ML EM Estimation of Fast Time-Varying OFDM-Type Channels

(Invited Paper)

Souheib Ben Amor[1], Sofiène Affes[1], and Faouzi Bellili[2]

[1]INRS-EMT, Université du Québec, Montréal, QC, Canada, Emails: {souheib.ben.amor, affes}@emt.inrs.ca

[2]Department of Electrical and Computer Engineering, University of Manitoba, Winnipeg, MB, Canada, Email: faouzi.bellili@umanitoba.ca

***Abstract*—In this paper, we investigate the problem of fast time-varying multipath channel estimation over orthogonal frequency-division multiplexing (OFDM)-type transmissions. We do so by tracking each complex gain variation using a polynomial-in-time expansion. To that end, we derive the log-likelihood function (LLF) in the non-data-aided (NDA) case. Since the LLF is extremely nonlinear, we opt for the expectation maximization (EM) concept to find its global maximum. Simulation results show that the new estimator is able to converge to the global maximum within few iterations only and to provide accurate estimates for all multipath gains, thereby resulting in significant BER and link-level throughput gains.**

***Index Terms*—Maximum likelihood (ML), expectation maximization (EM), channel estimation, time-varying channel (TVC), OFDM.**

I. Introduction

Orthogonal frequency-division multiplexing (OFDM) showed its effectiveness in current 4^{th} generation wireless technology (4G). A scalable variety of CP-OFDM is already included in 5^{th} generation (5G) new radio (NR) standards by the 3^{rd} Generation Partnership Project (3GPP) [1]. The adopted waveform will include multiple sub-carrier spacings that depend on the type of deployments and service requirements. Despite its attractive features such as robustness to frequency selective channels, OFDM-type radio interface technologies (RITs) are already very sensitive to time-varying channels due to the resulting loss of orthogonality between the subcarriers. Channel estimation, hence, becomes at very high mobility a daunting task [2].

So far, a number of channel estimation techniques have been reported in literature. All available solutions go under one of two major categories: i) the data-aided (DA) approach where the transmitted symbols are assumed to be perfectly known at the receiver. Such solutions provide higher performance at a significant cost in terms of overhead; ii) the blind or non-data-aided (NDA) approach where the receiver has no a priori information about the transmitted sequence. Hence, accurate channel tracking is possible at the receiver with minimal overhead.

Under the DA category, and for fast time-varying channels, many techniques use a basis expansion model (BEM) to estimate the equivalent discrete-time channel taps [3-5]. In [3], BEM methods such as Karhunen-Loeve BEM were designed with low mean square error (MSE). They are, however, sensitive to statistical channel mismatch. The complex-exponential BEM, also proposed in [3], is independent from channel statistics but then suffers from large modeling errors. The polynomial BEM (P-BEM) investigated in [4] provides accurate performance, but only at low Doppler. In [5], the complex gain variation of each path was approximated by a polynomial function of time then estimated by least square (LS) technique. This solution offers accurate performance even at high Doppler. However, it requires that the number of paths be lower than the inserted pilot symbols in each OFDM time slot.

Under the NDA category, time-varying channel estimation was also investigated over OFDM-type radio access. In [9], the authors used the discrete Legendre polynomial BEM along with the space alternating generalized expectation maximization-maximum a posteriori probability SAGE-MAP technique to estimate the channel coefficients in the time domain. Whereas the authors of [6] opted for the EM technique to estimate the signal-to-noise ratio (SNR) in the case, however, of single-carrier SISO transmissions. Both the EM and LS techniques were also proposed in [7] and [8], respectively, to estimate, however, the SNR over single-carrier SIMO systems.

In this paper, we develop an iterative ML EM estimator of fast time-varying channels over OFDM-type radio interfaces. By relying on the polynomial approximation of the multipath channel gains in [10] and introducing instead of LS the powerful EM technique, our solution offers a much more accurate ML-type acquisition of the polynomial coefficients and the resulting time-varying channel gains. To avoid local convergence that is inherent to EM-type iterative algorithms, we initialize it with the LS technique. Moreover, despite its accurate performances under high interference level, the ML EM technique provides even more accurate channel gains estimates with the use of inter carrier interference (ICI) cancellation technique.

The rest of the paper is organized as follows: In Section II, we introduce the system model. In Section III, we derive the new ML EM solution of the underlying estimation problem. In Section IV, we run exhaustive computer simulations to assess the performance of the proposed fast time-varying channel estimator. Finally, we draw out some concluding remarks in Section V.

