Sigma}(i, u, v) = \frac{\binom{N-1}{n}(-1)^n \tau_\Omega^n \alpha_{I_i}^{N_v m_{I_i}} B(N_u m_i, N_v m_{I_i})^{-1}}{N_u m_i \Gamma(N_v m_{I_i}) \Gamma(\delta_n + N_v m_{I_i} + N_u m_i)}. \quad (22)$$

Proof: The result follows by using $\Psi(a, b; z) \underset{z \to \infty}{\approx} z^{-a}$ and $\Phi_2(c, b_1, b_2, c-1; x, y, z) \underset{x,y \to 0}{\approx} z^{-c}$ along with some series manipulations.

Corollary 2: The diversity and coding gains of multiple-antenna CRNs with AF relaying are, respectively, given by

$$G_d = \min(N_s m_1, N_d m_2), \quad (23)$$

$$G_a = \begin{cases} \Delta(1, s, s)^{-\frac{1}{G_d}} & N_s m_1 < N_d m_2; \\ (\Delta(1, s, s) + \Delta(2, r, d))^{-\frac{1}{G_d}}, & N_s m_1 = N_d m_2; \\ \Delta(2, r, d)^{-\frac{1}{G_d}}, & N_s m_1 > N_d m_2; \end{cases} \quad (24)$$

where

$$\Delta(i, u, v) = \frac{aN}{2\sqrt{\pi}} \sum_{n=0}^{N-1} \sum_{\Omega(n, N_v m_{I_i})} \frac{\widetilde{\Sigma}(i, u, v) \Gamma(\frac{1}{2} + N_u m_i)}{\left(\frac{b m_{I_i} \lambda_i (n+1)}{2 m_i \lambda_{I_i}}\right)^{N_u m_i}}. \quad (25)$$

Proof: Since the asymptomatic SER in (21) is dominated by the first and second summations, then re-expressing the SER in (21) as $P_e^{l\infty} = (G_a \bar{\gamma})^{-G_d}$, where G_d is the diversity order, and G_a is the array gain [15], yields the desired result.

$$
\begin{aligned}
P_e^l = &\frac{a\sqrt{b}N}{2\sqrt{2\pi}}\Bigg[\sum_{n=0}^{N-1}\sum_{\Omega(n,N_s m_{I_1})}\sum_{l=0}^{N_s m_{I_1}-1}\frac{\Sigma(1,s,s)}{\sqrt{\frac{m_1\lambda_{I_1}}{m_{I_1}\lambda_1(n+1)\bar{\gamma}}}}\Gamma\left(l+\frac{1}{2}+N_s m_1\right)\Psi\left(N_s m_1+\frac{1}{2}+l,\frac{3}{2},\frac{b m_{I_1}\lambda_1(n+1)\bar{\gamma}}{2m_1\lambda_{I_1}}\right)\\
&+\sum_{n=0}^{N-1}\sum_{\Omega(n,m_{I_2})}\sum_{l=0}^{m_{I_2}-1}\frac{\Sigma(2,r,d)_{N_r=1}}{\sqrt{\frac{m_2\lambda_{I_2}}{m_{I_2}\lambda_2(n+1)\bar{\gamma}}}}\Gamma\left(l+\frac{1}{2}+N_d m_2\right)\Psi\left(N_d m_2+\frac{1}{2}+l,\frac{3}{2},\frac{b m_{I_2}\lambda_2(n+1)\bar{\gamma}}{2m_2\lambda_{I_2}}\right)\\
&-N\sum_{n,p=0}^{N-1}\widetilde{\sum}\sum_{t=0}^{N_s m_{I_1}-1}\sum_{l=0}^{m_{I_2}-1}\frac{\Sigma(1,s,s)}{\left(\frac{m_1\lambda_{I_1}}{m_{I_1}\lambda_1(n+1)\bar{\gamma}}\right)^{-t-N_s m_1}}\frac{\Sigma(2,r,d)_{N_r=1}}{\left(\frac{m_2\lambda_{I_2}}{m_{I_2}\lambda_2(p+1)\bar{\gamma}}\right)^{-l-N_d m_2}}\Gamma\left(l+t+\frac{1}{2}+N_s m_1+N_d m_2\right)\\
&\Phi_2\left(N_s m_1+N_d m_2+l+t+\frac{1}{2};N_s m_1+l,N_d m_2+t;N_s m_1+N_d m_2+l+t-\frac{1}{2};\frac{m_1\lambda_{I_1}}{m_{I_1}\lambda_1(n+1)\bar{\gamma}},\frac{m_2\lambda_{I_2}}{m_{I_2}\lambda_2(p+1)\bar{\gamma}},\frac{b}{2}\right)\Bigg], \quad (15)
\end{aligned}
$$

