

Resource Allocation in Heterogeneous Small-Cell Networks with Interference Avoidance Admission

Vahid Asghari[†], Oussama Rhouma[†], Sofiène Affes[†], and Ali Ghrayeb[‡]

[†]INRS-EMT, University of Quebec, Montreal, QC, Canada

Email: {vahid, oussama.rhouma, affes}@emt.inrs.ca

[‡]Texas A&M University at Qatar

Email: ali.ghrayeb@qatar.tamu.edu

Abstract—In this paper, we consider a heterogeneous small-cell network (HSCNet) with co-channel deployment and investigate the throughput performance of the proposed admission control algorithm based on a new received interference constraint at the small-cell base station (SCBS) nodes. In the considered co-channel heterogeneous network, by utilizing the proposed admission algorithm based on the amount of received interference generated at the neighboring macro users, the new user is admitted at the small-cell network (SCNet) while ensuring that no harmful interference is caused to the adjacent macro users. For the proposed admission algorithm, we investigate the achievable effective capacity performance of the SCNet subject to maximizing the throughput performance of the SCNet. Finally, we sustain our theoretical analysis by numerical results.

I. INTRODUCTION

An important feature of 4G networks is the integration of different heterogeneous networks and achieving seamless connectivity [1], [2]. In fact, a heterogeneous network (Het-Net) is a network that could combine different radio access technologies (RATs) and/or multi-tier cells (i.e., cells with different coverage settings and footprints or topologies). In the latter case of single-RAT multi-tier HetNet, small cells (i.e., femto, pico, micro) are introduced on the top of conventional macro cells [3]. In this case, hyper-dense deployment of heterogeneous small-cell networks (HSCNets) increases the number of cells in a given area thereby providing a huge capacity gain by bringing small-cell base stations closer to the mobile devices. Small cell (SC) technology has been shown to have the potential to improve both coverage and capacity, especially in indoor and very crowded environments [1], [4]. The primary advantage of deploying SC technology such as in Micro and Femto cells, is to improve the throughput performance of the indoor users due to the shorter transmission ranges and having a backhaul link to connect to the network gateway [1]. A SC access point is merely a small cellular base station that is installed in a home or small business area. The SC base station (SCBS) is normally connected to the service provider's network, e.g., a macrocell base station (MBS), via a broadband link such as a digital subscriber line.

A major challenge, however, in deploying such SCNets is the interference that results from sharing the same spectrum between the SCs and the macrocell networks [1], [5]. In fact, as SCs get closer to each other, this increases not only the desired

signal strength but also the interference from other cells. This interference must be eliminated or reduced to avoid degrading the overall network throughput, and a sophisticated mobility management mechanism is needed as the number of cell-edge users, suffering from severe interference from the neighboring cells, will naturally grow. A relatively easier solution is to split the frequency band between different SCs in heterogeneous systems. However, given the frequency spectrum scarcity, it would not, by far, be the most spectrum efficient solution [6]. A more efficient solution is to adopt a co-channel operation approach. Yet one has to cope with the interference to/from the MBS that might seriously degrade the network's overall performance [7].

In most admission algorithms, normally channel assignment must be done, which consists of the admission of new users in the network based on the available resources and QoS requirements [8]. In co-channel HSCNets, the admission algorithm needs to consider not only the QoS requirements of the admitted users, but also it needs to consider the effect of this assignment on the QoS of existing macro users (MUs) and also SC users (SCUs) in the networks. There are a few studies in the context of admission algorithms for HSCNets. For instance, an admission algorithm for heterogeneous networks is developed in [9] based on the current load and service mix in each available network. In [10], assuming dynamic QoS requirements in heterogeneous networks, an admission algorithm has been investigated based on the fuzzy-logic theory to find the optimal admission thresholds and to keep the handoff dropping probability at a low level. Furthermore, the problem of admission control in HetNets has also been studied in [11] in which high priority users (with strict QoS requirements) were considered along with the regular users (with no QoS requirement) and the proposed algorithm was developed based on a resource allocation optimization problem.

