Large Scale Opportunistic Antenna and User Selection in AF Relay Networks with Interference

Imène Trigui, Sofiène Affes, and Alex Stéphenne
INRS-EMT, 800, de la Gauchetière Ouest, Bureau 6900, Montréal, H5A 1K6, Qc, Canada.

***Abstract*—In this paper, the asymptotic performance of multiuser multiple-antenna (MU-MIMO) relay networks employing opportunistic scheduling and operating in the presence of rayleigh fading and co-channel interference is investigated. Notwithstanding the system complexity, due to the newly found complementary moment generating function (CMGF) transform, an exact expression for the capacity, when the antenna counts at the source and the user number are allowed to grow unbound, is obtained. The large scale analysis embodies popular observations, so far intuitively or empirically disclosed, through analytically insightful new formulas. An interesting aspect of this analysis comes from an altered view of multiuser diversity in the context of cellular systems. Previously, multiuser diversity capacity gain has been known to grow as $O(\ln\ln(K))$, from selecting the maximum of K exponentially-distributed powers. Because interference aware scheduling is considered, we find instead that the gain is $O(\ln(K^{1/Q}))$ where Q is the number of interferers. Simulation results indicate a rather fast convergence to the asymptotic limits with the system's size, thereby demonstrating the practical importance of the scaling results.**

***Index Terms*—Amplify-and-forward, MIMO systems, multiuser diversity, opportunistic scheduling, co-channel interference.**

I. INTRODUCTION

Driven by the surge of shared data volume and connected devices, multiuser multiple-antenna (MU-MIMO) relaying networks have drawn recently a significant attention, as a promising solution to cope with the necessities of more efficient and larger networks. Aiming to enhance multiuser capacity, multiple antenna communications have been actually identified as a key enabling technique to secure the unprecedented data deluge these large networks are deemed to convey [1]- [2]. As such, there has been prominent activity in the past decade toward understanding the fundamental system capacity limits of such architectures, notably when interference-limited, the ultimate nature of future cellular networks [3]- [4].

Many contributions spearheaded this line of research by considering the combination of cooperative and multiuser diversities in the context of single-antenna communications [5]- [7]. It has been shown in multiuser dual-hop amplify-and-forward (AF) relaying networks, that the end-to-end signal-to-noise ratio (SNR) is an inadequate criterion for reduced-feedback approaches with threshold-based SNR. Alternatively, the second-hop SNR which requires less attendant in complexity at the relay, turns out to be more promising in terms of achieved capacity.

Work supported by the CREATE PERSWADE Program of NSERC and a Discovery Accelerator Supplement Award from NSERC.

Aiming to further increase the system capacity and reliability, another line of work dedicated to multiuser relay-assisted networks with multi-antenna communications is longing for understanding such systems [8]- [9]. In [9], capacity is achieved by allowing the communication, through an AF relay, of multiple antenna devices and a source, using receive antenna diversity. It has been shown, only through empirical trials, that spatial diversity at the destinations deteriorates the system capacity and, further, burdens the feedback cost.

Although these works have made great strides toward understanding MU-MIMO relay-assisted communications, they all rely on the absence of the harmful effect of co-channel interference (CCI). The recognition of the interference-limited nature of emerging communication systems, such as heterogeneous cellular networks, has motivated several works to investigate the impact of CCI on the performance of relay networks for different fading models and communication setups [10]- [13]. In [11], a novel analytical capacity expression for two-hop multiple antenna AF relaying systems have been proposed. The more general case of multihop interference-limited communications has also been treated in [12]. However the works in [10] and [13] only consider a single-user scenario. So far, CCI assessment in the context of multiuser relaying networks has only recently been considered in [14] by harnessing on opportunistic scheduling. This work, however, provides only bounds on the system capacity without characterizing its scaling laws.

The ultimate goal of this paper is to quantify more accurately the capacity of MU-MIMO relay-assisted networks if straightforward opportunistic scheduling is employed among users and transmit selection is employed among sources's antennas in a cellular environment. More importantly, it shows that understanding how CCI affects multiuser and spatial diversity gains become feasible by exploiting some newly derived scaling laws.

Notation: $\dagger$ is the transpose complex conjugate operator. $\stackrel{d}{=}$ denotes an equality in distribution. $\mathcal{CN}(\mu,\sigma^2)$ is a complex Gaussian Random Variable (RV) with mean μ and variance σ^2. $E\{\cdot\}$ is the expectation operator. $M_X(s)=E\{e^{-sX}\}$ is the MGF of RV X. $F_X(z)=Pr(X\leq z)$ is the Cumulative Distribution Function (CDF) of X and $F_X^{(c)}(z)=1-F_X(z)$ is its Complementary Cumulative Distribution Function (CCDF). $M_X^{(c)}(s)=1-sM_X(s)$ is the complementary MGF of X while $f_X(x)$ is its Probability Density Function (PDF). $\Gamma(z,x)$ and ${}_2\mathrm{F}_1(a,b,c,x)$ denote the upper incomplete gamma func-

978-1-4673-9814-5/16/$31.00 ©2016 IEEE

tion [15, Eq.(8.350.2)] and the Gauss hypergeometric function [15, Eq.(9.100)], respectively. In turn, the second-type Bessel function of order ν and the exponential integral function are, respectively, denoted by $\mathrm{K}_{\nu}(x)$ and $\mathrm{E_i}(x)$ [15, Eq.(8.222.1)]. $\mathrm{W}(a, b, z)$ is the Whittaker hypergeometric function [15, Eq.(9.222.1)]. "$\chi(2p)$" denotes a chi-square random variable with $2p$ degrees of freedom. For two functions $f(n)$ and $g(n)$, $f(n) = O(g(n))$ means $\lim_{n \longrightarrow \infty} |f(n)/g(n)| < \infty$.