The notations adopted in this paper are as follows. Vectors

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and matrices are represented in lower- and upper-case bold fonts, respectively. Moreover, $\{.\}^T$ and $\{.\}^H$ denote the conjugate and Hermitian (i.e., transpose conjugate) operators. The Euclidean norm of any vector is denoted as $\|.\|$. For any matrix $\mathbf{X}$, $[\mathbf{X}]_q$ and $[\mathbf{X}]_{l,k}$ denote its q^{th} column and $(l,k)^{th}$ entry, respectively. For any vector $\mathbf{x}$, diag$\{\mathbf{x}\}$ refers to the diagonal matrix whose elements are those of $\mathbf{x}$. Moreover, $\{.\}^*$, $\angle\{.\}$, and $|.|$ return the conjugate, angle, and modulus of any complex number, respectively. Finally, $\mathbb{E}\{.\}$ stands for the statistical expectation, j is the imaginary number (i.e., $j^2=-1$), and the notation $\triangleq$ refers to definitions.

II. System Model

Consider an OFDM single input single output (SISO) system with N subcarriers and a cyclic prefix (CP) of a length N_{cp}. The wireless link between the transmitter and receiver is modeled as a multipath Rayleigh fading channel as follows:

$$h(t,\tau)=\sum_{l=1}^{L}\alpha_l(t)\delta(\tau-\tau_l T_s), \tag{1}$$

where L is the number of paths. For each path, the delay τ_l is normalized by the sampling period T_s and the complex gain $\alpha_l(t)$ is generated with variance σ_l^2. The multipath power profile (i.e., the channel) is assumed to be normalized (i.e., $\sum_{l=1}^{L}\sigma_l^2=1$). We adopt the approximation in [10] of the sampled complex gain of the l^{th} path within the duration of N_c consecutive OFDM blocks, $\boldsymbol{\alpha}_l=[\alpha_l(-N_{cp}T_s),\ldots,\alpha_l(N_bN_c-N_{cp}-1)]^T$, by a polynomial model of order N_c-1 as follows:

$$\alpha_l(pT_s)\approx\sum_{d=1}^{N_c}c_{d,l}p^{(d-1)}+\zeta_l[p], \tag{2}$$

where $p\in[-N_{cp},\ldots,N_bN_c-N_{cp}-1]$, $\mathbf{c}_l=[c_{1,l},c_{2,l},\ldots,c_{N_c,l}]^T$ is the approximating polynomial coefficients vector of the l^{th} path and $\zeta_l[p]$ is the approximation error. $T=N_bT_s$ denotes the OFDM block duration where $N_b=N+N_{cp}$. At the destination, after removing the CP and applying a $N-$point fast Fourier transform (FFT), the collected OFDM symbols at each local approximation window of N_c OFDM blocks (i.e., $n=1,2,\ldots,N_c$), can be written in a matrix form as follows:

$$\mathbf{y}_k \;=\; \mathbf{H}_k\mathbf{a}_k+\mathbf{w}_k, \tag{3}$$

where $\mathbf{y}_k=[y_k[1],y_k[2],\ldots,y_k[N]]^T$ is the received k^{th} OFDM block, and $\mathbf{w}_k=[w_k[1],w_k[2],\ldots,w_k[N]]^T$ is a complex white Gaussian noise vector with covariance σ^2I_N where I_N is the N-dimensional identity matrix. The N transmitted symbols during the k^{th} OFDM block, $\mathbf{a}_k=[a_k[1],a_k[2],\ldots,a_k[N]]^T$, are generated randomly from a $M-$ary constellation alphabet, $\mathcal{C}^M$, with a probability $\{P[a_m]=\frac{1}{M}\}_{a_m\in\mathcal{C}^M}$. The $N\times N$ matrix, $\mathbf{H}_k$, is the channel frequency response whose elements are given by:

$$[\mathbf{H}_k]_{m,n}=\frac{1}{N}\sum_{l=1}^{L}\left[e^{-j2\pi\left(\frac{k-1}{N}-\frac{1}{2}\right)\tau_l}\sum_{q=0}^{N-1}\alpha_{k,l}(qT_s)e^{j2\pi\frac{n-m}{N}q}\right], \tag{4}$$

where $\{\alpha_{k,l}(qT_s)\}_{q=kN_b}^{N_b+N-1}$ are the complex gains of the l^{th} path within the duration of the k^{th} OFDM block. With the above approximation [10], the polynomial coefficients, $\mathbf{c}_l$, of l^{th} path and corresponding to the N_c consecutive OFDM blocks can be obtained using the time average of the channel gain over the effective duration of each OFDM time slot ($\{\bar{\alpha}_{k,l}=\frac{1}{N}\sum_{q=kN_b}^{kN_b+N-1}\alpha_{k,l}(qT_s)\}_{k=0}^{N_c-1}$) as follows:

$$\mathbf{c}_l=\mathbf{T}^{-1}\bar{\boldsymbol{\alpha}}_l, \tag{5}$$

where:

$$\bar{\boldsymbol{\alpha}}_l=[\bar{\alpha}_{0,l},\bar{\alpha}_{2,l},\ldots,\bar{\alpha}_{N_c-1,l}]^T,$$
$$\mathbf{T}=\begin{pmatrix}1 & \frac{N-1}{2} & \frac{(N-1)(2N-1)}{6}\\ 1 & \frac{N-1}{2}+N_b & \frac{(N-1)(2N-1)}{6}+(N-1)N_b+N_b^2\\ 1 & \frac{N-1}{2}+2N_b & \frac{(N-1)(2N-1)}{6}+2(N-1)N_b+4N_b^2\end{pmatrix}.$$

Using these coefficients, the samples of the complex gain of each channel path over the interval $[-N_{cp},\ldots,N_bN_c-N_{cp}-1]$, $\mathbf{c}_l=[c_{1,l},c_{2,l},\ldots,c_{N_c,l}]$, can be obtained as follows:

$$\boldsymbol{\alpha}_l=\mathbf{S}^T\mathbf{c}_l, \tag{6}$$

where $\mathbf{S}$ is $N_c\times N_bN_c$ matrix whose elements are given by:

$$\left\{\left\{[\mathbf{S}]_{d,p'}=(p'-N_{cp}-1)^{d-1}\right\}_{p'=1}^{N_bN_c}\right\}_{d=1}^{N_c}. \tag{7}$$

The channel gains can be estimated in (6) from the channel coefficient estimates whose estimation in (5) ultimately requires an estimate for the channel gain time averages vector $\bar{\boldsymbol{\alpha}}$.

In [10], $\bar{\boldsymbol{\alpha}}$ is estimated by DA LS over N_p pilot symbols inserted in each OFDM block. Two more processing blocks of i) iterative ICI cancellation and ii) frequency-domain smoothing (to take advantage of the previous N_c-1 estimates of $\{\bar{\alpha}_{k,l}\}_{k=0}^{N_c-2}$) then follow to improve estimation accuracy and speed up convergence. However, increasing performance requires a relatively large number of pilot symbols per block.

In the following, we address the problem of estimating $\bar{\boldsymbol{\alpha}}$ using all data symbols available at each OFDM block, not only pilots. By doing so, we develop a new ML-type EM solution that is able to significantly improve performance while keeping the same overhead or otherwise reducing it. Accuracy can be further enhanced as in [10] by suppressing the ICI components from the received signal.

III. NDA ML EM Channel Gains Estimation

The probability density function (pdf) of the received samples $\{\{y_k(n)\}_{n=1}^{N}\}_{k=0}^{N_c-1}$ conditioned on the transmitted symbol $a_k[n]$ and parametrized by $\boldsymbol{\psi}_k=\left[[\bar{\alpha}_{k,1},\bar{\alpha}_{k,2},\ldots,\bar{\alpha}_{k,L}],\sigma^2\right]^T$, is expressed as follows:

$$p(y_k(n)|a_k[n]=a_m;\boldsymbol{\psi}_k)=\frac{1}{2\pi\sigma^2}\exp\left\{\frac{-1}{2\sigma^2}\left|y_k(n)-a_m[\mathbf{H}_k]_{n,n}\right|^2\right\}, \tag{8}$$

where:

$$[\mathbf{H}_k]_{n,n} = \frac{1}{N}\sum_{l=1}^{L}\left[e^{-j2\pi\left(\frac{n-1}{N}-\frac{1}{2}\right)\tau_l}\sum_{q=0}^{N-1}\alpha_{l,k}(qT_s)\right], \quad (9)$$

Note here that we neglect the effect of the ICI components and we also assume that normalized delays,$\{\tau_l\}_{l=1}^{L}$, are perfectly known to the receiver. The n^{th} diagonal element of the matrix $\mathbf{H}_k$ in (9) can also be written as follows:

$$[\mathbf{H}_k]_{n,n} = \bar{\boldsymbol{\varphi}}_k^T\mathbf{F}_n, \quad (10)$$

where $\bar{\boldsymbol{\varphi}}_k = [\bar{\alpha}_{k,1}, \bar{\alpha}_{k,2}, \ldots, \bar{\alpha}_{k,L}]^T$ and $\mathbf{F}_n$ is a vector containing the elements of the n^{th} row of the $N \times L$ matrix $\mathbf{F}$ defined as:

$$[\mathbf{F}]_{m,l} = e^{-j2\pi\left(\frac{m-1}{N}-\frac{1}{2}\right)\tau_l}. \quad (11)$$