In this paper, we consider a heterogeneous small-cell network (HSCNet) where the macros and SCs operate in a co-channel scenario where any new user, e.g., either MUs or SCUs, can be served by the nearby macro or SC base stations if they satisfy some predefined admission requirements. In this regard, we focus on investigating the admission control algorithms in HSCNets when a new user appears within the transmission area of a SC network (SCNet). In particular, we propose an interference avoidance-based admission control algorithm for the co-channel HSCNets while maximizing the throughput performance of the SCNet. In this context, we investigate the

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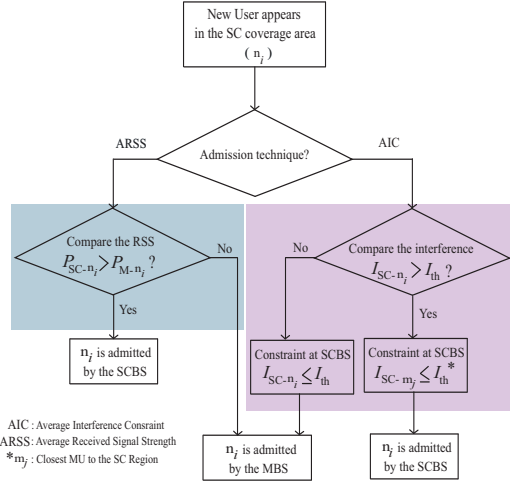


Figure 1: Flowchart of the admission algorithms.

performance of the proposed admission algorithm based on the interference avoidance in terms of the effective capacity of the SCNet while ensuring that no harmful interference is caused to the macro network. In this regard, considering a typical downlink scenario at the SCNet, we first investigate the optimal resource allocation schemes based on the proposed admission algorithm and under some QoS constraint. Then, we evaluate the efficiency of the proposed admission algorithm and its impact on network and user performances through numerical results and comparisons.

II. SYSTEM AND CHANNEL MODELS

We consider a typical downlink scenario in a heterogenous small-cell network (HSCNet) with co-channel deployment with one macro base station (MBS) and a number of macro users (MUs) that are distributed within the MBS transmission area. We also consider a pair of SC base station (SCBS) and a SC user (SCU) which utilizes the spectrum band originally assigned to the macro network. The SCBS is connected to the MBS via a dedicated wired digital subscriber line (DSL) backhaul link which can be used to exchange information between macro and small-cell communication systems¹. We assume a co-channel (spectrum-sharing) HSCNet in which new users, e.g., MUs or SCUs, may receive service by the nearby macro or small-cell (SC) BSs if they satisfy the defined admission requirements. In this context, we focus on the admission control scenarios in co-channel HSCNets when a new user appears within the coverage area of a SC transmission. In particular, we first propose an admission algorithm based on the average interference constraint (AIC) at the SCBS. Then, we investigate the performance of our proposed admission algorithm in terms of the achievable effective capacity of the SC communication system. It is worth noting that we also compared the proposed AIC-based algorithm with the conventional technique based on the average received signal strength (ARSS) through system-level simulations in which

¹Note that this connection can be provided through the service gateway unit (SGU) which mandates the use of either of the network infrastructures to appropriately route and serve the traffic to and from these networks [2].

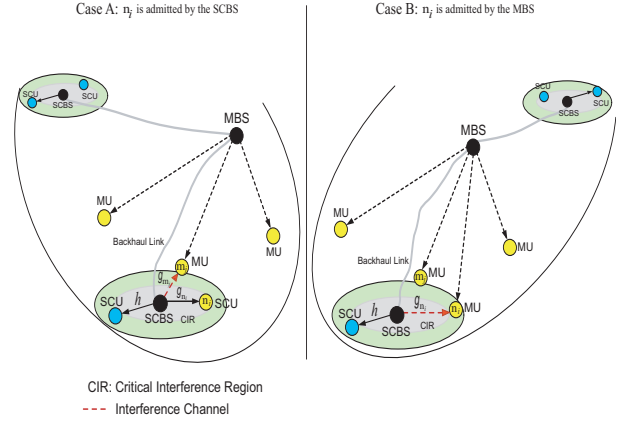


Figure 2: Illustration of the proposed AIC-based admission algorithm.