II. System Model

We consider a downlink of an AF multi-user half-duplex relay network consisting of K user nodes (U_k) for $k \in \{1, \ldots, K\}$, one source node (S), and one relay node (R). User nodes are spatially distributed single-antenna terminals. The source and relay are MIMO-enabled, and are equipped with M and N antennas, respectively. Here, we assume that M is significantly larger than N. Further, M and K can grow without bound. We further assume that the multi-user relay network of interest is affected by perfectly synchronized co-channel interfering signals from other sources belonging to adjacent-cell wireless systems. In the first hop the relay is aware of the channel matrix from the source to the relay, denoted by $\mathbf{H}$, and, hence, is able to schedule the transmission of the strongest source antenna. The signal model of the first-hop downlink channel is, hence, given by

$$y = \mathbf{w}^{\dagger} \left(\sqrt{\rho} \mathbf{H}^{(i)} x + \sqrt{\mu} \sum_{l=1}^{L} \mathbf{f}_l x_l \right), \qquad i = 1, \ldots, M, \quad (1)$$

where (i) denotes that the ith column is selected out of M in the full propagation matrix $\mathbf{H} \sim \mathcal{C}N_{N \times M}(\mathbf{0}_{N \times M}, \mathbf{I}_N \times \mathbf{I}_M)$ that accounts for the independent, fast Rayleigh fading. In (1), the variables ρ and μ contains the transmit powers of the source and interferences, respectively, and $\mathbf{f}_l$ is the channel vector between the relay the and the lth interferer. Again, $\mathbf{f}_l$ is the i.i.d. fast fading. Moreover, $\mathbf{w}^{\dagger} = \mathbf{H}^{(i)} / \|\mathbf{H}^{(i)}\|$ is the receive-MRC vector at the relay. Next, the latter employs an amplification factor, which is designed to constrain the instantaneous transmit power on its received signal vector. This amplification factor can then be written as

$$G_R = \sqrt{\frac{\lambda}{\left(\rho |\mathbf{H}^{(i)}|^2 + \mu \sum_{l=1}^{L} |\mathbf{w}^{\dagger} \mathbf{f}_l|^2 \right)}}, \quad (2)$$

where λ is the transmission power at the relay. After amplification at the relay, the received signal at user node (U_k) can be written as

$$z_k = \mathbf{w}_k^{\dagger} \mathbf{g}_k G_R y + \sqrt{\nu} \sum_{j=1}^{Q_k} g_{k,j} x_{k,j}, \quad j = 1, \ldots, K, \quad (3)$$

where $\mathbf{g}_k$ for $k \in \{1, \ldots, K\}$ is the channel vector between the kth user and the relay. Further, $g_{k,j}$ is the channel coefficient between the kth user and the interference j with transmit power ν. To exploit the multiuser diversity of the system, we consider an interference-aware user scheduling algorithm in which users are selected based on their respective SIRs, i.e.,

$$k = arg \max_{j=1,\ldots,K} \frac{\lambda}{\nu} \frac{|\mathbf{g}_k|^2}{\sum_{j=1}^{Q_k} |g_{k,j}|^2}. \quad (4)$$

Next, by substituting (1) and (2) into (3), the instantaneous end-to-end SIR of the MIMO AF dual hop interference aware user scheduling is given by

$$\gamma = \frac{\frac{Y^{(M)}}{\mu \sum_{l=1}^{L} |\mathbf{w}^{\dagger} \mathbf{f}_l|^2} Z^{(K)}}{1 + \frac{Y^{(M)}}{\mu \sum_{l=1}^{L} |\mathbf{w}^{\dagger} \mathbf{f}_l|^2} + Z^{(K)}}, \quad (5)$$

where $Y^{(M)} = \rho \max_{i=1,\ldots,M} |\mathbf{H}^{(i)}|^2$ and $Z^{(K)} = \max_{j=1,\ldots,K} \frac{\lambda}{\nu} \frac{|\mathbf{g}_k|^2}{\sum_{j=1}^{Q_k} |g_{k,j}|^2}$. The average achievable ergodic capacity of the multi-user AF MIMO relay network can be computed as follows

$$C = \frac{1}{2\ln(2)} \int_0^{\infty} s e^{-s} M_{X^{(M)}}^{(c)}(s) M_{Z^{(K)}}^{(c)}(s) ds, \quad (6)$$

where $X^{(M)} = \frac{Y^{(M)}}{\mu \sum_{l=1}^{L} |\mathbf{w}^{\dagger} \mathbf{f}_l|^2}$. Moreover, in (6), we define the CMGF of X as

$$M_X^{(c)}(s) \triangleq \int_0^{\infty} e^{-sx} F_X^{(c)}(x) dx. \quad (7)$$

Interested readers are referred to [13, Theorem 1] for the proof of (6), omitted here for conciseness.

III. Large Scale Capacity Analysis

In this section the asymptotic ergodic capacity with massive antenna selection and interference-aware user scheduling, i.e., when the numbers of antennas at the source M and user number K grow large. The number of relay antennas N is kept, however, small and finite, a consideration deemed in line with the current wireless communication landscape, which portray a tremendous increase in the number of wireless terminals of various types yet only a small increase in the number of wireless infrastructure (relay) support. Since using (6), we first derive the asymptotic CMGFs of $X^{(M)}$ and $Z^{(K)}$ in closed-forms.

A. Asymptotic CMGF of $X^{(M)}$

Let $Y_i = \rho |\mathbf{H}^{(i)}|^2 \overset{d}{=} \rho \chi(2N)$ with PDF and CDF given, respectively by, $f_Y(x) = \frac{(\frac{x}{\rho})^{N-1}}{N!} e^{-\frac{x}{\rho}}$ and $F_Y(x) = 1 - \frac{\Gamma(N, \frac{x}{\rho})}{\Gamma(N)}$. In what follows we show that the distribution of $Y^{(M)} = \max_{i=1,\ldots,M} Y_i$ approaches Gumbel distribution, $\exp[-\exp(-\frac{x - b_M}{\rho})]$, as the number of the source antennas, M, is increased, where $b_M = F_Y^{-1}(1 - \frac{1}{M})$. First of all we show that

$$\frac{1 - F_Y(x)}{f_Y(x)} = \frac{\Gamma(N, \frac{x}{\rho})}{(\frac{x}{\rho})^{N-1} e^{-\frac{x}{\rho}}} \underset{x \to \infty}{\longrightarrow} \rho > 0. \quad (8)$$

In this paper, a homogeneous network in which all users are clustered together, whereby $\lambda_j = \lambda, j = 1 \ldots K$ are considered. This policy guarantees a uniform user experience and saves valuable energy at terminals.