$$
\text{where}\quad \Sigma(i,u,v)_{n,l}=\frac{\binom{N-1}{n}(-1)^n\tau_\Omega^n\left(\frac{m_{I_i}}{\lambda I_i}\right)^{N_v m_{I_i}}(1-N_v m_{I_i})_l(N_u m_i)_l}{N_u m_i\Gamma(N_v m_{I_i})B(N_u m_i,N_v m_{I_i})\Gamma(\delta_n+N_v m_{I_i}+N_u m_i)(1+N_u m_i)_l}. \quad (16)
$$

V. Illustrative Numerical Results

Fig. 1 confirms that the theoretical results match perfectly their empirical counterparts, hence confirming their correctness. It also suggests significant capacity improvement when increasing the number of antennas, more so at the destination (i.e., Nd) than at the source (Ns). Moreover, it clearly appears that the capacity gap due to the increase of the number of PUs diminishes as the number of antennas increases. This implies that a MIMO CRN is able to maintain its performance in dynamic environments where PUs vary in number when the antenna arrays are relatively large.

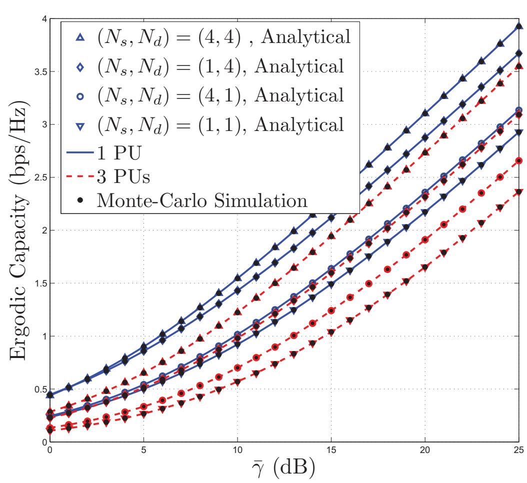

Fig. 1. The impact of the S-R MIMO link size (N_s, N_d) and PUs number N on the ergodic capacity of two-hop AF MIMO CRNs, with $m_1 = m_2 = 0.7$, $m_{I_1} = m_{I_1} = 1$, $\lambda_1 = \lambda_2 = 2$ dB, $\lambda_{I_1} = \lambda_{I_2} = 1$ dB.

Fig. 2 plots the SER lower bound and its true value via computer simulations. We can readily note that the lower bound remains sufficiently tight across the entire SNR range of interest, meaning that it is able to serve as an effective approximation for the exact SER. As expected, the SER increases with N while the diversity gain remains unchanged.

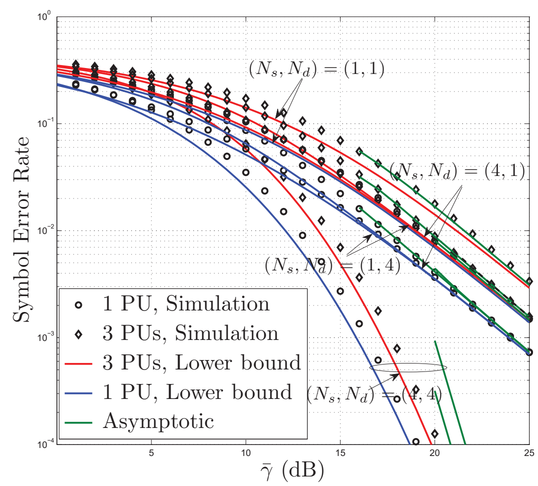

Fig. 2. The impact of the S-R MIMO link size (N_s, N_d) and PUs number N on the SER of two-hop AF MIMO CRNs, with $m_1 = m_2 = 1.5$, $m_{I_1} = m_{I_1} = 1$, $\lambda_1 = \lambda_2 = 2$ dB, $\lambda_{I_1} = \lambda_{I_2} = 1$ dB, $a = 0.5$, $b = 1$.