because of the limited space, we present them in our future publications. In order to better explain the proposed AIC-based algorithm, we first explain the conventional ARSS algorithm. In the conventional admission technique using ARSS, the admission control algorithm compares the received signal power from the associated macro and SC base stations at the new user, and decides accordingly to admit this user as a new SCU or MU. The admission algorithm based on ARSS is illustrated in Fig. 1, where P_{SC-n_i} and P_{M-n_i} are the average received powers at the new user n_i from the SCBS and MBS, respectively. In particular, if $P_{SC-n_i} > P_{M-n_i}$, this means that the new user n_i has a stronger signal reception from the SCBS than from the MBS. Hence, n_i is admitted by the SCBS as a SCU. Otherwise, n_i is admitted by the MBS as a MU. Note that the impact of co-channel interference is not considered by the ARSS algorithm. On the other hand, we propose an admission algorithm at the SC communication network, namely the AIC-based admission, which accounts for the amount of average received interference generated at the new user. As illustrated in Fig. 1, the average received interference from the SCBS at the new user, I_{SC-n_i} , is compared with a predefined interference threshold, I_{th} . Then, as shown in Fig. 2-A, if $I_{SC-n_i} > I_{th}$, this means that n_i is inside the *critical interference region* (CIR²) and it is admitted by the SCBS as an additional SCU, while the SCBS transmit power must adhere to the interference constraint at the closest MU, m_j , i.e., $I_{SC-m_j} \leq I_{th}$. Otherwise, as shown in Fig. 2-B, if $I_{SC-n_i} \leq I_{th}$, this means that n_i is outside the CIR region and it is admitted by the MBS as a MU, while the SCBS transmission must satisfy the interference constraint at n_i , i.e., $I_{SC-n_i} \leq I_{th}$ ³.

In our system model, we consider a time division (TD) multiple access scenario at the SCs transmission in which each SCBS wishes to send messages to SCUs with individual QoS constraints. We assume that, at the SCBS, the data sequences generated for each SCU are divided into frames with the same

²CIR is the area within the SCBS transmission region in which the interference at the non-SCUs is greater than a certain threshold.

³It is worth noting that we assume n_i to be always the closest user to the SCBS right before m_j .

time duration, T , as the data-link layer. These frames are initially stored in individual buffers before they are transmitted through the wireless channel. We consider a discrete-time flat-fading channel with perfect channel state information (CSI) at the SCBS and the SCUs. As shown in Fig. 2, we assume that the channel power gain between the SCBS and the SCU is given by $h[n]$, where n is the time index of the block, and it follows the exponential distribution with mean δ_h^2 . We also define the channel power gain between the SCBS and the new user n_i by $g_{n_i}[n]$ and the one between the SCBS and the closest MU m_j by $g_{m_j}[n]$. We assume that g_{n_i} and g_{m_j} are exponentially distributed with mean $\delta_{g_{n_i}}^2$ and $\delta_{g_{m_j}}^2$, respectively. We consider that the interference received at the SCUs inside the SC from the neighboring SCBSs and MBSs can be modeled as an additive zero-mean Gaussian noise⁴ with variance δ_n^2 . Moreover, the received signal bandwidth at the SCUs is denoted by B .

Obviously the QoS provided by a SCBS to the new admitted users is affected by the adopted admission control algorithm. In the following, our aim is to investigate the performance of our proposed AIC-based admission algorithm subject to maximizing the achievable effective capacity of the SCNets in case of fixed power transmission at the SCBS.

III. RESOURCE ALLOCATION SCHEME FOR AIC-BASED ADMISSION ALGORITHM

The concept of effective capacity, proposed in [12], is defined as the maximum arrival rate that a given service process can support in order to guarantee a QoS requirement specified by a delay-exponent parameter θ given as [13]

$$\theta = - \lim_{x \rightarrow \infty} \frac{\ln(\Pr(q(\infty) > x))}{x}, \quad (1)$$

where $q(n)$ is the transmit buffer length at time n . Note that we consider the same QoS constraint with the same delay exponent θ for both SCUs and MUs. In this regard, considering uncorrelated block fading channels where the service process $\{R[n], n = 1, 2, \dots\}$ is also uncorrelated, SCBS's maximum supported arrival rate given that the delay constraint is satisfied defines the effective capacity over the block-fading channel as [14]:

$$C_{\text{eff}}(\theta, \xi) = \frac{-1}{\theta} \ln(E_h[e^{-\theta R}]), \quad (2)$$

where $E_x[\cdot]$ is the statistical average with respect to x and R is the stochastic service process rate assumed to be stationary and ergodic⁵. Now, considering the AIC-based admission algorithm explained in Fig. 1, we investigate the achievable effective capacity of the SC system with fixed transmit power at the SCBS in both cases where $I_{\text{SC}-n_i} \leq I_{\text{th}}$ and $I_{\text{SC}-n_i} > I_{\text{th}}$.

Case I: First, we start with the case where the average received interference received at the new user, n_i , is less than the interference constraint limit value, i.e., $I_{\text{SC}-n_i} \leq I_{\text{th}}$, and

⁴Validity of this assumption is sustained by the fact of employing interference mitigation techniques, e.g., "interference cancelation" and "interference suppression", in co-channel heterogenous networks as explained in [1], [2].