By injecting (10) back into (8), we obtain the following result:

$$p(y_k(n)|a_k[n] = a_m; \boldsymbol{\psi}_k) = \frac{1}{2\pi\sigma^2}\exp\left\{\frac{-1}{2\sigma^2}\left|y_k(n) - a_m\bar{\boldsymbol{\varphi}}_k^T\mathbf{F}_n\right|^2\right\}. (12)$$

Now, by averaging (12) over the alphabet, the pdf of the received samples can be written as follows:

$$p(y_k(n);\boldsymbol{\psi}_k) = \frac{1}{2M\pi\sigma^2}\sum_{m=1}^{M}\exp\left\{-\frac{1}{2\sigma^2}\left|y_k(n) - a_m\bar{\boldsymbol{\varphi}}_k^T\mathbf{F}_n\right|^2\right\}. (13)$$

It is obvious at this stage that maximizing (13) with respect to $\boldsymbol{\psi}_k$ is analytically intractable. Thus, we will resort to the EM concept to find the maximum of the multidimensional likelihood function (LF). First, we define the log-LF (LLF), $\boldsymbol{L}(\boldsymbol{\psi}_k|a_k[n] = a_m) \triangleq \ln(p(y_k(n)|a_k[n] = a_m; \boldsymbol{\psi}_k)$, of $y_k(n)$ conditioned on the transmitted symbol $a_k[n]$ for the k^{th} OFDM symbol which can be written as:

$$\boldsymbol{L}(\boldsymbol{\psi}_k|a_k[n] = a_m) = -\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\Big(|y_k(n)|^2 + \left|a_m\bar{\boldsymbol{\varphi}}_k^T\mathbf{F}_n\right|^2 - 2\Re\left\{y_k(n)^* a_m\bar{\boldsymbol{\varphi}}_k^T\mathbf{F}_n\right\}\Big). (14)$$

During the first step of the EM algorithm, known as the "*Expectation step*", we start by computing the expectation of LLF in (14) over all possible transmitted symbols, $\{a_m\}_{m=1}^{M}$. Then, the resulting expectation is maximized with respect to the unknown coefficient $\boldsymbol{\psi}_k$. By relying on an initial guess, $\widehat{\boldsymbol{\psi}}_k^{(0)}$, of the channel estimates, the cost function, at the r^{th} EM iteration is given by:

$$\boldsymbol{Q}\left(\boldsymbol{\psi}_k|\widehat{\boldsymbol{\psi}}_k^{(r-1)}\right) = \sum_{n=1}^{N}E_{a_m}\left\{\boldsymbol{L}(\boldsymbol{\psi}_k|a_k[n] = a_m)\Big|y_k(n); \widehat{\boldsymbol{\psi}}_k^{(r-1)}\right\}, (15)$$

where $E_{a_m}\{.\}$ is the expectation over all possible transmitted symbols $\{a_m\}_{m=1}^{M}$ and $\widehat{\boldsymbol{\psi}}_k^{(r-1)} = \left[\widehat{\boldsymbol{\varphi}}_k^{(r-1)}, \widehat{\sigma^2}_k^{(r-1)}\right]^T$ contains the estimate of $\boldsymbol{\psi}_k$ at the $(r-1)^{th}$ EM iteration. The equation in (15) can be further simplified as follows:

$$\boldsymbol{Q}\left(\boldsymbol{\psi}_k|\widehat{\boldsymbol{\psi}}_k^{(r-1)}\right) = -N\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\left(Z_{2,k} + \sum_{n=1}^{N}\gamma_{n,k}^{(r-1)}\left|\bar{\boldsymbol{\varphi}}_k^T\mathbf{F}_n\right|^2 - 2\beta_{n,k}^{(r-1)}\right), (16)$$

where:

$$Z_{2,k} = \sum_{n=1}^{N}|y_k(n)|^2, \quad (17)$$

$$\gamma_{n,k}^{(r-1)} = E_{a_m}\left\{|a_m|^2\Big|y_k(n); \bar{\boldsymbol{\varphi}}_k^{(r-1)}\right\}, \quad (18)$$

$$\beta_{n,k}^{(r-1)} = E_{a_m}\left\{\Re\left\{y_k(n)^* a_m\bar{\boldsymbol{\varphi}}_k^T\mathbf{F}_n\right\}\Big|y_k(n); \bar{\boldsymbol{\varphi}}_k^{(r-1)}\right\}. (19)$$

