SNR Estimation Over SIMO Channels From Linearly Modulated Signals

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***Abstract*—In this paper, we address the problem of data-aided (DA) and nondata-aided (NDA) per-antenna signal-to-noise ratio (SNR) estimation over wireless single-input multiple-output (SIMO) channels from linearly modulated signals. Under constant channels and additive white Gaussian noise (AWGN), we first derive the DA maximum-likelihood (ML) SNR estimator in closed-form expression. The performance of the DA ML estimator is analytically carried out by deriving the closed-form expression of its bias and variance. Besides, in order to compare its performance with the fundamental limit, we derive the DA Cramér-Rao lower bound (CRLB) in closed-form expression. In the NDA case, the expectation-maximization (EM) algorithm is derived to iteratively maximize the log-likelihood function. The performance of the NDA ML estimator is empirically assessed using Monte Carlo simulations. Moreover, we introduce an efficient algorithm, which applies to any one/two-dimensional M-ary constellation, to numerically compute the NDA CRLBs. In this paper, the noise components are assumed to be spatially uncorrelated over all the antenna elements and temporally white. In both cases, we show that our new inphase and quadrature I/Q-based estimators offer substantial performance improvements over the single-input single-output (SISO) ML SNR estimator due to the optimal usage of the statistical dependence between the antenna branches, and that it reaches the corresponding CRLB over a wide SNR range. We also show that the use of the I/Q-based ML estimators can lead to remarkable performance improvements over the moment-based estimators for the same antenna-array size. Moreover, it is shown that SIMO configurations can contribute to decreasing the required number of iterations of the EM algorithm.**

***Index Terms*—Cramér-Rao lower bound (CRLB), data-aided (DA), nondata-aided (NDA), single-input multiple-output (SIMO), signal-to-noise ratio (SNR) estimation.**

I. Introduction

MANY modern communication systems require accurate signal-to-noise ratio (SNR) estimates for the optimal usage of radio resources [1]–[3]. For instance, the knowledge of the SNR is a requirement in many applications in order to perform efficient signal detection, power control or adaptive modulation schemes. Roughly speaking, SNR estimators may be divided into two major categories: data-aided (DA) and nondata-aided (NDA). In contrast to DA methods which rely on the *a priori* perfect knowledge of the transmitted symbols to facilitate the estimation process, NDA techniques base the estimation process only on the received samples and they do not, therefore, impinge upon the whole throughput of the system.

In both cases, SNR estimates can be obtained from the inphase and quadrature (I/Q) components of the received signal or simply from its magnitude (i.e., envelope). They are, respectively, referred to as I/Q-based and envelope-based SNR estimators. So far, for linearly modulated signals, various SNR estimation techniques have been reported in the literature for application over flat fading channels in traditional single-input single-output (SISO) transmissions [4], [5]. These include the maximum-likelihood (ML) I/Q-based estimator [6], [7] and the ML envelope-based estimator [8], [9]. In both cases, the analytical derivation of the NDA ML estimator was recognized there to be mathematically intractable, and the numerical computations of the ML SNR estimates were carried out using the iterative expectation-maximization (EM) algorithm [10]. On the other hand, it has been recently shown in [11] for the first time with moment-based estimators, termed there as M_4, that the exploitation of the statistical dependence offered by SIMO systems can lead to remarkable improvements in SNR estimation accuracy over current state-of-the-art techniques. However, contrarily to the I/Q-based estimators, it is well known that the envelope-based SNR estimators do not exploit the whole information carried by the received signal.

Motivated by these facts, we develop in this paper per-antenna maximum-likelihood SNR estimators which exploit the entire information carried by the signal (i.e., I/Q-based estimators) as well as the rich statistical dependence experienced by multiple receiving antennas (i.e., SIMO estimators). In fact, we derive a closed-form solution for the SIMO DA ML SNR estimator. Moreover, we analytically assess its exact performance by deriving its mean and variance in closed form. Then, in order to assess the absolute performance of the DA ML estimator, we derive the closed-form expression of the CRLB for the per-antenna DA SNR estimates over SIMO channels (i.e., DA CRLB).