⁵Hereafter, for notational simplicity, we omit the time index n whenever it is clear from the context.

hence, it is admitted by the MBS. In this case, n_i is considered as a MU, and therefore, there is only one SCU, f , within the SC transmission region and the stochastic service rate, R_f , can be obtained according to

$$R_f = TB \ln \left(1 + \frac{P_{\text{SC}} h}{\delta_n^2} \right), \quad (3)$$

where P_{SC} denotes the SCBS fixed transmit power per user at a SCNet. Making use of the service rate associated to the SCU, we can obtain the achievable effective capacity as presented by the following proposition.

Proposition 1. *The achievable effective capacity expression corresponding to the case of fixed power transmission at the SCBS using the AIC-based admission algorithm with $I_{\text{SC}-n_i} \leq I_{\text{th}}$, can be characterized according to*

$$C_{\text{eff}}(\theta, \xi) = \frac{-1}{\theta} \ln \left(\frac{\delta_n^2}{P_{\text{SC}} \delta_h^2} e^{\frac{\delta_n^2}{P_{\text{SC}} \delta_h^2}} E_{\theta TB} \left(\frac{\delta_n^2}{P_{\text{SC}} \delta_h^2} \right) \right), \quad (4)$$

where $E_n(x)$ denotes the exponential integral function with exponent parameter n [15].

Proof. Cf. [16]. ■

Case II: Now, if $I_{\text{SC}-n_i} > I_{\text{th}}$, the new user, n_i , is admitted by the SCBS as a new SCU. In this case, given the fact that two users, i.e., f and n_i , are assumed within the SC system, the SCBS should adopt a sophisticated resource allocation technique to serve these SCUs. In this paper, we assume that the SCBS utilizes the available information such as channel power gains (h and g_{n_i}) and the QoS constraint (θ), to find the optimal resource allocation scheme and finally, characterize the achievable effective capacity of the SC communication system.

As in [17], the problem of obtaining an optimal resource allocation scheme can be defined by the problem of finding a transfer function in the h - g_{n_i} plane, $Z(h)$, such that the user f is served by the SCBS when $g_{n_i} < Z(h)$ and the user n_i is served by the SCBS when $g_{n_i} > Z(h)$. Particularly, the optimal resource allocation scheme consists of a set of points on the boundary of the achievable capacity region such that the effective capacity of user n_i is maximized, while the effective capacity of user f is fixed. Accordingly, to obtain these points on the boundary of the achievable capacity region, we form the following optimization problem

$$C_{\text{eff}}^{\max}(\theta, \xi) = \max_{Z(h)} \{ \xi C_f(\theta, \xi) + \bar{\xi} C_{n_i}(\theta, \xi) \}, \quad (5)$$

where ξ ($\bar{\xi} = 1 - \xi$) denotes the weight allocated to the SCU f with $\xi \in [0, 1]$ and $C_f(\theta, \xi)$ and $C_{n_i}(\theta, \xi)$ are defined as

$$C_f(\theta, \xi) = \frac{-1}{\theta} \ln(E_{h, g_{n_i}}[e^{-\theta R_f}]), \quad (6)$$

$$C_{n_i}(\theta, \xi) = \frac{-1}{\theta} \ln(E_{h, g_{n_i}}[e^{-\theta R_{n_i}}]), \quad (7)$$

where R_f is given in (3) and

$$R_{n_i} = TB \ln \left(1 + \frac{P_{\text{SC}} g_{n_i}}{\delta_n^2} \right). \quad (8)$$

Following the above discussion, since TD multiple access scenario is considered at the SCBS transmission, only one user can be served at time n . This means, when $R_f > 0$, it follows that $R_{n_i} = 0$ and vice-versa. Accordingly, the maximization problem in (5) can be rewritten as

$$C_{\text{eff}}^{\max}(\theta, \xi) = \max_{Z(h)} \left\{ \frac{-\xi}{\theta} \ln \left(\mathbb{E}_{h, g_{n_i} | R_f=0} [e^{-\theta R_f} X_2] + \mathbb{E}_{h, g_{n_i} | R_f>0} [e^{-\theta R_f} X_1] \right) - \frac{\bar{\xi}}{\theta} \ln \left(\mathbb{E}_{h, g_{n_i} | R_{n_i}=0} [e^{-\theta R_{n_i}} X_1] + \mathbb{E}_{h, g_{n_i} | R_{n_i}>0} [e^{-\theta R_{n_i}} X_2] \right) \right\}, \quad (9)$$

where the indicator function $X_i \forall i = 1, 2$ is defined as follows:

$$X_1 = \begin{cases} 1 & g_{n_i} < Z(h) \\ 0 & g_{n_i} > Z(h) \end{cases}, \quad X_2 = \begin{cases} 1 & g_{n_i} > Z(h) \\ 0 & g_{n_i} < Z(h) \end{cases}. \quad (10)$$

