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Energy-Efficient Distributed Amplify-and-Forward Beamforming for Wireless Sensor Networks

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Abstract—**In this paper, we consider an amplify-and-forward (AF) beamformer that achieves a dual-hop communication from a source to a receiver in highly-scattered environments, through a wireless sensor network (WSN) comprised of K independent and autonomous nodes. The AF beamforming weights are derived to maximize the received signal-to-noise ratio (SNR) subject to a constraint over the nodes' total transmit power. We verify that the so-obtained SNR-optimal AF beamformer (OB) implementation requires from the small-battery powered WSN nodes a prohibitive power cost. Exploiting the polychromatic structure of scattered channels, we develop a novel polychromatic (i.e., multi-ray) distributed AF beamformer (P-DB) that performs nearly as well as OB while requiring much less power consumption at each node. Furthermore, we prove that the proposed P-DB always outperforms two other benchmarks: the monochromatic (i.e., single-ray) DB (M-DB) which neglects scattering, and the bichromatic (i.e., two-ray) DB (B-DB) which relies on an efficient polychromatic channel approximation by two rays that is valid for small angular spreads (AS).**

I. Introduction

One of the main challenges in wireless sensor networks (WSNs) is establishing an energy-efficient and reliable communication link between a remote access point and sensor nodes which are typically small battery-powered primitive devices. As such, many cooperative communication schemes have been introduced to significantly reduce the energy required at each node through cooperation [1]-[5]. Indeed, in a cooperative WSN, nodes play often a central role in the signal transmission flow by processing the received signals from the source and then forwarding them to the destination. Many techniques have been so far developed to process the signals at the sensors node. Among them is amplify-and-forward (AF) which reduces processing at each node to a simple multiplication of the received signal by properly selected beamforming weights, thereby avoiding decoding or other sophisticated techniques that are excessively complex for implementation at the primitive sensors [4]. These weights are crucial not only to achieving predefined objectives, but also to complying with real-world constraints and restrictions, making their design an active research subject.

When designing the AF weights, [1]-[5] have ignored the scattering phenomenon present in almost all real-world environments and consequently assumed a simple single-ray (i.e., monochromatic) channel. Unfortunately, the so-obtained monochromatic distributed AF beamformers (M-DB) usually experience a performance degradation in scattered channels that are multi-ray (i.e., polychromatic) and, hence, characterized by an angular spread (AS) [6]-[8]. It has been indeed shown that M-DB performance slightly deteriorates in areas where the AS is small and becomes unsatisfactory when the latter grows large [8]-[10]. In order to tackle this problem, [9] and [10] have proposed a bichromatic (i.e., two-ray) DB (B-DB) which accounts for scattering by an efficient two-ray approximation of the polychromatic channel at relatively low AS. The latter outperforms M-DB and, further, reaches optimality for small to moderate AS values in lightly- to moderately-scattered environments. However, B-DB experiences a severe performance degradation in highly-scattered environments where AS values are relatively large. Besides to M-DB and B-DB, a third technique exists in the literature. Known as optimal AF beamformer (OB), this technique is able to guarantee optimal performance for any AS values and, hence, in all real-world environments including the highly-scattered ones [11]. However, in contrast to M-DB and B-DB, OB weights requires full knowledge of all sensors channels' and coordinates. Hence it requires huge information exchange between nodes that translates into sensor power depletion, making OB unsuitable for WSNs. This work aims then to design a new energy-efficient distributed DB that guarantees, as OB, the optimality for any AS values without depleting, as M-DB and B-DB, the sensors' power.

In this paper, we consider an OB that achieves a dual-hop communication from a source to a receiver in highly-scattered environments through a WSN comprised of K independent and autonomous sensor nodes. The OB weights are derived to maximize the received signal-to-noise ratio (SNR) subject to a constraint over the nodes' total transmit power. We verify that OB implementation requires a prohibitive power cost from small-battery powered sensors. Exploiting, the polychromatic structure of scattered channels, we develop a novel DB that performs nearly as well as OB while requiring much less power consumption at each sensor. Furthermore, we prove that our polychromatic DB (P-DB) always outperforms M-DB, which neglects scattering, and that it is more robust against this phenomenon than B-DB whose performance significantly deteriorates in highly-scattered environments.

II. System model

Our system consists of a WSN comprised of K uniformly and independently distributed single-antenna sensor nodes in a disc $D(O,R)$, a source S, and a receiver Rx, all located in the same plane. We assume that there is no direct link from S to Rx due to high pathloss attenuation. (r_k, ψ_k) denote the polar coordinates of the k-th node and (A_s, ϕ_s) denote those of the source. Without loss of generality, we assume that $\phi_s = 0$ and $A_s \gg R$ (i.e., source located in the far field). Furthermore, the following assumptions are considered throughout the paper:

A1) A number of scatterers located in the same plane containing $D(O,R)$ generate from the transmit signal L rays (i.e., spatial chromatics with reference to their angular distribution).