Unfortunately, for the NDA approach, an analytical derivation of the ML solution turns out to be fairly complex from the computational point of view. Therefore, we resort to the EM algorithm [10] which has been so far widely used in signal parameter estimation to numerically find the ML estimates. Its performance will also be numerically assessed using Monte Carlo simulations. Furthermore, we introduce in this paper, a functional approach that allows the efficient numerical computation

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of the CRLBs for NDA SNR estimates over SIMO channels from any one/two-dimensional M-ary constellation-modulated signal. Using the normalized mean square error (NMSE) as a performance measure, we will show that the DA estimator performs better than the NDA estimator especially in the low SNR region and that the two estimators exhibit the same performance over a wide range of medium-to-high SNR values.

The structure of the rest of this paper is as follows. In Section II, we introduce the equivalent baseband model of the signal. The ML SNR estimator for the DA approach, its bias, its variance and the corresponding CRLBs will be derived in Section III. In Section IV, the EM-based ML SNR estimator for the NDA approach is developed and the NDA CRLBs are derived. Simulation results are discussed in Section V and some concluding remarks are drawn out in Section VI.

II. System Model

We consider an array of N_a receiving antenna elements. Over the observation interval, the SIMO channel is supposed to be with constant gain coefficients $\{S_i\}_{i=1,2,\ldots,N_a}$. We assume that the same noise power, $2\sigma^2$, is experienced over all the antenna elements. Assuming an ideal receiver with perfect time and carrier frequency synchronization, the received signal at the output of the matched filter, on the i^{th} antenna element, is given by

$$r_i(n) = S_i a(n) e^{j\phi_i} + \sqrt{2\sigma^2} w_i(n), \quad n = 1, 2, \ldots, N, \quad i = 1, 2, \ldots, N_a \quad (1)$$

where, at time index n, $a(n)$ is the transmitted symbol, and $r_i(n)$ is the corresponding received sample on the i^{th} antenna element. The noise components $w_i(n)$, assumed spatially white, are modelled by zero-mean Gaussian random variables with independent real and imaginary parts, each of variance 1/2 and N stands for the size of the observation window. Moreover, the transmitted symbols are assumed to be independent and identically distributed (iid) and drawn from any linear M-ary constellation. The constellation power is assumed to be normalized to one, i.e., $\mathrm{E}\{|a(n)|^2\} = 1$ where $|.|$ returns the norm of any complex number and $\mathrm{E}\{.\}$ stands for statistical expectation. The parameter ϕ_i accounts for a nonrandom known phase shift introduced by the channel, i.e., estimated in practice,[1] but assumed perfectly known here for simplicity. Therefore, we will henceforth consider the derotated received signal $\{y_i(n) = r_i(n) e^{-j\phi_i}\}_{i=1}^{N_a}$:

$$y_i(n) = S_i a(n) + \sqrt{2\sigma^2} \eta_i(n), \ \ n = 1, 2, \ldots, N, \ \ i = 1, 2, \ldots, N_a \quad (2)$$

where the derotated noise components $\{\eta_i(n) = w_i(n) e^{-j\phi_i}\}_{i=1}^{N_a}$ have the same probability distribution as $\{w_i(n)\}_{i=1}^{N_a}$, i.e., both $\eta_i(n)$ and $w_i(n)$ are complex zero-mean AWGN samples with the same variance. In the sequel, for simplicity, we refer to the "*derotated received samples*" by simply "*the received samples*". Based on the N received samples on each antenna branch, the true per-antenna SNRs that we wish to estimate are given by

$$\rho_i = \frac{S_i^2}{2\sigma^2}, \quad i = 1, 2, \ldots, N_a. \quad (3)$$

From (3), we see that there are $N_a + 1$ parameters, $\{S_i\}_{i=1,2,\ldots,N_a}$ and σ^2, which are involved in the derivation of the ML per-antenna SNR estimators and the corresponding CRLBs. These parameters can be gathered in the following parameter vector:

$$\boldsymbol{\theta} = \left[S_1, S_2, \ldots, S_{N_a}, \sigma^2\right]^T \quad (4)$$

where $[.]^T$ stands for the transpose operator. Moreover, the received samples on all the antenna elements, at a given instant n, can be more conveniently written in the following vectorial form:

$$\boldsymbol{y}(n) = \left[y_1(n), y_2(n), \ldots, y_{Na}(n)\right]^T, \quad n = 1, 2, \ldots, N. \quad (5)$$