Next to find b_M, we apply the asymptotic expansion of the incomplete Gamma function given by [15, Eq.8.257] leading to

$$1 - F_Y(b_M) = \frac{b_M}{\rho}^{N-1} e^{-\frac{b_M}{\rho}}(1 + O(1/b_M)) = \frac{1}{M}$$
$$\Longrightarrow \frac{b_M}{\rho} - (N-1)\ln\left(\frac{b_M}{\rho}\right) = \ln(M), \quad (9)$$

and implying that

$$b_M = \rho\ln(M) + \rho(N-1)\ln\ln(M) + O(\ln\ln\ln(M)). \quad (10)$$

Substituting (8) and (10) verifies that

$$\frac{d}{dx}\left[\frac{1-F_Y(x)}{f_Y(x)}\right]_{x=b_M} = 0. \quad (11)$$

Therefore, the asymptotic distribution of $Y^{(M)}$ is Gumbel type [18, Th. 10.5.2] with the constant b_M as defined in (10).

Since the relay is subject to L interferences assumed for sake of tractability and conciseness to be i.i.d with mean power μ, the CCDF of the first hop SIR $X^{(M)} = \frac{Y^{(M)}}{\mu\sum_{l=1}^{L}|\mathbf{w}^\dagger\mathbf{f}_l|^2}$ is obtained as

$$F^{(c)}_{X^{(M)}}(x) = \int_0^\infty F^{(c)}_{Y_M}(xy) f_{\mu\sum_{l=1}^{L}|\mathbf{w}^\dagger\mathbf{f}_l|^2}(y)dy, \quad (12)$$

where for L i.i.d zero mean and unit variance interferences at the relay we have $f_{\sum_{l=1}^{L}|\mathbf{w}^\dagger\mathbf{f}_l|^2}(y) = \frac{y^{L-1}e^{-y}}{\Gamma(L)}$. Accordingly, we get the CCDF of $X^{(M)}$ as

$$F^{(c)}_{X^{(M)}}(x) = \frac{\mu^{-L}}{\Gamma(L)}\int_0^\infty y^{L-1}e^{-\frac{y}{\mu}}\left(1 - e^{-e^{-\frac{yx-b_M}{\rho}}}\right)dy. \quad (13)$$

By letting $t = e^{-\frac{yx}{\rho}}$, we have $y = -\frac{\rho}{x}\ln(t)$. Furthermore, for notational simplicity, we define $\zeta = e^{\frac{b_M}{\rho}}$, thus (13) can be rewritten as

$$F^{(c)}_{X^{(M)}}(x) = \frac{(-1)^{L-1}\left(\frac{\rho}{\mu x}\right)^L}{\Gamma(L)}\left(\int_0^{\frac{4}{\zeta}}\ln(t)^{L-1}t^{\frac{\rho}{\mu x}-1}\left(1 - e^{-\zeta t}\right)dt + \int_{\frac{4}{\zeta}}^1 \ln(t)^{L-1}t^{\frac{\rho}{\mu x}-1}dt\right). \quad (14)$$

Recalling (10), we can easily see that the limit of the first term on the R.H.S. of (14) becomes vanishingly small as $\lim_{M\longrightarrow\infty}\frac{4}{\zeta} \approx \frac{4}{M} = 0$. Furthermore, the limit of the second term can be simplified using [15, Eq. (3.381.1)] as

$$F^{(c)}_{X^{(M)}}(x) \overset{(a)}{\simeq} 1 - \frac{\Gamma\left(L, \frac{\beta_M^2}{x}\right)}{\Gamma(L)}, \quad (15)$$

where $\beta_M = \sqrt{\frac{b_M - \rho\ln(4)}{\mu}}$. In turn, a closed-form expression of the CMGF of the fist hop SIR $X^{(M)}$ when M geos to infinity follows form plugging (15) into (7) as

$$\begin{aligned} M^{(c)}_{X^{(M)}}(s) &= \int_0^\infty e^{-sx}\left(1 - \frac{\Gamma\left(L, \frac{\beta_M^2}{x}\right)}{\Gamma(L)}\right)dx \\ &= \frac{1}{s} - 2\sum_{n=0}^{L-1}\frac{\beta_M^{n+1}}{n!}s^{\frac{n-1}{2}}K_{n-1}\left(2\beta_M\sqrt{s}\right) \end{aligned} \quad (16)$$

after using [15, Eq.(8.352.2)] and [15, Eq.(3.471.9)].

B. Asymptotic CMGF of $Z^{(K)}$

Here we present of the asymptotic distribution of the maximum of K i.i.d SIRs denoted by $Z^{(K)} = \frac{\lambda}{\nu}\max_{j=1,\ldots,K}\frac{|\mathbf{h}_j^R|^2}{\sum_{l=1}^{Q}|u_l|^2}$. Before embarking on the proof, it is worth examining the SIRs $Z_j = \frac{|\mathbf{h}_j^R|^2}{\sum_{l=1}^{Q}|u_l|^2}$. We have $Z_j \overset{d}{=} \frac{\chi(2N)}{\chi(2Q)}, j = 1\ldots, K$ who are K i.i.d random variables with CDF

$$F_Z(x) = \frac{x^N\ {}_2\mathrm{F}_1(N+Q, N, 1+N, -x)}{NB(N,Q)}, \quad (17)$$

implying that $F_{Z^{(K)}}(x) = [F_Z(\frac{\nu}{\lambda}x)]^K$. First, we show that

$$\begin{aligned} \lim_{x\to\infty}\frac{1-F_Z(x)}{1-F_Z(tx)} &= \lim_{x\to\infty}\frac{t^{-N}(1+tx)^{N+Q}}{(1+x)^{N+Q}} \quad (18) \\ &= t^Q, \quad (19) \end{aligned}$$

where to get from (18) to (19) we use the Hospital rule after substituting the hypergeometric function by its equivalent ${}_2\mathrm{F}_1(a,b;b+1;z) = bz^{-b}B_z(b,1-a)$ where $B_z(c,d)$ is the Beta function [15, Eq.(8.380.1)]. This implies that the limit distribution of $Z^{(K)}$ lies in the domain of maximal attraction of Fréchet distribution [18, Theroem 10.5.2], i.e.