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The latter form a polychromatic propagation channel [8]-[12] wherein the l-th chromatic is characterized by its complex amplitude $\alpha_l = \rho_l e^{j\varphi_l}$ and angle deviation θ_l [12]. All θ_ls, φ_ls, and ρ_ls are mutually independent. All rays have equal power $1/L$ (i.e., $\mathrm{E}\left\{|\alpha_l|^2\right\} = 1/L$). The $\theta_l,\ l = 1,\ldots,L$ are i.i.d. zero-mean random variables with a symmetric probability density function (pdf) $p(\theta)$ and variance σ_θ^2 [8]-[12]. The former is called scattering or angular distribution while the latter is commonly known as angular spread (AS). Please note that, in this work, we are particularly interested in highly-scattered environments (i.e. large AS values).

A2) The source sends a narrow-band unit-power signal s. Noises at receiver and nodes are zero-mean Gaussian random variables with variances ${\sigma_n}^2$ and ${\sigma_v}^2$, respectively.

A3) The nodes' forward channels to the receiver $[\mathbf{f}]_k,\ k = 1,\ldots,K$ are zero-mean unit-variance circular Gaussian random variables. The source signal, nodes' channels, and noise are all mutually independent.

A4) Each node is aware of its own coordinates, forward and backward channels, as well as the wavelength λ while being oblivious to the locations and the channels of *all* other nodes in the WSN [1]-[5], [9].

A1 and $A_s \gg R$ yield

$$[\mathbf{g}]_k = \sum_{l=1}^{L} \alpha_l e^{-j\frac{2\pi}{\lambda} r_k \cos(\theta_l - \psi_k)}, \tag{1}$$

where $[\mathbf{g}]_k$ is the k-th node's backward channel gain. If the scattering effect was neglected (i.e., $\sigma_\theta \longrightarrow 0$) to assume a monochromatic propagation channel (i.e., $\theta_l = 0$), $[\mathbf{g}]_k$ would be reduced to

$$[\mathbf{g}_{\mathrm{mono}}]_k = e^{-j(2\frac{\pi}{\lambda}) r_k \cos(\psi_k)}, \tag{2}$$

the conventional steering vector very well-known in the array-processing literature [1]-[5], [8]-[11]. We show later that such an assumption may hinder the M-DB performance.

From (1), the power received at the k-th node will be subject to a variation, due to the summation of L chromatics at each particular channel realization. This is actually a form of fading. According to the Central Limit Theorem and A1, when L is large enough, $[\mathbf{g}]_k$ is a zero-mean Gaussian random variable and, hence, is nothing but a Rayleigh channel. Please note that there is no line-of-sight (LOS) component in our channel model. If LoS were assumed, the channel would become Rician, as $[\mathbf{g}]_k$ would be a non-zero mean Gaussian random variables.

The communication link between the source S and the receiver Rx is established using the following dual-hop cooperative schemes: In the first time slot, the source sends its signal s to the nodes which receive faded and noisy mixtures of it. In the second time slot, each node multiplies its signal with a properly selected weight and relays the resulting signal. The received signal at the nodes in the first time slot is

$$\mathbf{x} \ = \ s\mathbf{g} + \mathbf{v}, \tag{3}$$

where $\mathbf{g} \triangleq [[\mathbf{g}]_1 \ldots [\mathbf{g}]_K]^T$ and $\mathbf{v}$ is the nodes' noise vector. The nodes transmitted signal vector in the second time slot is given by

$$\mathbf{y} \ = \ \mathbf{w}^\star \odot \mathbf{x}, \tag{4}$$

where $\mathbf{w} \triangleq [w_1 \ldots w_K]$ is the AF beamforming vector with w_k being the k-th node's weight. The received signal at Rx is then

$$\begin{aligned} r &= \mathbf{f}^T\mathbf{y} + n \\ &= s\mathbf{w}^H\mathbf{h} + \mathbf{w}^H(\mathbf{f}\odot\mathbf{v}) + n, \end{aligned} \tag{5}$$

where $\mathbf{h} \triangleq \mathbf{f} \odot \mathbf{g}$ with $\mathbf{f} \triangleq [[\mathbf{f}]_1 \ldots [\mathbf{f}]_K]^T$ and n is the receiver noise.

Several approaches may be adopted to design these AF weights such as minimizing the nodes' power subject to the received quality of service constraint, minimizing the noise power while maintaining a given beamforming response level, or maximizing the received signal-to-noise ratio (SNR) subject to the total transmit power constraint [13]. Only the latter is of interest in this paper.