Then, considering the entire observation window, the received signal can be written in the following $N_a \times N$ matrix form:

$$\boldsymbol{Y} = \left[\boldsymbol{y}(1), \boldsymbol{y}(2), \ldots, \boldsymbol{y}(N)\right]. \quad (6)$$

III. DA ML SNR Estimators and CRLBs

In this section, we suppose perfect *a priori* knowledge of the transmitted data. We will firstly derive the DA ML solution for the per-antenna SNR estimates in closed-form expression. Then, we will derive the exact expression of the new estimator's bias and variance. Finally, we will derive the closed-form expression of the corresponding DA CRLBs.

A. Closed-Form Solution for Per-Antenna DA ML SNR Estimation

DA approaches can only generate estimates when known data are available. The particular application will impose whether or not this limitation is objectionable. Nevertheless, in most cases, there is no additional penalty by using SNR estimators that employ known data in transmission systems that already use training sequences for equalizer or synchronizer training. Assuming the noise components to be spatially white, the received samples are also iid. Therefore, it can be seen that the probability density function (pdf) of the received matrix $\boldsymbol{Y}$ parameterized by $\boldsymbol{\theta}$ is given by

$$P[\boldsymbol{Y}; \boldsymbol{\theta}] = \prod_{n=1}^{N} \prod_{i=1}^{Na} \frac{1}{2\pi\sigma^2} \exp\left(-\frac{|y_i(n) - S_i a(n)|^2}{2\sigma^2}\right). \quad (7)$$

Consequently, applying the logarithm to (7), the log-likelihood function, denoted as $L(\boldsymbol{\theta}) = \ln(P[\boldsymbol{Y}; \boldsymbol{\theta}])$, develops into

$$L(\boldsymbol{\theta}) = \sum_{n=1}^{N} \sum_{i=1}^{N_a} -\ln(2\pi\sigma^2) - \frac{|y_i(n)|^2}{2\sigma^2} - \frac{S_i^2}{2\sigma^2} |a(n)|^2 + \frac{2S_i}{2\sigma^2} \Re\left\{y_i(n)^* a(n)\right\} \quad (8)$$

[1]Note here that I/Q-based methods are sensitive to phase uncertainties (carrier/phase offsets, phase noise) whereas envelope-based methods are robust in this sense.

where $\{.\}^*$ indicates complex conjugation and $\Re\{.\}$ stands for the real part of any complex number. Then, setting the partial derivatives of (8) with respect to $\{S_i\}_{i=1}^{N_a}$ and σ^2 to zero yields the DA ML solutions as follows:

$$\widehat{S}_i=\frac{1/N\sum_{n=1}^{N}\Re\{y_i(n)^*a(n)\}}{1/N\sum_{n=1}^{N}|a(n)|^2}, \quad i=1,2,\ldots,N_a \tag{9}$$

$$\widehat{\sigma^2}=\frac{1}{2NN_a}\sum_{n=1}^{N}\sum_{i=1}^{N_a}\left|y_i(n)-\widehat{S}_i a(n)\right|^2. \tag{10}$$

On the i^{th} antenna branch the closed-form solution for the DA ML SNR estimator is simply obtained as

$$\widehat{\rho_i}=\frac{\widehat{S_i}^2}{2\widehat{\sigma^2}} \quad i=1,2,\ldots,N_a \tag{11}$$

where $\widehat{S_i}$ and $\widehat{\sigma^2}$ are given by (9) and (10), respectively. Note that the DA ML solution holds regardless of the modulation order or type. Next, we derive the exact bias and variance of the DA ML estimator (11).