Proposition 2. *Considering the fact that the function $\ln(x)$ is a monotonically increasing function of x , the maximization problem in (9) can be simplified into the following minimization problem as*

$$C_{\text{eff}}^{\max}(\theta, \xi) = \min_{Z(h)} \left\{ \frac{\xi}{\theta} \ln \left(\int_0^\infty d_1(h, Z(h)) dh \right) + \frac{\bar{\xi}}{\theta} \ln \left(\int_0^\infty d_2(h, Z(h)) dh \right) \right\}, \quad (11)$$

where

$$d_1(h, Z(h)) = \frac{e^{-\frac{h}{\delta_h^2}}}{\delta_h^2} \left(e^{-\frac{Z(h)}{\delta_{g_{n_i}}^2}} + \left(1 + \frac{P_{\text{SC}} h}{\delta_n^2} \right)^{-\theta T B} \left(1 - e^{-\frac{Z(h)}{\delta_{g_{n_i}}^2}} \right) \right), \quad (12)$$

$$d_2(h, Z(h)) = \frac{e^{-\frac{h}{\delta_h^2}}}{\delta_h^2} \left(1 - e^{-\frac{Z(h)}{\delta_{g_{n_i}}^2}} + G(Z(h)) \right), \quad (13)$$

and

$$G(Z(h)) = \frac{(\delta_n^2)^{\theta T B} e^{\frac{\delta_n^2}{P_{\text{SC}} \delta_{g_{n_i}}^2}} (\delta_n^2 + P_{\text{SC}} Z(h))^{1-\theta T B}}{P_{\text{SC}} \delta_{g_{n_i}}^2} \times E_{\theta T B} \left(\frac{\delta_n^2 + P_{\text{SC}} Z(h)}{P_{\text{SC}} \delta_h^2} \right). \quad (14)$$

Proof. Cf. [16]. ■

Now, considering that the optimization problem in (11) is a variational problem [18], we adopt the Euler-Lagrange technique to solve the minimization problem and find the optimal resource allocation scheme, $Z(h)$, as presented in Theorem 1.

Theorem 1. *The optimal resource allocation scheme that consists of a set of points on the boundary of the achievable*

capacity region, is given by

$$Z(h) = \frac{\delta_n^2}{P_{\text{SC}}} \times \left(\left(1 + \frac{\xi D_2}{\bar{\xi} D_1} \left(\left(1 + \frac{P_{\text{SC}} h}{\delta_n^2} \right)^{-\theta T B} - 1 \right) \right)^{\frac{-1}{\theta T B}} - 1 \right), \quad (15)$$

where $D_i \forall i = 1, 2$ is defined as

$$D_i \triangleq \int_0^\infty d_i(h, Z(h)) dh, \quad (16)$$

where d_1 and d_2 are given in (12) and (13), respectively.

Proof. Cf. [16]. ■

By considering $\xi \in [0, 1]$, D_1 and D_2 in (15) for a given value of QoS constraint θ , we can obtain a class of optimal allocation schemes in which can be used to calculate the achievable effective capacity in (11). In fact, initiating D_1 and D_2 to some positive values and making use of (15), there exists an optimal allocation function $Z(h)$ for a given values of ξ and θ . On the other hand, it is noted that D_1 and D_2 are both depend on the optimal allocation function $Z(h)$ through (16) (see also (12) and (13)). In this regard, algorithm 1 is provided to calculate the optimal allocation function $Z(h)$ using an iterative technique. It is worth noting that it is observed that the proposed iterative method is a convergent algorithm and generally converges in about five to eight iterations.

Algorithm 1 Calculate the optimal resource allocation scheme, $Z(h)$.