III. SNR-OPTIMAL AF BEAMFORMER (OB)

The SNR-optimal AF beamformer (OB) $\mathbf{w}_{\mathrm{O}}$ must satisfy the following optimization problem:

$$\mathbf{w}_{\mathrm{O}} = \arg\max \xi_{\mathbf{w}} \quad \text{s.t.} \quad P_{\mathrm{Total}} \leq P_{\max}, \tag{6}$$

where $\xi_{\mathbf{w}}$ is the SNR achieved using $\mathbf{w}$ and

$$P_{\mathrm{Total}} = (1+\sigma_v^2)\left\|\mathbf{w}\right\|^2, \tag{7}$$

is the nodes' total transmit power. From (5), $\Omega_{\mathbf{w}}$ is given by

$$\Omega_{\mathbf{w}} = \frac{P_{\mathbf{w}}^s}{P_{\mathbf{w}}^n}, \tag{8}$$

where $P_{\mathbf{w}}^n = \sigma_v^2 \mathbf{w}^H \boldsymbol{\Delta}\mathbf{w} + \sigma_n^2$ and $P_{\mathbf{w}}^s = |\mathbf{w}^H\mathbf{h}|^2$ are, respectively, the desired and noise powers with $\boldsymbol{\Delta} \triangleq \mathrm{diag}\{|[\mathbf{f}]_1|^2 \ldots |[\mathbf{f}]_K|^2\}$. Please note that in order to guarantee its optimality, $\mathbf{w}_{\mathrm{O}}$ must satisfy the constraint in (6) with equality. Otherwise, we could find $\varepsilon > 1$ such that $\mathbf{w}_\varepsilon = \varepsilon\mathbf{w}_{\mathrm{O}}$ verifies $(1+\sigma_v^2)\left\|\mathbf{w}_\varepsilon\right\|^2 = P_{\max}$. In such a case, since $d\Omega_{\mathbf{w}_\varepsilon}/d\varepsilon > 0$ for any $\varepsilon > 0$, the SNR achieved using $\mathbf{w}_\varepsilon$ would be higher than that achieved using $\mathbf{w}_{\mathrm{O}}$. This would then contradict the optimality of the latter. As such, (6) could be equivalently written as

$$\mathbf{w}_{\mathrm{O}} = \arg\max \frac{\mathbf{w}^H\mathbf{h}\mathbf{h}^H\mathbf{w}}{\sigma_v^2\mathbf{w}^H\tilde{\boldsymbol{\Delta}}\mathbf{w}} \quad \text{s.t.} \quad (1+\sigma_v^2)\left\|\mathbf{w}\right\|^2 = P_{\max}, \tag{9}$$

where $\tilde{\boldsymbol{\Delta}} = \boldsymbol{\Delta} + \kappa\mathbf{I}$ and $\kappa = \sigma_n^2(1+\sigma_v^2)/\left(\sigma_v^2 P_{\max}\right)$. It can be shown that the OB solution of (9) is given by

$$\mathbf{w}_{\mathrm{O}} = \left(\frac{P_{\max}}{K\left(1+\sigma_v^2\right)\xi}\right)^{\frac{1}{2}} \tilde{\boldsymbol{\Delta}}^{-1}\mathbf{h}, \tag{10}$$

where $\xi = \left(\mathbf{h}^H\tilde{\boldsymbol{\Delta}}^{-2}\mathbf{h}\right)/K$. It follows from (10) that OB weights depend on locally unavailable information at every node, namely $[\mathbf{g}]_k,\ k = 1,\ldots,K$ and $[\mathbf{f}]_k,\ k = 1,\ldots,K$ as well as $P_{\max}/K$ and $\sigma_n^2/P_{\max}$. The OB implementations requires that the nodes estimate and share their forward and backward channels. Let us denote by p the power required to broadcast any given real in the network. The overall power consumed by the WSN during this process is then $2(Kp+1)$. This amount increases linearly with K and becomes rapidly prohibitive as the nodes' number is typically large in WSNs. Accordingly, OB is unsuitable for such network. This motivates us to develop in the next section a new AF beamformer whose implementation requires much less power cost, thereby complying with the WSNs power constraint.

IV. Proposed polychromatic DB (P-DB)

To reduce the excessively large power cost incurred by OB implementation, the prospective AF beamformer must certainly avoid any information exchange between the nodes. To this end, one should substitute ξ with a quantity locally computable at all nodes. In order to preserve the optimality of the solution in (10), such a quantity must also well-approximate ξ. In this work, we propose to use $\xi_{\mathrm{D}} = \lim_{K\to\infty} \xi$ instead of ξ. Let us first take an in-depth look at the latter. ξ could be expressed as

$$\xi \;=\; \frac{1}{K}\sum_{k=1}^{K} \frac{|[\mathbf{f}]_k|^2}{\left(|[\mathbf{f}]_k|^2+\kappa\right)^2} \sum_{l=1}^{L}\sum_{m=1}^{L} \alpha_l \alpha_m^* e^{j4\pi \sin\left(\frac{\theta_l-\theta_m}{2}\right) t_k}, \tag{11}$$

where

$$t_k = \frac{r_k}{\lambda}\sin\left(\frac{(\theta_l+\theta_m)}{2} - \psi_k\right). \tag{12}$$