B. Exact Expressions for the Bias and Variance of the DA ML Estimator

In this section we focus on the exact performance of the DA ML estimator. At high SNR, this analysis holds as well for the NDA ML estimator that will be developed in the next section. The closed-form expression of the performance criterion can lead to a significant speed-up factor in the assessment of the SNR estimator relative to computer simulations and can allow accurate system design improvement and optimization. In this context, we show in Appendix A that the bias and the variance of the DA ML SNR estimator are given by the following expressions:

$$\mathrm{E}\{\widehat{\rho_i}\}=\frac{N_aN}{N_a(N-1)-1}\left(\rho_i+\frac{1}{2N}\right) \tag{12}$$

$$\mathrm{Var}\{\widehat{\rho_i}\}=\frac{N_a^2N^2\left(\rho_i^2+\rho_i\left(\frac{2N_a(N-1)}{N}-\frac{1}{N}\right)+\frac{N_a(N-1)}{2N^2}-\frac{1}{4N^2}\right)}{(N_a(N-1)-1)^2(N_a(N-1)-2)}. \tag{13}$$

Another major goal of calculating the exact mean of $\widehat{\rho_i}$ is to improve our estimator performance by calculating and removing its bias. It is interesting to note that the DA ML estimator is biased with

$$\mathrm{Bias}\{\widehat{\rho_i}\}=\frac{N_a+1}{N_a(N-1)-1}\rho_i+\frac{N_a}{2N_a(N-1)-2}. \tag{14}$$

It is also clearly seen that increasing N or N_a can significantly reduce the bias. Yet to eliminate this bias, one could use the estimator $\widehat{\rho_i}^{\text{ub}}=((N_a(N-1)-1)/N_aN)\widehat{\rho_i}-(1/2N)$ instead of $\widehat{\rho_i}$. However, in most applications, N is sufficiently large so that the bias will be negligible. The variance of the unbiaised estimator is given by

$$\mathrm{Var}\left\{\widehat{\rho_i}^{\text{ub}}\right\}=\frac{1}{(N_a(N-1)-2)}\left(\rho_i^2+\rho_i\left(\frac{2N_a(N-1)}{N}-\frac{1}{N}\right)+\frac{N_a(N-1)}{2N^2}-\frac{1}{4N^2}\right). \tag{15}$$

Now examining the analytical form of the normalized mean square error (NMSE) defined as

$$\mathrm{NMSE}(\widehat{\rho_i})=\frac{E\left\{(\hat{\rho}_i-\rho_i)^2\right\}}{\rho_i^2} \tag{16}$$

we can easily verify that asymptotically, as long as $N\gg 1$, the NMSE of the unbiased estimator is given by

$$\mathrm{NMSE}\left(\widehat{\rho_i}^{\text{ub}}\right)=\frac{\mathrm{Var}\left\{\widehat{\rho_i}^{\text{ub}}\right\}}{\rho_i^2}\underset{N\gg 1}{\longrightarrow}\frac{1}{N_aN}+\frac{2}{N\rho_i} \tag{17}$$

where $\mathrm{Var}\{\widehat{\rho_i}^{\text{ub}}\}$ is given by (15). It will be seen in the next subsection that this is in fact the expression of the normalized CRLB in DA scenarios, which means that our DA ML estimator is asymptotically efficient [the minimum variance unbiased (MVU) estimator]. Note also from (17) that the NMSE is inversely proportional to N and N_a.

C. Derivation of SNR DA CRLBs Over SIMO Channels

In this subsection, based on the assumptions made so far, we will derive the CRLBs for the SNR estimates in the DA case (where all the transmitted symbols are assumed to be perfectly known to the receiver) from any AWGN-corrupted linearly modulated signal. To do so, we consider the following parameter vector:

$$\begin{aligned}\boldsymbol{\alpha}&=[\alpha_1,\alpha_2,\ldots,\alpha_{N_a+1}]^T\\&=\left[\rho_1,\rho_2,\ldots,\rho_{N_a},\sigma^2\right]^T.\end{aligned} \tag{18}$$