$$[F_Z(a_K x)]^K = e^{-x^{-Q}}, x \geq 0, \quad (20)$$

where a_K is a normalizing parameter defined such that $F_Z(a_K) = 1 - \frac{1}{K}$. In order to find a_K while keeping the analytical complexity tractable, we concentrate the following analysis on cases (i) $Q = 1, \forall\ N$, and (ii) $N = 1, \forall\ Q$. The arbitrary N, Q case can be handled using bounding techniques but thwarting the paper goal of exact capacity analysis. In case (i), by exploiting the ${}_2\mathrm{F}_1$ reduction formulas ${}_2\mathrm{F}_1(b,a;a;z) = (1-z)^{-b}$, we show that a_K satisfies

$$a_K \underset{K\to\infty}{\approx} NK. \quad (21)$$

In case (ii), we have $N = 1$ thereby enabling the simplification of $F_Z(x)$ relying on the fact that ${}_2\mathrm{F}_1(1,b;2;z) = \frac{(1-z)^{-b}-1}{(b-1)z}$. The parameter a_k is therefore obtained as follows

$$a_K \underset{K\to\infty}{\approx} K^{1/Q} - 1. \quad (22)$$

Substituting a_K into (20), we obtain

$$F_{Z^{(K)}}(x) = \begin{cases} e^{-\frac{NK\lambda}{\nu x}}, & Q = 1, \forall\ N \\ e^{-\left(\frac{\left(K^{1/Q}-1\right)\lambda}{\nu x}\right)^Q}, & N = 1, \forall\ Q \end{cases} \quad (23)$$

As for the CMGF of Z^K, replacing (23) into (7) and performing some algebraic manipulations, we get

$$M^{(c)}_{Z^{(K)}}(s) \overset{(a)}{\approx} \begin{cases} \frac{1}{s}\left(1 - e^{-\frac{NK\lambda}{4\mu}s}\right), & Q = 1, \forall\ N \\ \frac{1}{s}\left(1 - e^{-\frac{\lambda}{\mu}\left(\frac{K^{\frac{1}{Q}}-1}{4^{\frac{1}{Q}}}\right)s}\right), & N = 1, \forall\ Q \end{cases} \quad (24)$$

where (a) follows from the fact that $1 - e^{-x} \underset{x\geq 4}{\approx} 1$.

we assume that all users have the same statistical behavior with equal interference number $Q_j = Q, j = 1\ldots K$ and similar interference distribution.

C. Ergodic Capacity

In this subsection, the asymptotic ergodic capacity expression is derived in closed-form.

Theorem 1: The asymptotic capacity of the MU-MIMO relay network under two hop AF relaying and interference-aware user scheduling is given by

$$C = \frac{1}{2\ln(2)}\left(\ln\left(\alpha_K\right) - e^{\frac{\beta_M^2}{\alpha_K}}\Gamma\left(0,\frac{\beta_M^2}{\alpha_K}\right) - e^{\beta_M^2}\mathrm{E_i}\left(-\beta_M^2\right)\right.$$
$$\left.-\sum_{n=1}^{L-1}\frac{\beta_M^n}{n}\left(e^{\frac{\beta_M^2}{2}}W_{-\frac{n}{2},\frac{n-1}{2}}\left(\beta_M^2\right) - \frac{e^{\frac{\beta_M^2}{2\alpha_K}}}{\alpha_K^{n/2}}\mathrm{W}_{-\frac{n}{2},\frac{n-1}{2}}\left(\frac{\beta_M^2}{\alpha_K}\right)\right)\right), \quad (25)$$

where $\beta_M = \sqrt{\frac{b_M - \rho\ln(4)}{\mu}}$ and $\alpha_K = 1 + \frac{\widehat{a}_K\lambda}{\nu}$. with

$$\widehat{a}_K = \begin{cases} \frac{NK}{4}, & Q=1, \forall\ N \\ \frac{K^{\frac{1}{Q}}-1}{4^{\frac{1}{Q}}}, & N=1, \forall\ Q \end{cases} \quad (26)$$

Proof: Plugging (16) and (24) into (6) yields

$$C = \frac{1}{2\ln(2)}\left(\int_0^\infty \frac{e^{-s}}{s}\left(1-e^{-\frac{\widehat{a}_K\lambda}{\mu}s}\right)ds\right.$$
$$\left.-2\beta_M\Phi_0\left(\frac{\widehat{a}_K\lambda}{\nu},\beta_M\right) - 2\sum_{n=1}^{L-1}\frac{\beta_M^{n+1}}{n!}\Phi_n\left(\frac{\widehat{a}_K\lambda}{\nu},\beta_M\right)\right), \quad (27)$$

whereby

$$\Phi_n(a,b) \underset{n\geq 0}{=} \int_0^\infty e^{-s}s^{\frac{n-1}{2}}\left(1-e^{-as}\right)K_{n-1}\left(2b\sqrt{s}\right)ds. \quad (28)$$

A closed-form expression for Φ_0 is obtained, after several manipulations, as

$$\Phi_0(a,b) = \frac{1}{2b}\left(e^{\frac{b^2}{1+a}}\Gamma\left(0,\frac{b^2}{1+a}\right) + e^{b^2}\mathrm{E_i}\left(-b^2\right)\right). \quad (29)$$

Furthermore, applying [15, Eq.6.643], a closed-from expression for $\Phi_{n_{n\geq1}}$ is obtained as

$$\Phi_{n_{n\geq1}}(a,b) = \frac{\Gamma(n)}{2b}\left(e^{\frac{b^2}{2}}W_{-\frac{n}{2},\frac{n-1}{2}}\left(b^2\right) - \frac{e^{\frac{b^2}{2(1+a)}}}{(1+a)^{n/2}}W_{-\frac{n}{2},\frac{n-1}{2}}\left(\frac{b^2}{1+a}\right)\right). \quad (30)$$

Finally, substituting (29) and (30) into (27) and resorting to [15, Eq. 3.421.5] completes the proof.