Exploiting the strong law of large numbers along with the statistical independence of r_k, ψ_k, and $[\mathbf{f}]_k$, we obtain

$$\begin{aligned} \xi_{\mathrm{D}} \;&=\; \lim_{K\to\infty} \xi \\ &\xrightarrow{p1}\; \beta_1 \sum_{l=1}^{L}\sum_{m=1}^{L} \alpha_l \alpha_m^* \vartheta\left(\theta_l-\theta_m\right), \end{aligned} \tag{13}$$

where

$$\begin{aligned} \beta_1 \;&=\; \mathrm{E}\left\{|[\mathbf{f}]_k|^2 / \left(|[\mathbf{f}]_k|^2+\kappa\right)^2\right\} \\ &=\; -(1+\kappa)e^{\kappa}\mathrm{Ei}(-\kappa) - 1, \end{aligned} \tag{14}$$

and $\vartheta(\phi) = \mathrm{E}\left\{e^{j4\pi\sin(\phi/2)t_k}\right\}$. To derive the latter in closed-form expression, t_k's pdf $f_{t_k}(t)$ is required. The latter is actually related to the nodes' spatial distribution. Two main distributions frequently used in AF beamforming: the Uniform and Gaussian ones.

It can be shown that f_{t_k} is given by [1], [2]

$$f_{t_k} = \begin{cases} \frac{2\lambda}{R\pi}\sqrt{1-\left(\frac{\lambda}{R}t\right)^2}, & -\frac{R}{\lambda} \le t \le \frac{R}{\lambda} \quad \text{Uniform} \\ \frac{\lambda}{\sqrt{2\pi}\sigma} e^{-\frac{(\lambda t)^2}{2\sigma^2}}, & -\infty \le t \le \infty \quad \text{Gaussian} \end{cases}, \tag{15}$$

where σ^2 denotes the variance of the Gaussian random variables corresponding to the nodes' cartesian coordinates. Exploiting (15), we obtain

$$\vartheta(\phi) = \begin{cases} 2\frac{J_1\left(4\pi\frac{R}{\lambda}\sin(\phi/2)\right)}{4\pi\frac{R}{\lambda}\sin(\phi/2)}, & \phi \ne 0 \\ 1, & \phi = 0 \end{cases} \;\text{Uniform} \\ \quad e^{-8\left(\pi\frac{\sigma}{\lambda}\sin(\phi/2)\right)^2}, \quad \text{Gaussian} \,. \tag{16}$$

The substitution of ξ with ξ_{D} in (10) yields

$$\mathbf{w}_{\mathrm{P}} = \left(\frac{P_{\max}}{K\left(1+\sigma_v^2\right)\xi_{\mathrm{D}}}\right)^{\frac{1}{2}} \tilde{\boldsymbol{\Delta}}^{-1}\mathbf{h}, \tag{17}$$

where the $\mathbf{w}_{\mathrm{P}}$ is the beamforming vector of our proposed polychromatic distributed AF beamformer (P-DB). A straightforward inspection of (17) reveals that the k-th node's weight $[\mathbf{w}_{\mathrm{P}}]_k$ depends solely on its forward and backward channels. Therefore, P-DB implementation does not require any information exchange between the nodes and, hence, its overall power cost is $3p$. This cost is actually incurred by the broadcast over the WSN of $P_{\max}/K$, $\sigma_n^2/P_{\max}$, and R or σ depending on the nodes' spatial distribution. Our proposed P-DB provides then a power gain of $2(K+1)/3$ against OB and, hence, is more suitable for WSNs. Indeed, using P-DB in lieu of OB in WSNs would not only extend their lifetime significantly, but also guarantee that the communication power cost does not grow with K. Furthermore, we will show in the sequel that the proposed P-DB performs nearly as well as OB even for a relatively small number of nodes. We will also compare it with two other DB benchmarks, namely M-DB which ignores scattering and B-DB whose design relies on a polychromatic channel's approximation by two chromatics at $\pm\sigma_\theta$ when the latter is relatively small. It can be easily shown that M-DB beamforming vector $\mathbf{w}_{\mathrm{M}}$ is given by

$$\mathbf{w}_{\mathrm{M}} = \left(\frac{P_{\max}}{K\left(1+\sigma_v^2\right)\beta_1}\right)^{\frac{1}{2}} \tilde{\boldsymbol{\Delta}}^{-1}\mathbf{a}(0), \tag{18}$$

where $\mathbf{a}(\phi) \triangleq \left[[\mathbf{a}(\theta)]_1 \ldots [\mathbf{a}(\theta)]_K\right]^T$ with $[\mathbf{a}(\theta)]_k = [\mathbf{f}]_k e^{-j(2\pi/\lambda)r_k\cos(\theta-\psi_k)}$. In turn, $\mathbf{w}_{\mathrm{BD}}$ is given by

$$\mathbf{w}_{\mathrm{BD}} = \left(\frac{P_{\max}}{K\left(1+\sigma_v^2\right)\beta_1}\right)^{\frac{1}{2}} \frac{\tilde{\boldsymbol{\Delta}}^{-1}\left(\mathbf{a}(\sigma_\theta)+\mathbf{a}(-\sigma_\theta)\right)}{\left(1+\vartheta\left(2\sigma_\theta\right)\right)}. \tag{19}$$

It follows from (18) and (19) that both $\mathbf{w}_{\mathrm{M}}$ and $\mathbf{w}_{\mathrm{BD}}$ depends on the information commonly available at each node. Therefore, their implementation requires a negligible power cost that does not grow with K, making them very suitable for WSNs.