- 1: Initiate D_1 and D_2 to some positive values, e.g., $D_1 = D_2 = 1$.
 - 2: For the assumed value of ξ and θ , calculate $Z(h)$ using the formula given in (15).
 - 3: Knowing (16), update the values of D_1 and D_2 by using $Z(h)$ obtained in step 2.
 - 4: Repeat steps 2 and 3 until convergence occurs.
 - 5: End.
-

Finally, having obtained the optimal allocation function $Z(h)$ and calculating D_1 and D_2 given in (16), the achievable effective capacity of the SCNet in the second case where $I_{\text{SC}-n_i} > I_{\text{th}}$ can be obtained according to

$$C_{\text{eff}}(\theta, \xi) = \frac{\xi}{\theta} \ln(D_1) + \frac{\bar{\xi}}{\theta} \ln(D_2). \quad (17)$$

IV. ILLUSTRATIVE NUMERICAL RESULTS

In this section, we perform the obtained allocation policy for the proposed admission algorithm at the SCNet transmission and provide numerical results in terms of the achievable effective capacity at the SCNet. In our numerical results, we consider $\delta_h^2 = \delta_n^2 = 0$ dB, $H_{\text{SC}} = H_{n_i} = H_{m_j} = 1$, $\alpha = 2$, $T = 100$ msec and $B = 10$ KHz. It is worth noting that in our figures, we assume that $d_{n_i} < d_{m_j}$ with $d_{m_j} = 1.2$.

In Figs. 3 and 4, considering the AIC-based admission algorithm, the achievable effective capacity regions for the

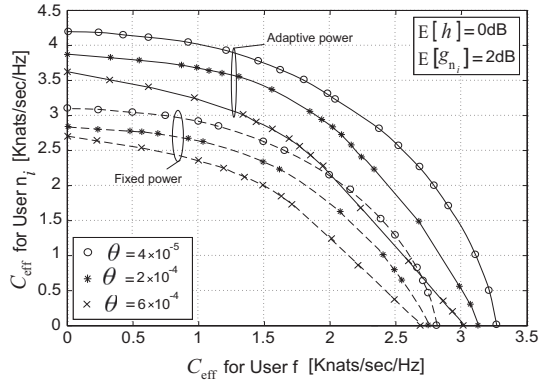


Figure 3: Achievable effective capacity region for the SCUs at the SCBS and for various values of service delay exponent, θ .

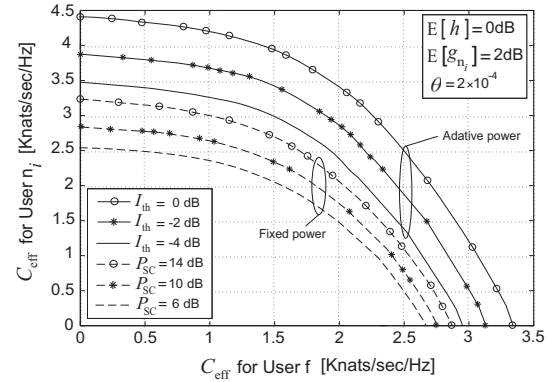


Figure 4: Achievable effective capacity region for the SCUs at the SCBS and for various values of P_{SC} .

two SCUs, f and n_i , in a SCNet are shown for different values of QoS exponent constraint. In these figures, we set $d_{n_i} = 0.9$ and $I_{th} = -2$ dB. In Fig. 3, the variation of the QoS exponent θ is investigated when we set $P_{SC} = 10$ dB. We observe that the effective capacity of the SCNet decreases as the QoS exponent increases. On the other hand, in Fig. 4, we set $\theta = 2 \times 10^{-4}$ and illustrate the effect of the transmit power, P_{SC} , at the SCBS transmitter on the effective capacity achieved by the SCUs while utilizing the AIC-based admission algorithm. In our results, we observe that by increasing the transmit power P_{SC} at the SCBS, the achievable effective capacity of SCUs increases, while adhering to the co-channel interference constraint at the neighboring MUs.

V. CONCLUDING REMARKS

In this paper, a HSCNet with co-channel deployment was considered and we studied the throughput performance of our proposed admission control algorithm based on the maximum tolerable interference limit at the neighboring macro users. In this regard, we proposed the admission algorithm which is operating based on the average received interference generated at the neighboring MUs. To evaluate the throughput performance of the proposed admission algorithm, we obtained the associated optimal resource allocation scheme at the SCBS transmitter for the case of fixed power transmission scheme such that the achievable effective capacity performance of the SCNet was maximized subject to satisfying a statistical delay QoS constraint. Finally, the numerical results were provided to illustrate the performance and benefits of using the proposed algorithm in the HSCNets. Indeed, it was shown that by utilizing the proposed admission algorithm in a HSCNet with co-channel deployment, the new users can be admitted to serve by the SCNet while no harmful interference is caused to the macro network.

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