IV. Scaling Laws

In an effort to understand the impact of key parameters on the system capacity and gain more physical insights, in this section, we look into the scaling laws to extract two important design parameters: 1) multiuser gain due to interference awareness, and capacity losses due to increased dimensionality of interference.

Corollary 1 (Case (i), M and $K\to\infty$): If K grows faster than $\ln(M)$ (i.e., $\lim_{M,K\longrightarrow\infty} K/\ln(M) = \infty$, which include the case K=M), it holds that

$$C \approx \frac{1}{2\ln(2)}\left(\ln\left(\frac{\rho}{\mu}\left(\ln(M)+(N-1)\ln\ln(M)\right)\right)+\gamma - H_{L-1}\right), \quad (31)$$

where γ is the Euler-Mascheroni constant [15, Eq.(8.367.1)] and H_n is the harmonic number of order n.

Proof: As $\frac{K}{\ln(M)} \longrightarrow \infty$, we have $\beta_M \underset{M\longrightarrow\infty}{\longrightarrow} \infty$ and $\frac{\beta_M}{\alpha_K} \underset{M,K\longrightarrow\infty}{\longrightarrow} 0$, then the following approximations hold: $\Gamma(0,z) \underset{z\longrightarrow0}{=} -\ln(z) - \gamma$, $e^z \underset{z\longrightarrow0}{=} 1 + o(z)$, and $e^zE_i(-z) \underset{z\longrightarrow\infty}{=} 0$, $e^{z/2}W_{a,b}(z) \underset{z\longrightarrow\infty}{\approx} z^a$, $z^{n/2}e^{z/2}W_{-n/2,b}(z) \underset{z\longrightarrow0}{=} 0$. Considering all these facts, C reduces after several manipulations to

$$C \approx \frac{1}{2\ln(2)}\left(\ln\left(\beta_M^2\right) + \gamma - \sum_{n=1}^{L-1}\frac{1}{n}\right), \quad (32)$$

proving thereby (31). The capacity then becomes independent of the relay-to-destination channel parameters. Besides, by using massive MIMO-enabled source and relay, the received power of each user node can be scaled down inversely proportional to the number of antennas at the relay without incurring any performance penalty.

Remark 1 (Increased Dimensionality of Interference): The capacity of large-scale MU-MIMO (with $K \gg \ln(M)$) in the presence of a large number of interferers L behaves as

$$2^{2C} \approx \frac{\rho}{\mu}\frac{\ln(M)+(N-1)\ln\ln(M)}{L}. \quad (33)$$

Proof: Resorting to the approximation of the Harmonic number as L goes large given by

$$H_{L-1} = \gamma + \ln(L-1) + O(L^{-2}), \quad (34)$$

it follows from (31) that

$$C \approx \frac{1}{2\ln(2)}\left(\ln\left(\rho\left(\ln(M)+(N-1)\ln\ln(M)\right)\right) - \ln(\mu L)\right), \quad (35)$$

yielding (33) after using some logarithmic identities. It can be inferred from Remark 1 that the capacity loss due to an increased dimensionality of spatial interference is much more pronounced than any improvement resulting form adding more active transmit antennas at the source. This is due to the fact that C decreases with $\ln(L)$ while it increases with $\ln\ln(M)$, respectively. In view of this it is not surprising that recent research on spatial multiplexing in cellular systems has reached the common conclusion that adding more transmit antennas or data streams at each base station can actually decrease the capacity at low SIR due to the increased dimensionality of spatial interference [4], [16].

Corollary 2 (Case (ii) and $\ln(M^{1/L}) \gg K^{1/Q}-1$):

If $Q \gg \left\lfloor \frac{\ln(K)}{\ln\ln(M)} \right\rfloor$, where $\lfloor x \rfloor$ denotes the integer part of x, then it holds that

$$C \approx \frac{1}{2\ln(2)}\left(\ln\left(1+\frac{\lambda}{\nu}\left(K^{\frac{1}{Q}}-1\right)\right) - L\frac{\frac{\lambda}{\nu}\left(K^{\frac{1}{Q}}-1\right)}{\frac{\rho}{\mu}\ln(M)}\right). \tag{36}$$

Proof: According to (25), the growth of $K^{1/Q}-1$ with respect to $\ln(M)$ is crucial for the capacity scaling law. In fact it is easy to show that, for $\left\lfloor \frac{\ln(K)}{\ln\ln(M)} \right\rfloor \ll Q$, we have

$$\frac{\beta_M^2}{\alpha_K} \underset{M,K\longrightarrow\infty}{\approx} \frac{\ln(M)}{K^{1/Q}-1} \longrightarrow \infty. \tag{37}$$

Then it follows immediately that all the terms but the first in the first term on the R.H.S of (25) become vanishingly small as M and K grow large with $\frac{\ln(M)}{K^{1/Q}-1} \longrightarrow \infty$. In fact, considering that $e^z\Gamma(0,z) \underset{z\longrightarrow\infty}{=} \frac{1}{z}$, $e^z E_i(-z) \underset{z\longrightarrow\infty}{=} -\frac{1}{z}$, and $z^{n/2}e^{z/2}W_{-n/2,\frac{n-1}{2}}(z) \underset{z\longrightarrow\infty}{=} 1-\frac{n}{z}$, it follows after some manipulations that

$$\begin{aligned} C \;\approx\; & \frac{1}{2\ln(2)}\left(\ln\left(1+\frac{\lambda}{\nu}\left(K^{\frac{1}{Q}}-1\right)\right) - \right.\\ & \left. L\left(\frac{1+\frac{\lambda}{\nu}(K^{\frac{1}{Q}}-1)}{\frac{\rho}{\mu}\ln(M)} - \frac{1}{\frac{\rho}{\mu}\ln(M)}\right)\right), \end{aligned} \tag{38}$$

thereby concluding the proof. More importantly, despite imposing (37), the latter appears to be insufficient to ensure the multiuser diversity gain in (36). In fact, C is a monotonically increasing function of K if and only if the following condition holds

$$\ln(M^{1/L}) \gg K^{1/Q}-1, \tag{39}$$

where (39) verifies $\frac{dC}{dK} > 0$.