V. The performance of P-DB

Let $\bar{\Omega}_{\mathbf{w}} = \mathrm{E}\left\{P_{\mathbf{w}}^s / P_{\mathbf{w}}^n\right\}$ denote the achieved average SNR (ASNR) using the AF beamforming vector $\mathbf{w}$. Note that the expectation is taken with respect to $[\mathbf{f}]_k$, r_k, ψ_k, α_l and θ_l. Unfortunately, $\bar{\Omega}_{\mathbf{w}}$ for $\mathbf{w} \in \{\mathbf{w}_{\mathrm{P}}, \mathbf{w}_{\mathrm{O}}, \mathbf{w}_{\mathrm{M}}\}$ is untractable in closed form, to the best of our knowledge. We propose then to adopt the average-signal-to-average-noise ratio (ASANR) $\tilde{\Omega}_{\mathbf{w}} = \mathrm{E}\left\{P_{\mathbf{w}}^s\right\} / \mathrm{E}\left\{P_{\mathbf{w}}^n\right\}$ as a performance measure instead to compare the proposed P-DB against its benchmarks.

A. Proposed P-DB vs M-DB

Following derivation steps similar to those in [22, Appendix A] and exploiting the fact that, according to A1, we have $\mathrm{E}\left\{\alpha_l^* \alpha_m\right\}$ equal to $1/L$ if $l = m$ and 0 otherwise, we obtain

$$\mathrm{E}\left\{P_{\mathbf{w}_{\mathrm{P}}}^s\right\} = \frac{P_{\max}}{\left(1+\sigma_v^2\right)\beta_1}\left(\beta_2 + (K-1)\beta_3^2\right), \tag{20}$$

where

$$\begin{aligned} \beta_2 \;&=\; \mathrm{E}\{|[\mathbf{f}]_k|^4 / (|[\mathbf{f}]_k|^2+\kappa)^2\} \\ &=\; 1+\kappa+\kappa(2+\kappa)e^{\kappa}\mathrm{Ei}(-\kappa), \end{aligned} \tag{21}$$

and

$$\begin{aligned} \beta_3 \;&=\; \mathrm{E}\left\{|[\mathbf{f}]_k|^2 / \left(|[\mathbf{f}]_k|^2+\kappa\right)\right\} \\ &=\; 1+\kappa e^{\kappa}\mathrm{Ei}(-\kappa). \end{aligned} \tag{22}$$

On the other hand, in order to derive $\mathrm{E}\left\{P_{\mathbf{w}_{\mathrm{P}}}^n\right\}$, one should first take the expectation over r_ks, ψ_ks, and $[\mathbf{f}]_k$s yielding to

$$\begin{aligned} \mathrm{E}\left\{P_{\mathbf{w}_{\mathrm{P}}}^n\right\} &= \mathrm{E}_{\alpha_l,\theta_l}\left\{\frac{\sigma_v^2 P_{\max}\beta_2 \sum_{l,m=1}^{L}\alpha_l\alpha_m^*\vartheta\left(\theta_l-\theta_m\right)}{\left(1+\sigma_v^2\right)\xi_{\mathrm{D}}}\right\} \\ &\quad + \sigma_n^2 \\ &= \sigma_v^2 \frac{P_{\max}\beta_2}{\left(1+\sigma_v^2\right)\beta_1} + \sigma_n^2. \end{aligned} \tag{23}$$

Accordingly, the achieved ASANR using the proposed P-DB is given by

$$\tilde{\Omega}_{\mathbf{w}_{\mathrm{P}}} = \frac{\beta_2 + (K-1)\beta_3^2}{\sigma_v^2\left(\beta_2 + \kappa\beta_1\right)}. \tag{24}$$

It can be observed from (24) that $\tilde{\Omega}_{\mathbf{w}_{\mathrm{P}}}$ linearly increases with the number of nodes K and does not depend on the AS σ_θ. This means that the proposed P-DB's performance is not affected by the scattering phenomenon even in highly-scattered environments (i.e., large σ_θ values) .