The CRLB for the DA SNR estimates on the i^{th} antenna element is given by

$$\mathrm{CRLB}_{\mathrm{DA}}(\rho_i)=\left[\boldsymbol{I}_{\mathrm{DA}}^{-1}(\boldsymbol{\alpha})\right]_{i,i} \tag{19}$$

where $\boldsymbol{I}_{\mathrm{DA}}(\boldsymbol{\alpha})$ denotes the DA Fisher information matrix (FIM) whose entries are defined as

$$[\boldsymbol{I}_{\mathrm{DA}}(\boldsymbol{\alpha})]_{i,j}=-\mathrm{E}\left\{\frac{\partial^2\ln\left(P[\boldsymbol{Y};\boldsymbol{\alpha}]\right)}{\partial\alpha_i\partial\alpha_j}\right\} \tag{20}$$

where $P[\boldsymbol{Y};\boldsymbol{\alpha}]$ was defined in (7) but rewritten here in terms of $\{\rho_i\}_{i=1}^{N_a}$ and σ^2 as follows:

$$P[\boldsymbol{Y};\boldsymbol{\alpha}]=\prod_{n=1}^{N}\prod_{i=1}^{Na}\frac{1}{2\pi\sigma^2}\exp\left(-\frac{\left|y_i(n)-\sqrt{2\rho_i\sigma^2}a(n)\right|^2}{2\sigma^2}\right). \tag{21}$$

At this stage, it can be easily verified that $\mathrm{E}\{\partial^2\ln(P[\boldsymbol{Y};\boldsymbol{\alpha}])/\partial\rho_i\partial\rho_j\}=0$, for $i\neq j$. Moreover, for ease of notation, we denote the remaining FIM elements defined in (20) as follows:

$$c=-E\left\{\frac{\partial^2\ln\left(P[\boldsymbol{Y};\boldsymbol{\alpha}]\right)}{\partial{\sigma^2}^2}\right\}, \tag{22}$$

$$a_i=-E\left\{\frac{\partial^2\ln\left(P[\boldsymbol{Y};\boldsymbol{\alpha}]\right)}{\partial\rho_i^2}\right\}, \quad 1\le i\le N_a, \tag{23}$$

$$b_i=-E\left\{\frac{\partial^2\ln\left(P[\boldsymbol{Y};\boldsymbol{\alpha}]\right)}{\partial\rho_i\partial\sigma^2}\right\}, \quad 1\le i\le N_a \tag{24}$$

where $\ln(P[\boldsymbol{Y};\boldsymbol{\alpha}])$ is the log-likelihood function of the system which immediately develops form (21) as

$$\ln\left(P[\boldsymbol{Y};\boldsymbol{\alpha}]\right) = \sum_{n=1}^{N}\sum_{i=1}^{N_a} -\ln(2\pi\sigma^2) - \frac{|y_i(n)|^2}{2\sigma^2} - \rho_i|a_n|^2 + 2\sqrt{\frac{\rho_i}{2\sigma^2}}\Re\left\{y_i(n)^* a(n)\right\}. \quad (25)$$

Note that $c = [\boldsymbol{I}_{\mathrm{DA}}(\boldsymbol{\alpha})]_{N_a+1,N_a+1}$, $b_i = \{[\boldsymbol{I}_{\mathrm{DA}}(\boldsymbol{\alpha})]_{i,N_a}\}_{i=1,2,\cdots,N_a}$, and $a_i = \{[\boldsymbol{I}_{\mathrm{DA}}(\boldsymbol{\alpha})]_{i,i}\}_{i=1,2,\cdots,N_a}$ and, therefore, the $((N_a+1)\times(N_a+1))$ dimensional FIM can be written as

$$\boldsymbol{I}_{\mathrm{DA}}(\boldsymbol{\alpha}) = \begin{pmatrix} a_1 & 0 & 0 & \cdots & 0 & b_1 \\ 0 & a_2 & 0 & \cdots & 0 & b_2 \\ 0 & 0 & \ddots & \ddots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & 0 & \cdots & 0 & a_{N_a} & b_{N_a} \\ b_1 & b_2 & \cdots & \cdots & b_{N_a} & c \end{pmatrix}. \quad (26)$$