Corollary 3 (Case (ii) and $\ln(M^{1/L}) \ll K^{1/Q}-1$):

When $\frac{\ln(M)}{K^{1/Q}-1} \longrightarrow 0$, valid for small to moderate values of Q, the scaling law for arbitrary L is given by

$$C \approx \frac{1}{2\ln(2)}\left(\ln\left(\frac{\rho}{\mu}\ln(M)\right)+\gamma - H_{L-1} - \frac{1}{L}\frac{\frac{\rho}{\mu}\ln(M)}{\frac{\lambda}{\nu}(K^{1/Q}-1)}\right), \tag{40}$$

Proof: : (40) is obtained in the same line of (31). The forth term in the R.H.S of (40) follows form resorting to the series expansion of the Whittaker function near zero. It is easy to prove that (40) is an increasing function of M only if

$$\ln(M^{1/L}) \ll K^{1/Q}-1, \tag{41}$$

thereby avoiding impeding the diversity gain.

Remark 2 (Interference Oblivious vs Interference Aware Multiuser Scheduling):

Neglecting the second term on the R.H.S of (36) yields

$$\begin{cases} 2^{2C} \overset{(a)}{\approx} \frac{\lambda}{\nu}K^{1/Q}, & \text{for small } Q \\ 2^{2C}-1 \overset{(b)}{\approx} \frac{\lambda}{\nu}\frac{\ln(K)}{Q}, & \text{for large } Q. \end{cases} \tag{42}$$

where (a) follows form the fact that $\ln(1+x) \approx \ln(x)$ when x is large enough (valid for small Q). Nevertheless, when Q is large, $\ln(1+x) \approx \ln(x)$ does not hold true. Instead, we have

$$K^{1/Q}-1 \underset{Q\gg 1}{\approx} \frac{\ln(K)}{Q}, \tag{43}$$

thereby yielding (b).

From (31) and (36), if the CSI is estimated in the second hop instead of SIR, the capacity will scale as $\ln\ln(K) - H_{Q-1}$ when Q is small. The SIR-based scheduling outperforms then its CSI-based (interference oblivious) counterpart, since $\ln\left(K^{1/Q}\right) > \ln\ln(K) - H_{Q-1}$ always holds when $K \gg 1$. This proves the advantage of having global network CSI knowledge, namely through cooperation where the CSI is shared between the transmitting nodes so that the interference generated in other cells is taken into consideration. When Q is large, interference-aware user scheduling still outperforms its interference-oblivious counterpart especially at low SIR. However, it exhibits the loglog behavior of the CSI-based scheme leading to slow increase of C as K goes to infinity. Obviously, when this slight gain is hindered by the overhead cost due to large Q, CSI-based scheduling proves to be a more attractive alternative.

Remark 3 (Capacity Losses Due to Interference):

Consider a two-hop AF relaying with interference-aware scheduling subject to L_1, Q_1 and L_2, Q_2 interferences at the relay and each destination, respectively. For tractability and without loss of generality, we assume that $M = K$ and $\frac{\rho}{\mu} = \frac{\lambda}{\nu}$. Then the capacity losses due to increased interference dimensionality can be computed as

$$\begin{cases} \delta_C = \frac{\ln\left(\frac{Q_1}{Q_2}\right) - \frac{L_1}{Q_1} + \frac{L_2}{Q_2}}{2\ln(2)}, & \ln(M^{1/L_i}) \ll K^{1/Q_i}-1 \\ \delta_C = \frac{H_{L_1-1} - H_{L_2-1} - \frac{Q_1}{L_1} + \frac{Q_2}{L_2}}{2\ln(2)}, & \ln(M^{1/L_i}) \gg K^{1/Q_i}-1 \end{cases} \tag{44}$$

where $i \in \{1,2\}$.

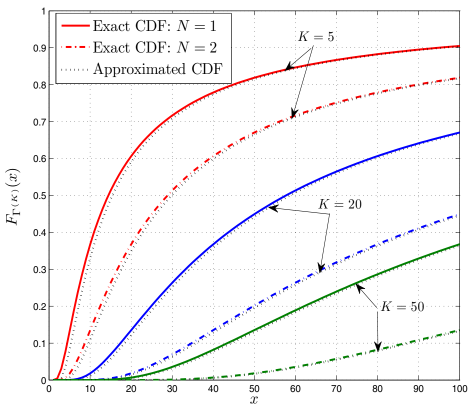
Proof: : (44) is obtained by resorting to (36) and (40) after invoking (43) along with some manipulations.

Equations (36) and (40) analytically unarguably confirm the common intuitive observation that AF relaying performance is ultimately the performance of one of its hops. This dominance is usually the output of the system's parametric objective function that quantifies the link strength. So far, in the literature, several functions defining the link's strength were proposed, namely, the antenna number, the fading shape, or the product of both. Oblivious to the interference number, these rules turn out to be totally inaccurate in interference-limited environments. This work remedies, for the first time this shortcoming. Indeed, the obtained results not only exemplify the interference nature of the considered system, but also analytically quantify through, simple formulas, the capacity loss due to interference.