Let us now turn our attention to the ASANR $\tilde{\Omega}_{\mathbf{w}_{\mathrm{M}}}$ achieved using M-DB. Following the same approach above, one can show that

$$\tilde{\Omega}_{\mathbf{w}_{\mathrm{M}}} = \frac{\beta_2 + (K-1)\beta_3^2 \int_\Theta p(\theta)\vartheta^2(\theta)\, d\theta}{\sigma_v^2\left(\beta_2 + \kappa\beta_1\right)}, \tag{25}$$

where Θ is the span of the pdf $p(\theta)$ over which the integral is calculated[1]. Since $\vartheta(0) = 1$ regardless of the nodes' spatial distribution, it follows from (24) and (25) that $\tilde{\Omega}_{\mathbf{w}_{\mathrm{M}}} = \tilde{\Omega}_{\mathbf{w}_{\mathrm{P}}}$ when scattering does not exist (i.e., $\sigma_\theta = 0$). Indeed, in such a case, $\mathbf{w}_{\mathrm{P}} = \mathbf{w}_{\mathrm{M}} \sum_{l=1} \alpha_l / \sqrt{\sum_{l=1}\alpha_l \sum_{m=1}\alpha_m^*}$ and, therefore, $P_{\mathbf{w}_{\mathrm{P}},s} = P_{\mathbf{w}_{\mathrm{M}},s}\sum_{l=1}\alpha_l \sum_{m=1}\alpha_m^*$. Using the fact that $\mathrm{E}\left\{\sum_{l=1}\alpha_l \sum_{m=1}\alpha_m^*\right\} = 1$, we have $\mathrm{E}\left\{P^s_{\mathbf{w}_{\mathrm{P}}}\right\} = \mathrm{E}\left\{P_{\mathbf{w}_{\mathrm{M}},s}\right\}$. Furthermore, it is straightforward to show that $P^n_{\mathbf{w}_{\mathrm{P}}} = P^n_{\mathbf{w}_{\mathrm{M}}}$ when $\sigma_\theta = 0$ and, hence, M-DB achieves the same performance as the proposed P-DB when there is no scattering. This is in fact hardly surprising since the monochromatic channel assumption made for M-DB design is valid in such a case. Nevertheless, assuming that nodes are uniformly distributed and $p(\theta) = 1/(2\sqrt{3}\sigma_\theta)$, it can be shown for relatively small AS that

$$\tilde{\Omega}_{\mathbf{w}_{\mathrm{M}}} \simeq \frac{\beta_2 + (K-1)\beta_3^2\, {}_3F_4\!\left(\frac{1}{2}, 2, \frac{3}{2}; \frac{3}{2}, 2, 2, 3, -12\pi^2\left(\frac{R}{\lambda}\right)^2\sigma_\theta^2\right)}{\sigma_v^2\left(\beta_2 + \kappa\beta_1\right)}, \tag{26}$$

where ${}_3F_4\left(\frac{1}{2}, 2, \frac{3}{2}; \frac{3}{2}, 2, 2, 3, -12\pi^2(R/\lambda)^2 x^2\right)$ is the hypergeometric function that decreases with x and reaches its peak at 0. It can be then inferred from (26), that the ASANR achieved by M-DB decreases when the AS σ_θ and/or R/λ increases. This is in contrast with the proposed P-DB whose ASANR remains constant for any σ_θ and R/λ. Consequently, the proposed P-DB is more robust against scattering than M-DB whose design ignores the presence of scattering thereby resulting in a channel mismatch that causes severe ASANR deterioration.

B. Proposed P-DB vs B-DB

It can be shown that the achieved ASANR using B-DB is [9]

$$\begin{aligned}\tilde{\Omega}_{\mathbf{w}_{\mathrm{BD}}} \;=\; & \frac{2\beta_2 + \left((K-1)\beta_3^2\right)/\left(1+\vartheta\left(2\sigma_\theta\right)\right)}{\sigma_v^2\left(\beta_2+\kappa\beta_1\right)} \\ & \frac{\int_\Theta p(\theta)\left(\vartheta\left(\theta+\sigma_\theta\right) + \vartheta\left(\theta - \sigma_\theta\right)\right)^2 d\theta}{\left(1+\vartheta\left(2\sigma_\theta\right)\right)}.\end{aligned} \tag{27}$$

According to (27), regardless of the nodes' spatial distribution, $\tilde{\Omega}_{\mathbf{w}_{\mathrm{BD}}}$ boils down to its maximum level $\tilde{\Omega}_{\mathbf{w}_{\mathrm{P}}}$, when there is no scattering (i.e., $\sigma_\theta = 0$). As has been shown in [9] and [10], since B-DB is able to achieve its maximum ASANR level for small to moderate AS values such as in lightly- to moderately-scattered environments, it turns out that the proposed P-DB and its B-DB counterpart achieve the same ASANR in such environments. Nevertheless, when σ_θ is large such as in highly-scattered environments, using the fact that $\vartheta\left(2\sigma_\theta\right) \simeq 0$ for large σ_θ, one can easily show that

$$\lim_{K\to\infty} \frac{\tilde{\Omega}_{\mathbf{w}_{\mathrm{BD}}}}{\tilde{\Omega}_{\mathbf{w}_{\mathrm{P}}}} = \int_\Theta p(\theta)\left(\vartheta(\theta+\sigma_\theta) + \vartheta(\theta-\sigma_\theta)\right)^2 d\theta. \tag{28}$$

The ASANR gain achieved by the proposed P-DB against B-DB increases then with the latter, since the right-hand side (RHS) of (28) is a decreasing function of σ_θ. Consequently, in highly-scattered environments where the AS is large, the proposed P-DB outperforms B-DB whose performance deteriorates due to the channel mismatch.