Using the Bayes formula, the a posteriori probability of a_m , $P_{m,n,k}^{(r-1)} = P\left(a_m|y_k(n); \widehat{\boldsymbol{\psi}}_k^{(r-1)}\right)$, at the $(r-1)^{th}$ iteration is given by:

$$P\left(a_m|y_k(n); \widehat{\boldsymbol{\psi}}_k^{(r-1)}\right) = \frac{P[a_m]P\left(y_k(n)|a_m; \widehat{\boldsymbol{\psi}}_k^{(r-1)}\right)}{P\left(y_k(n); \widehat{\boldsymbol{\psi}}_k^{(r-1)}\right)}. (20)$$

Since the transmitted symbols are equiprobable (i.e., $P[a_m] = \frac{1}{M}$), we have the following result:

$$P\left(y_k(n); \widehat{\boldsymbol{\psi}}_k^{(r-1)}\right) = \frac{1}{M}\sum_{n=1}^{N}P\left(y_k(n)|a_m; \widehat{\boldsymbol{\psi}}_k^{(r-1)}\right). (21)$$

For normalized-energy constant-envelope constellations, note that we have $\gamma_{n,k}^{(r-1)} = 1$. Exploiting then the fact that $\bar{\boldsymbol{\varphi}}_k = \Re\{\bar{\boldsymbol{\varphi}}_k\} + \Im\{\bar{\boldsymbol{\varphi}}_k\}$ and $\mathbf{F}_n = \Re\{\mathbf{F}_n\} + \Im\{\mathbf{F}_n\}$, the cost function in (16) can be written as follows:

$$\boldsymbol{Q}\left(\boldsymbol{\psi}_k|\widehat{\boldsymbol{\psi}}_k^{(r-1)}\right) = -N\ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\Bigg(Z_{2,k} + \sum_{n=1}^{N}\gamma_{n,k}^{(r-1)} \times \left(\mathbf{F}_n^H\mathbf{G}_{1,k}\mathbf{F}_n + \Im\{\mathbf{F}_n\}^T\mathbf{G}_{2,k}\Re\{\mathbf{F}_n\} + \Re\{\mathbf{F}_n\}^T\mathbf{G}_{3,k}\Im\{\mathbf{F}_n\}\right) - 2\sum_{m=1}^{M}P_{m,n,k}^{(r-1)}\eta_{k,n}^{(m)}\Bigg), (22)$$

where:

$$\begin{aligned}
\mathbf{G}_{1,k} &= \Re\{\bar{\boldsymbol{\varphi}}_k\}\Re\{\bar{\boldsymbol{\varphi}}_k\}^T + \Im\{\boldsymbol{\varphi}_k\}\Im\{\bar{\boldsymbol{\varphi}}_k\}^T,\\
\mathbf{G}_{2,k} &= \Re\{\bar{\boldsymbol{\varphi}}_k\}\Im\{\bar{\boldsymbol{\varphi}}_k\}^T - \Im\{\bar{\boldsymbol{\varphi}}_k\}\Re\{\bar{\boldsymbol{\varphi}}_k\}^T,\\
\mathbf{G}_{3,k} &= \Im\{\bar{\boldsymbol{\varphi}}_k\}\Re\{\bar{\boldsymbol{\varphi}}_k\}^T - \Re\{\bar{\boldsymbol{\varphi}}_k\}\Im\{\bar{\boldsymbol{\varphi}}_k\}^T,\\
\eta_{k,n}^{(m)} &= \Re\{y_k(n)^* a_m\mathbf{F}_n^T\}\Re\{\bar{\boldsymbol{\varphi}}_k\} - \Im\{y_k(n)^* a_m\mathbf{F}_n^T\}\Im\{\bar{\boldsymbol{\varphi}}_k\}.
\end{aligned}$$