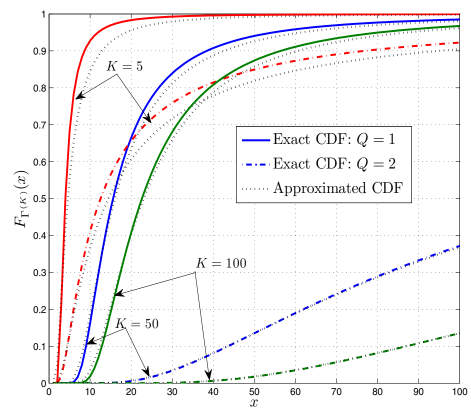
V. NUMERICAL AND SIMULATION RESULTS

Here, we provide some numerical examples to illustrate: 1) the tightness of the proposed approximations for large scale

These rules are usually obtained via a diversity order analysis by assuming equal per-hop ASIRs (cf. [13] and references therein).



(a)



(b)

Fig. 1. The exact and asymptotic distribution of $F_{Z(K)}$ for different values of N :(a) and Q :(b).

MU-MIMO relay networks; and 2) the impact of interference on spatial and multiuser diversity. The simulations set-up consists of an (M-N-K) MU-MIMO AF relaying system where the relay and each destination is subject to L and Q i.i.d. interferers, respectively. We also assume, without loss of generality, equal per-hop average SIRs $\frac{\rho}{\mu} = \frac{\lambda}{\nu}$ shorthanded in the plots as SIR.

Fig. 1 shows the exact and asymptotic CDFs of $Z^{(K)}$ for different values of N and Q. We observe that the asymptotic distribution in (23) is a good approximation even for small values of N and Q and the approximation becomes more accurate by increasing the value of K.

In Figs. 2 and 3, we illustrate the capacity for large M and K but finite and small relay antenna number N. The "approximation" curves refer to the average capacity using (25). These curves are in very good match with their stimulated counterparts, showing the accuracy and effectiveness of the proposed new approximations. We observe form Fig. 4 that the capacity exhibits the trend, $\ln(\ln(M)+(N-1)\ln\ln(M))+\gamma+H_{L-1}$, as predicted by Corollary 1. More importantly, the capacity loss due the increase of the number L of i.i.d interferences at the relay is consistent with the analysis. However, as the number of interference L exceeds a threshold, the capacity exhibits the trend in lemma 3.

In Figs, 4 and 5, the capacity of the two-hop opportunistic relaying scheme is shown as a function of increasing antenna number M and user number K for several numbers of interferences at the relay L and the users Q. The "approximation" curve refers to the average capacity using (25). These curves are in very good agreement with their stimulated counterparts. In Fig. 4 we observe that the capacity exhibits the trend predicted by Corollary 2 as long as $\ln(M^{1/L}) \gg K^{1/Q} - 1$. The latter condition is, however, not satisfied for $L = 3, Q = 4$ and $L = 4, Q = 5$ even though $Q > \left\lfloor \frac{\ln(K)}{\ln\ln(M)} \right\rfloor$ is always verified, impeding the inaccuracy of the scaling law in these cases. In Fig. 5 we observe that the capacity exhibits the trend predicted by (36) while we have neglected the second term of the R.H.S of (36) since $\frac{L(K^{\frac{1}{Q}}-1)}{\ln(M)} \longrightarrow 0$ when $L = 1$. Moreover, the capacity loss due to the increase of Q corroborates the analytical loss obtained using (44). Notice that, the tightness of the scaling law is poor when Q is near $\frac{\ln(K)}{\ln\ln(M)}$ which is equal to 4 when $K = M = 700$. In fact, when Q is at the vicinity of $\frac{\ln(K)}{\ln\ln(M^{1/L})}$, the capacity scaling law is more accurately predicted using (25).

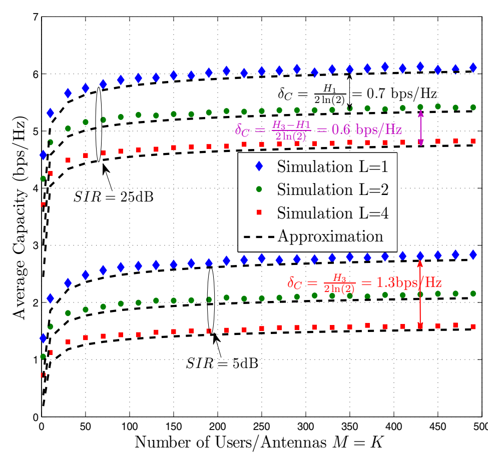


Fig. 2. System average capacity as a function of $M = K$ increasing when $Q = 1$ and $N = 2$ for different vales of L and $SIRs$.

VI. CONCLUSION

In this work, we have considered opportunistic scheduling design and analysis for two-hop MIMO AF relay networks in interference-limited environments. The scheme entails a two-hop communication protocol, in which an M-antenna source can communicate with K destinations only through a half-duplex multi-antenna relay. Through the asymptotic analysis for large M, K and simulations, it has been shown that interference-aware scheduling achieves an expanded multiuser diversity gain, however being conditioned. This analysis found that the multiuser diversity gain when $\ln(M^{1/L})$ grows

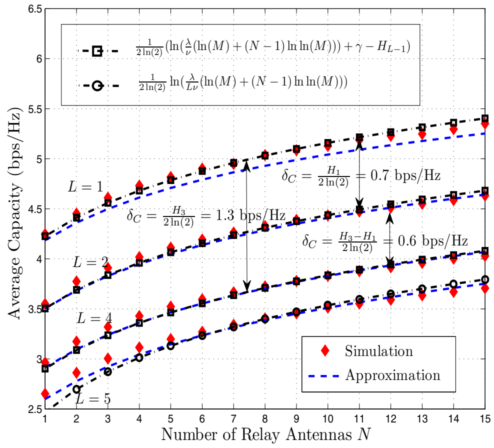


Fig. 3. System average capacity versus the relay antenna number N.