C. Proposed P-DB vs OB

The derivation of $\tilde{\Omega}_{\mathbf{w}_{\mathrm{O}}}$ in closed form turns out, unfortunately, to be impossible since $P^s_{\mathbf{w}_{\mathrm{O}}}$ and $P^n_{\mathbf{w}_{\mathrm{O}}}$ are complex functions of several random valuables. A very interesting result could, however, be obtained for large K. Indeed, one can show that

$$\begin{aligned}\lim_{K\to\infty} \frac{\tilde{\Omega}_{\mathbf{w}_{\mathrm{O}}}}{\tilde{\Omega}_{\mathbf{w}_{\mathrm{P}}}} &\xrightarrow{p1} \frac{\frac{(\beta_2+\kappa\beta_1)}{\beta_1}\mathrm{E}\left\{\left(\sum_{l,m=1}^{L}\alpha_l\alpha_m^*\vartheta(\theta_l - \theta_m)\right)\right\}}{\frac{\beta_2}{\beta_1} + \kappa} \\ &= 1,\end{aligned} \tag{29}$$

where the second line exploits the law of large numbers by which we can prove that

$$\lim_{K\to\infty} \frac{\mathbf{h}^H \tilde{\boldsymbol{\Delta}}^{-1}\mathbf{h}}{K} = \beta_3 \sum_{l,m=1}^{L} \alpha_l\alpha_m^*\vartheta\left(\theta_l - \theta_m\right), \tag{30}$$

and

$$\lim_{K\to\infty} \frac{\mathbf{h}^H \tilde{\boldsymbol{\Delta}}^{-1}\boldsymbol{\Delta}\tilde{\boldsymbol{\Delta}}^{-1}\mathbf{h}}{K} = \beta_2 \sum_{l,m=1}^{L} \alpha_l\alpha_m^*\vartheta\left(\theta_l - \theta_m\right). \tag{31}$$

It follows from (29) that the proposed P-DB is able to achieve the same ASANR as its OB counterparts and, therefore, achieves optimality for any AS value. This further proves the efficiency of the proposed P-DB.

Using the same method as in (29), one can easily show that

$$\lim_{K\to\infty} \frac{\tilde{\Omega}_{\mathbf{w}}}{\bar{\Omega}_{\mathbf{w}}} \xrightarrow{p1} 1, \tag{32}$$

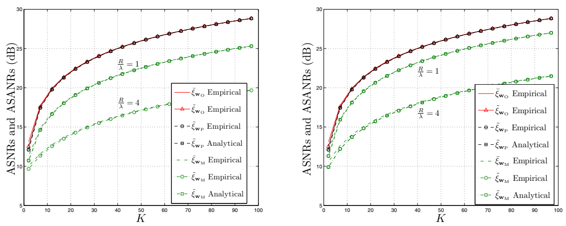
where $\mathbf{w} \in \{\mathbf{w}_{\mathrm{P}}, \mathbf{w}_{\mathrm{O}}, \mathbf{w}_{\mathrm{M}}\}$. Consequently, all the above results hold also for the ASNR as K grows large.

VI. Simulation Results

In this section, all the empirical average quantities are obtained by averaging over 10^6 realizations of all random variables. In all simulations, the number of chromatics is $L = 10$ and the noises' powers σ_n^2 and σ_v^2 are set to 10 dB below the source transmit power $p_s = 1$. We also assume that α_ls are circular Gaussian random variables and that the scattering distribution is Uniform (i.e., $p(\theta) = 1/(2\sqrt{3}\sigma_\theta)$). For fair comparisons between the Uniform and Gaussian nodes distributions, we choose $\sigma = R/3$ to guarantee in the Gaussian case that more than 99% of nodes are located in $D(O, R)$.

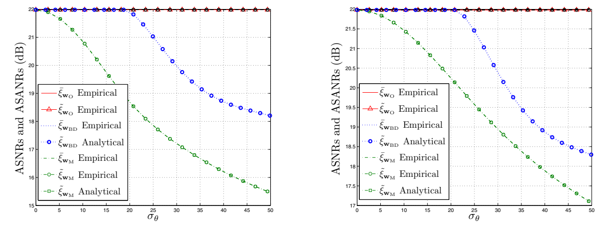
Fig. 1 displays the empirical and analytical ASNRs and ASANRs achieved by the proposed P-DB and its counterparts versus K for $\sigma_\theta = 20$ (deg) and $R/\lambda = 1, 4$. We consider

[1] In the Uniform and Gaussian distribution cases, $\Theta = [-\sqrt{3}\sigma_\theta, +\sqrt{3}\sigma_\theta]$ and $\Theta = [-\inf, +\inf]$, respectively.