This increase can be easily evaluated using (24). We also observe at high SNR that AF MIMO CRNs exhibit similar error rates with more antennas at the source or the destination.

VI. Conslusion

In this paper, new closed-from expressions for the ergodic capacity as well as the average error rate lower bound and its asymptotic value were derived for spectrum-sharing cognitive amplify-and-forward (AF) relay networks employing maximum ratio transmission/maximum ratio combining (MRT/MRC) schemes at the multiple-antenna source-destination pair. The findings of the paper are instructional on how the parameters of the secondary and/or primary networks affect the performance of the system.

References

[1] J. N. Laneman, D. N. C. Tse, and G. W. Wornell,“Cooperative diversity in wireless networks: efficient protocols and outage behaviour,” *IEEE Trans. Inform. Theory,* vol. 50, pp. 3062-3080, Dec. 2004.

[2] W. Chen, J. Montojo, A. Golitschek, C. Koutsimanis, and S. Xiaodong, “Relaying operation in 3GPP LTE: challenges and solutions,” *IEEE Commun. Mag.,* vol. 50, no. 2, pp. 156-162, Feb. 2012.

[3] A. Ghasemi and E. S. Sousa, “Fundamental limits of spectrum-sharing environments,” *IEEE Trans. Wireless Commun.,* vol. 6, no. 2, pp. 649-658, Feb. 2007.

[4] T. Q. Duong, D. B. da Costa, T. A. Tsiftsis, C. Zhong, and A. Nallanathan, “Outage and diversity of cognitive relaying systems under spectrum sharing environments in nakagami-m fading,” *IEEE Commun. Lett.,* vol. 16, no. 12, Dec. 2012.

[5] C. Zhong, T. Ratnarajah, and K.-K. Wong, “Outage analysis of decode-and-forward cognitive dual-hop systems with the interference constraint in Nakagami-m fading channels,” *IEEE Trans. Veh. Tech.*, vol. 60, no. 6, pp. 2875-2879, July 2011.

[6] T. Q. Doung, D. B. da Costa, M. Elkashlan, and V. N. Q. Bao, “Cognitive amplify-and-forward relay networks over Nakagami-m fading,” *IEEE Trans. Veh. Tech.*, vol. 61, no. 5, pp. 2368-2374, June 2012.

[7] H. Yu, W. Tang, and S. Li, “Outage probability and SER of amplify-and-forward cognitive relay networks,” *IEEE Wireless Commun. Lett.,* vol. 2, no. 2, Apr. 2014.

[8] A. M. Salhab, S. A. Zummo, “Cognitive amplify-and-forward relay networks with switch-and-examine relaying in Rayleigh fading Channels”,*IEEE Commun. Lett.,* vol. 18, no. 5, May 2014.

[9] P. L. Yeoh, M. Elkashlan, T. Q. Duong, N. Yang, and D. B. da Costa, “Transmit antenna selection for interference management in cognitive relay networks”, *IEEE Trans. Veh. Tech.*, accepetd for publication.

[10] S. Hua, H. Liu, M. Wu, and S. S. Panwar, “Exploiting MIMO antennas in cooperative cognitive radio networks”, in *IEEE INFOCOM,* Shanghai, China, 2011.

[11] H. Exton, *Multiple Hypergeometric Functions and Applications*; New York: Jhon Wiley, 1976.

[12] I. Trigui, S. Affes, and A. Stéphenne, ”On the ergodic capacity of amplify-and-forward relay channels with interference in Nakagami-m fading,” *IEEE Trans. Commun.*, vol. 61, no. 8, pp. 3136-3145, Aug. 2013.

[13] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products,* 5th Ed., San Diego, CA: Academic, 1994.

[14] M. R. Mckay, A. J. Grant, and I. B. Collings, “Performance analysis of MIMO-MRC in double-correlated Rayleigh environments,” *IEEE Trans. Commun.* , vol. 55, no. 3, pp. 497-507, Mar. 2007.

[15] Z. Wang and G. B. Giannakis, “A simple and general parameterization quantifying performance in fading channels,” *IEEE Trans. Commun.,* vol. 51, no. 8, pp. 1389-1398, Aug. 2003.