In the maximization step, we start differentiating the cost function in (22) with respect to $\Re\{\bar{\boldsymbol{\varphi}}_k\}$ and $\Im\{\bar{\boldsymbol{\varphi}}_k\}$ then setting it to zero to obtain the following results:

$$\sum_{n=1}^{N}\gamma_{n,k}^{(r-1)}\Big(\mathbf{J}_{1,n}\Re\{\bar{\boldsymbol{\varphi}}_k\} - \mathbf{J}_{2,n}\Im\{\bar{\boldsymbol{\varphi}}_k\}\Big) = \sum_{n=1}^{N}\boldsymbol{\mu}_{1,n}, \quad (23)$$

$$\sum_{n=1}^{N}\gamma_{n,k}^{(r-1)}\Big(\mathbf{J}_{1,n}\Im\{\bar{\boldsymbol{\varphi}}_k\} + \mathbf{J}_{2,n}\Re\{\bar{\boldsymbol{\varphi}}_k\}\Big) = -\sum_{n=1}^{N}\boldsymbol{\mu}_{2,n}, (24)$$

where:

$$\mathbf{J}_{1,n} = \Re\{\mathbf{F}_n\}\Re\{\mathbf{F}_n\}^T + \Im\{\mathbf{F}_n\}\Im\{\mathbf{F}_n\}^T, \quad (25)$$

$$\mathbf{J}_{2,n} = \Re\{\mathbf{F}_n\}\Im\{\mathbf{F}_n\}^T - \Im\{\mathbf{F}_n\}\Re\{\mathbf{F}_n\}^T, \quad (26)$$

$$\boldsymbol{\mu}_{1,n} = \sum_{m=1}^{M} P_{m,n,k}^{(r-1)} \Re\{y_k(n)^* a_m \mathbf{F}_n^T\}, \quad (27)$$

$$\boldsymbol{\mu}_{2,n} = \sum_{m=1}^{M} P_{m,n,k}^{(r-1)} \Im\{y_k(n)^* a_m \mathbf{F}_n^T\}. \quad (28)$$

Now, using the identity $\bar{\boldsymbol{\varphi}}_k = \Re\{\bar{\boldsymbol{\varphi}}_k\} + j\Im\{\bar{\boldsymbol{\varphi}}_k\}$ leads to the following result:

$$\sum_{n=1}^{N} (\mathbf{J}_{1,n} + j\mathbf{J}_{2,n})\gamma_{n,k}^{(r-1)} \bar{\boldsymbol{\varphi}}_k = \sum_{n=1}^{N} \boldsymbol{\mu}_{1,n} - j\boldsymbol{\mu}_{2,n}. \quad (29)$$

Hence, the estimated time average of the channel gains at the r^{th} iteration can be obtained as follows:

$$\widehat{\bar{\boldsymbol{\varphi}}}_k^{(r)} = \left(\sum_{n=1}^{N} \gamma_{n,k}^{(r-1)} (\mathbf{J}_{1,n} + j\mathbf{J}_{2,n})\right)^{-1} \times \quad (30)$$

$$\sum_{n=1}^{N} \left(\sum_{m=1}^{M} P_{m,n,k}^{(r-1)} y_k^*(n) a_m \mathbf{F}_n^T\right)^H. \quad (31)$$

Similarly, by differentiating the cost function in (22) with respect to σ^2, we obtain the following estimate of the noise variance:

$$\widehat{\sigma^2}^{(r)} = \frac{Z_{2,k} + \sum_{n=1}^{N} \left|\mathbf{F}_n^T \widehat{\bar{\boldsymbol{\varphi}}}_k^{(r-1)}\right| \gamma_{n,k}^{(r-1)} - 2\beta_{n,k}^{(r-1)}}{2N} \quad (32)$$

Finally, at convergence of the EM algorithm (i.e., after R_{EM} iterations), the channel estimates corresponding to N_c consecutive OFDM symbols are given by:

$$\widehat{\boldsymbol{\alpha}}_l = \mathbf{S}^T \widehat{\mathbf{c}}_l = \mathbf{S}^T \mathbf{T}^{-1} \widehat{\bar{\boldsymbol{\alpha}}}_l^{(R_{EM})}, \quad (33)$$

where $\widehat{\bar{\boldsymbol{\alpha}}}_l^{(R_{EM})} = [\widehat{\bar{\alpha}}_{l,1}^{(R_{EM})}, \widehat{\bar{\alpha}}_{l,2}^{(R_{EM})}, \ldots, \widehat{\bar{\alpha}}_{l,N_c}^{(R_{EM})}]^T$ is an estimation vector for the complex channel gain time averages of the l^{th} path over N_c OFDM data symbols. The channel gain estimates in (33) can be further as in [10] by implementing an iterative ICI suppression technique such as successive interference cancellation (SIC). Indeed, the channel estimates at the output of the EM technique can be used to reconstruct then remove the ICI components from the received signal and the resulting samples can be re-injected once again as new inputs of the new EM solution to increase accuracy. The process can be repeated over R_{ICI} iterations until no additional improvements can be achieved.