(a) Uniform distribution (b) Gaussian distribution

Fig. 1. The empirical ASNRs and ASANRs achieved using $\mathbf{w} \in \{\mathbf{w}_{\rm O}, \mathbf{w}_{\rm P}, \mathbf{w}_{\rm M}\}$ as well as the analytical ASANRs achieved using $\mathbf{w}_{\rm P}$ and $\mathbf{w}_{\rm M}$ vs. K for $\sigma_\theta = 20$ (deg) and $R/\lambda = 1, 4$ when the nodes' distribution is (a): Uniform and (b): Gaussian.



(a) Uniform distribution (b) Gaussian distribution

Fig. 2. The empirical ASNRs and ASANRs achieved using $\mathbf{w} \in \{\mathbf{w}_{\rm O}, \mathbf{w}_{\rm BD}, \mathbf{w}_{\rm P}, \mathbf{w}_{\rm M}\}$ as well as the analytical ASANRs achieved using $\mathbf{w}_{\rm P}$ and $\mathbf{w}_{\rm M}$ vs. σ_θ for $K = 20$ and $R/\lambda = 1$ when the nodes' distribution is (a): Uniform and (b): Gaussian.

both the Uniform and Gaussian spatial distributions in Fig. 1(a) and Fig. 1(b), respectively. We confirm, from these figures, that analytical $\tilde{\Omega}_{\mathbf{w}_{\rm P}}$ and $\tilde{\Omega}_{\mathbf{w}_{\rm M}}$ match perfectly their empirical counterparts. As can be observed from Fig. 1, the proposed P-DB outperforms M-DB in terms of achieved ASANR. Furthermore, the ASANR gain achieved using the proposed P-DB in lieu of the latter significantly increases when the ratio R/λ grows large. Moreover, the achieved ASANR using the proposed P-DB fits perfectly with that achieved using OB, which is unsuitable for WSNs, when K is in the range of 20 while it looses only a fraction of a dB when K is in the range of 5. Therefore, the proposed P-DB is able to reach optimality as OB while incurring much less power cost. Indeed, as shown in Section. IV, P-DB implementation requires 31.5 times less power than OB, making it a more suitable AF solution for WSNs applications. All these observations corroborate the theoretical results obtained in Section V.

Fig. 2 shows the empirical ASNRs and ASANRs achieved by the proposed P-DB and its counterparts versus the AS σ_θ for $K = 20$ and $R/\lambda = 1$. We consider the Uniform and Gaussian spatial distributions in Fig. 2(a) and Fig. 2(b), respectively. It can be observed from these figures that the ASANR achieved by M-DB decreases with σ_θ while that achieved by P-DB remains constant. This corroborates again the theoretical results obtained in Section V. Moreover, we observe from Fig. 2 that B-DB achieves the same ASNR as the proposed P-DB when σ_θ is relatively small such as in lightly- to moderately-scattered environments. Nevertheless, in highly-scattered environments where σ_θ is large (i.e., $\sigma_\theta \geq 20$ deg), the proposed P-DB outperforms B-DB whose performance further deteriorates as the AS grows large. This is hardly surprising since the two-ray channel approximation made for the B-DB's design is only valid for small σ_θ. Furthermore, it can be noticed from Figs. 2(a) and 2(b), that the ASNR gain achieved using the proposed P-DB instead of B-DB and M-DB can reach until about 4 (dB) and 6.5 (dB), respectively. From Figs 2(a) and 2(b), we also observe that the curves of $\bar{\Omega}_{\mathbf{w}_{\rm O}}$ and $\bar{\Omega}_{\mathbf{w}_{\rm P}}$ are indistinguishable. As highlighted above, this is due to the fact that both OB and the proposed P-DB constantly reach optimality.

VII. Conclusion

In this paper, we considered an OB that achieves a dual-hop communication from a source to a receiver in highly-scattered environments through a WSN comprised of K independent and autonomous sensor nodes. The OB weights are derived to maximize the received SNR subject to a constraint over the nodes' total transmit power. We verify that OB's implementation requires a prohibitive power cost from the small-battery powered nodes. Exploiting the polychromatic structure of scattered channels, we successfully develop a novel P-DB that performs nearly as well as OB while requiring much less power consumption at each sensor. Furthermore, we prove that our polychromatic DB (P-DB) always outperforms M-DB which neglects scattering and that it is more robust against this phenomenon than B-DB whose performance significantly deteriorates in highly-scattered environments.

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