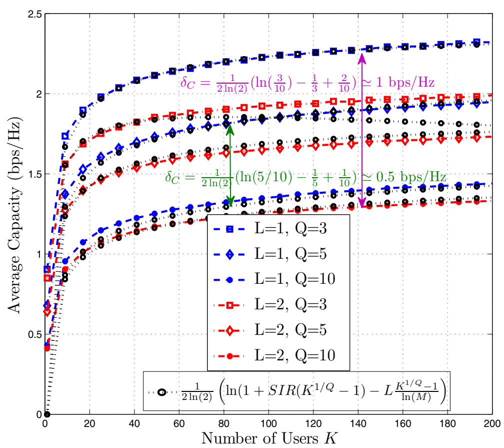


Fig. 4. System average capacity as a function of K increasing versus L, Q for $M = 1000$.

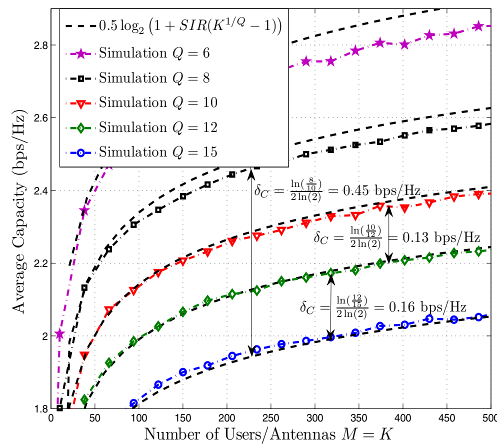


Fig. 5. System average capacity as a function of $M = K$ increasing when $L = 1$ and $N = 1$ for different values of Q.

faster than $K^{1/Q}$ is $\ln(K^{1/Q})$ whereas previous results have concluded that the maximum gain scales like $\ln\ln(K)$ when only an idealistic Rayleigh fading is considered. In this paper, the capacity losses due the increase of interferences numbers L and Q have been analytically quantified and verified via simulations.

REFERENCES

[1] C.J. Chen and L.C. Wang, "Enhancing coverage and capacity for multiuser MIMO systems by utilizing scheduling," *IEEE Trans. Wireless Commun.* vol. 5, no. 5, pp. 1148-1157, May 2006.

[2] P. Viswanath, D. N. C. Tse, and R. Laroia, "Opportunistic beamforming using dumb antennas," *IEEE Trans. Inform. Theory, vol.* 48, no. 6, pp. 1277-1294, June 2002.

[3] H. Dai, A. Molisch, and H. Poor, "Downlink capacity of interference-limited MIMO systems with joint detection," *IEEE Trans. Wireless Commun.,* vol. 3, no. 2, pp. 442-453, Mar. 2004.

[4] S. Catreux, P. F. Driessen, and L. J. Greenstein, "Attainable capacity of an interference-limited multiple-input multiple-output (MIMO) cellular system," *IEEE Trans. on Communications,* vol. 49, no. 8, pp. 1307-1311, Aug. 2001.

[5] X. Zhang, W. Wang, and X. Ji, "Multiuser diversity in multiuser two-hop cooperative relay wireless networks (TCRN): system model and performance analysis," *IEEE Trans. Veh. Technol.*, vol. 58, pp. 1031-1036, Feb. 2009

[6] H.J. Joung and C. Mun, "Capacity of multiuser diversity with cooperative relaying in wireless networks," *IEEE Commun. Lett.,* vol. 12, pp. 752-754, Oct. 2008.

[7] J.B. Kim and D. Kim, "Comparison of two SNR-based feedback schemes in multiuser dual-hop amplify-and-forward relaying networks," *IEEE Commun. Lett .,* vol. 12, pp. 557-559, Aug. 2008.

[8] W. Xu, X. Dong, W.S. Lu, "Joint precoding optimization for multiuser multi-antenna relaying downlinks using quadratic programming," *IEEE Trans Commun,* vol. 59, no. 5, pp. 12281235, oct. 2011.

[9] L. Yang and W. Liu, "On the capacity of MIMO relay wireless network with receive antenna selection," *IEEE Commun. Lett.*, vol. 15, no. 6, pp. 626-638, June 2011.

[10] G. Zhu, C. Zhong, H. A. Suraweera, Z. Zhang and C. Yuen, "Ergodic capacity comparison of different relay precoding schemes in dual-hop AF systems with co-channel interference," *IEEE Trans. Commun. ,* vol. 62, no. 7, pp. 2314-2328, July 2014.

[11] I. Trigui, S. Affes, and A. Stéphenne, "Ergodic capacity of two-hop multiple antenna AF systems with co-channel interference", *IEEE Wireless Commun. Let.*, vol. 4, no. 1, Feb. 2015.

[12] I. Trigui, S. Affes, and A. Stéphenne, "Ergodic capacity analysis for interference-limited AF multi-hop relaying channels in Nakagami-m fading," *IEEE Trans. Commun.,* vol. 61, no. 7, pp. 2726-2734, July 2013.

[13] I. Trigui, S. Affes, and A. Stéphenne, "On the ergodic capacity of amplify-and-forward relay channels with interference in Nakagami-m fading," *IEEE Trans. Commun.*, vol. 61, no. 8, pp. 3136-3145, Aug. 2013.

[14] Huang, F. Al-Qahtani, C. Zhong, Q. Wu, J. Wang, and H. Alnuweiri, "Performance analysis of multiuser multiple antenna relaying networks with co-channel interference and feedback delay," *IEEE Trans. Commun.* , vol. 62, no. 1, pp. 59-73, Jan. 2014

[15] I. S. Gradshteyn and I. M. Ryzhik, *Table of Integrals, Series and Products,* 5th Ed., San Diego, CA: Academic, 1994.

[16] R. Blum, "MIMO capacity with interference," *IEEE Journal on Sel. Areas in Communications,* vol. 21, no. 5, pp. 793-801, June 2003.

[17] M. R. Leadbetter and H. Rootzen, "Extremal theory for stochastic processes," *Ann. Probab.,* vol. 16, pp. 431-478, 1988.

[18] Gumbel, "Statistics of Extremes", New York: Columbia University Press, 1968.