IV. Simulation Results

In this section, we assess the performance of the new ML EM channel estimator at the component level in terms of the mean square error (MSE) of the channel gains, but also in terms of link-level bit error rate (BER) and throughput. In all simulations, we consider an OFDM RIT with $N = 128$ subcarriers, a central frequency $f_c = 5$ GHz, and a sampling frequency $T_s = 0.5\ \mu s$. The channel between the transmitter and receiver is modeled as a multipath Rayleigh fading channel and the complex gains $\{\alpha_l(t)\}_{l=1}^{L}$ are generated with variance $\{\sigma_l^2\}_{l=1}$ at any given Doppler using a uniform Jake's model. The channel parameters adopted in all simulations are summarized in the following table:

TABLE I
Channel Parameters

Path number	1	2	3	4	5	6
Average Power [dB]	-7.219	-4.219	-6.219	-10.219	-12.219	-14.219
Normalized Delay	0	0.4	1	3.2	4.6	10

We start by investigating the effect of the number of EM iterations on the estimation accuracy. To do so, we plot in Fig. 1 the MSE of the EM technique along with the low bound (LB) derived in [10] against R_{EM} at two different SNR levels.

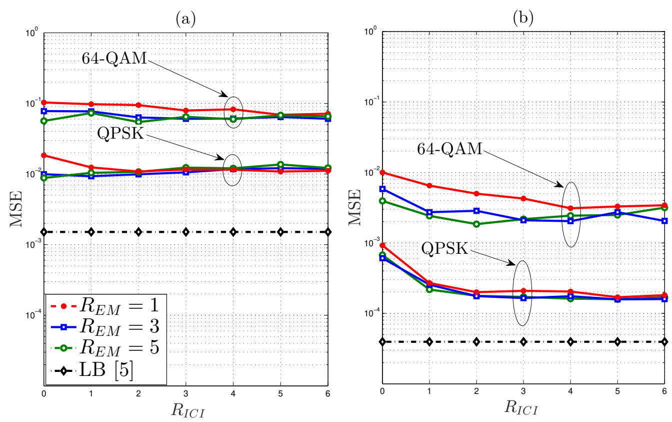

Fig. 1. MSE of ML EM vs its number of iterations with $N_c = 3$ and $N_p = 8$ at: (a) $SNR = 10$ dB, and (b) $SNR = 30$ dB.

Obviously, at a fixed SNR level, the convergence rate of the EM technique (R_{EM}) is affected by the ICI level corrupting the received samples. In fact, the EM technique is able to converge much faster when the ICI level is reduced with an ICI cancellation technique. For instance, when using QPSK modulation, ML EM is able to provide the same accuracy either with 1 or 5 EM iterations when ICI cancellation is applied. However, for high modulation order (e.g., 64-QAM) that are usually more sensitive to ICI component, the same technique requires at least 3 EM iterations to converge when no ICI cancellation procedure is used.

In Fig. 2, we assess the robustness of the proposed technique to ICI and compare it to both the LS technique and LB derived in [10]. We observe a clear advantage of the ML EM technique at both low and high Dopplers. We also observe that the ICI cancellation block enhances the performance of both techniques. However, the ML EM benefits from much larger gains and approaches the LB at high SNR values.

In Fig. 3, we see that the gap between the two techniques increases when reducing the number of pilots per OFDM block from $N_p = 16$ to $N_p = 8$, more so at high Doppler.

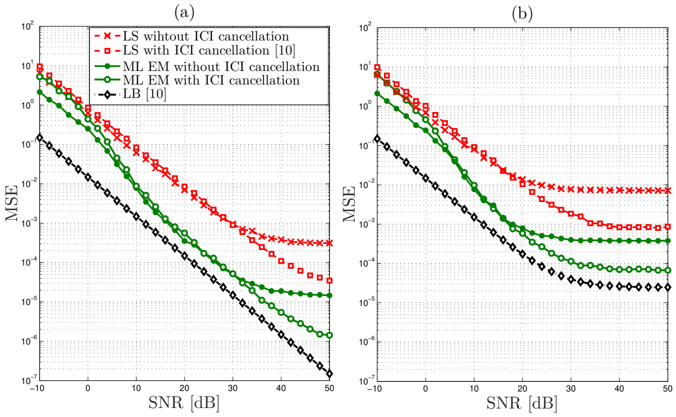

Fig. 2. MSE of ML EM and LS vs. the SNR with $N_c = 3$ and $N_p = 8$ at: (a) $F_D T = 0.02$, and (b) $F_D T = 0.1$.

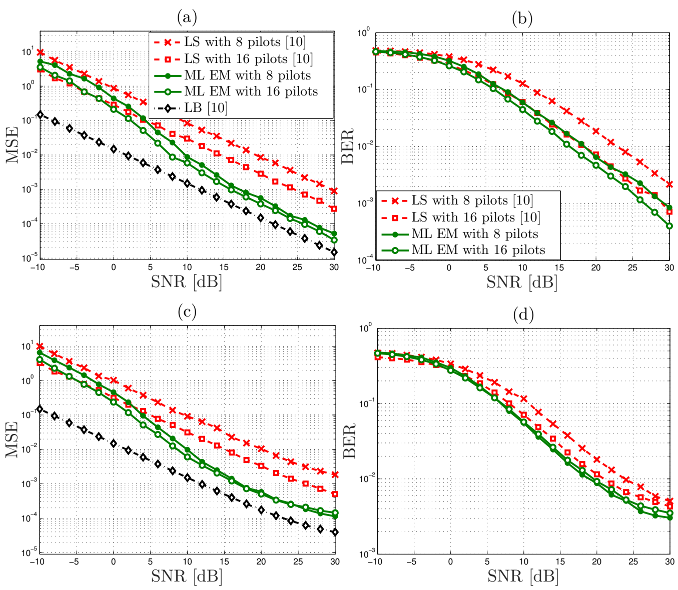

Fig. 3. MSE and BER of ML EM and LS vs. the SNR with $N_c = 3$ in terms of: (a) MSE at $F_D T = 0.02$, (b) BER at $F_D T = 0.02$, (c) MSE at $F_D T = 0.1$, and (b) BER at $F_D T = 0.1$.

Indeed, the LS technique's performance deteriorates when reducing N_p while the NDA technique keeps exactly the same performance at medium or high SNR values. Actually, the ML EM provides approximately the same BER as LS, yet with a lower number of pilots. Consequently, the new technique can achieve a higher throughput since the overhead is reduced by half.

In Fig. 4, we plot the link-level throughput curves of both ML EM and LS assuming the adoption of an adaptive (i.e., SNR-dependent) modulation scheme.

Here, we observe a clear advantage of the ML EM technique especially at high mobility (i.e., $F_D T = 0.1$) and higher modulation order (i.e., 16- and 64-QAM). Note that both techniques, ML EM and LS, use the SIC block to decode the received symbols. As reported previously, the LS technique provides less reliable channel estimates since it uses pilot symbols only. Those estimates, which are injected later in the SIC block, lead to a higher BER. Moreover, from Fig. 4 (b), we observe that the performance of the LS technique significantly deteriorates when the number of pilots is reduced by half from 16 to 8. Such behaviour stems from the fact that poor channel gain estimates results in less reliable ICI cancellation, especially at higher modulation orders. However, the ML EM maintains approximately the same performance in terms of MSE whether initialization is with $N_p = 8$ or 16 pilot symbols. Hence, it exhibits a higher link-level throughput than LS, more so at medium and high SNR levels, with best performance at high Doppler achieved with $N_p = 8$ pilots.

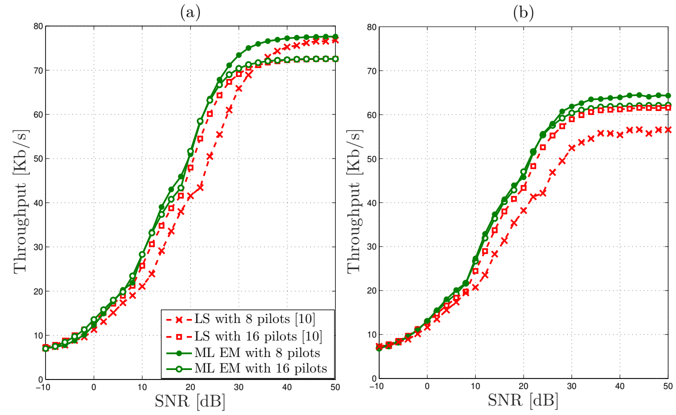

Fig. 4. Link-level throughput vs. the SNR for ML EM and LS at: (a) $F_D T = 0.02$, and (b) $F_D T = 0.1$.

V. Conclusion

In this paper, we addressed the problem of channel estimation under time-varying channels for OFDM systems. The proposed approach is based on a polynomial approximation of the complex channel gains and take advantage of all the available data symbols to provide reliable channel gains estimates. It is shown through simulations, that the proposed solution provides better performance than the DA technique. The latter translates into a net advantages of the proposed ML EM technique in terms of BER and link-level throughput gains, especially at medium and high SNRs, more so at relatively higher Doppler frequencies.

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