In particular, it is seen from (26) that the cross terms in the FIM corresponding to gains at different antennas are zero, meaning that the parameters ρ_i and ρ_j are uncoupled for $i \neq j$. It will be seen, in the next section, that this is not the case in NDA estimation. Now, as detailed in Appendix B, taking the i^{th} diagonal element of $\boldsymbol{I}_{\mathrm{DA}}^{-1}(\boldsymbol{\alpha})$, we show that the DA SNR CRLB on the i^{th} antenna element is given by:[2]

$$\mathrm{CRLB}_{\mathrm{DA}}(\rho_i) = \frac{\rho_i^2 + 2N_a\rho_i}{N_a N}, \quad i = 1,2,\ldots N_a. \quad (27)$$

First, note that (27) generalizes the result earlier introduced in [4, (64)], from SISO to SIMO configurations, for the case of one sample per symbol (i.e., $N_{ss} = 1$ where N_{ss} is used in [4] to refer to the number of samples per-symbol). Moreover, as mentioned previously, it can be verified from (17) and (27) that, for each antenna element, the asymptotic variance of our unbiased DA estimator coincides with its CRLB.

Furthermore, (27) leads to the discussion of the two following scenarios:

- **Scenario (a)**: CRLB for N_a antennas with N samples per antenna for which the bound is given by (27) and referred to as $\mathrm{CRLB}_{\mathrm{DA}}^{\mathrm{SIMO}}(\rho_i)$;
- **Scenario (b)**: CRLB on the i^{th} antenna element in a SISO configuration (single antenna) but with NN_a samples; for which the bound is obtained by first setting $N_a = 1$ in (27) then replacing N by NN_a to obtain

$$\mathrm{CRLB}_{\mathrm{DA}}^{\mathrm{SISO}}(\rho_i) = \frac{\rho_i^2 + 2\rho_i}{N_a N}. \quad (28)$$

The total number of samples in both scenarios is the same (NN_a). At high SNR, the ratio of both bounds goes to 1. However, as the SNR decreases, it turns out that the bound of scenario (b) is always below the one of scenario (a) and the advantage becomes larger as N_a increases, i.e.

$$\mathrm{CRLB}_{\mathrm{DA}}^{\mathrm{SISO}}(\rho_i) < \mathrm{CRLB}_{\mathrm{DA}}^{\mathrm{SIMO}}(\rho_i). \quad (29)$$

[2] Note here that the effect of having multiple antennas is to reduce the asymptotic floor that is attained by the normalized CRLB when $\mathrm{SNR} \to \infty$.

First, (29) can be intuitively explained by the fact that in scenario (a) we have more unknown parameters to be estimated (i.e., $S_1, S_2, \cdots, S_{N_a}$ and σ^2), than in scenario (b) (i.e., S_i and σ^2), from the same number of received samples in both scenarios (i.e., NN_a) and, hence, the CRLB of scenario (b) should be always smaller than the one of scenario (a). Hence, (29) may initially lead to the conclusion that it is preferable to have more samples with one antenna than more antennas. However, one should keep in mind that, in practice, the observation window size cannot be increased arbitrarily for the following two main reasons:

- First, in scenario (b), the observation interval size is larger (i.e., NN_a) over which the channel may not remain constant contrarily to a SIMO configuration with only N simultaneously received samples over each receiving antenna element (the observation window size is N over each antenna channel).
- Second, we are most often interested in instantaneous SNR estimation for real-time applications, which requires performing SNR estimation over short observation windows [N in scenario (a) instead of NN_a in scenario (b)]. In this case, SIMO configurations will always provide sufficiently accurate SNR estimates by exploiting spatial diversity, to have NN_a samples, over shorter observation windows than in traditional SISO configurations.

IV. NDA ML SNR Estimators and CRLBs

In this section, we will derive the NDA ML SNR estimators and the corresponding CRLB. In fact, when the transmitted symbols are assumed to be unknown, the probability function of the received vector $\boldsymbol{y}(n)$, parameterized by $\boldsymbol{\theta}$, can be obtained by averaging its DA pdf with respect to the data symbols as follows:

$$P\left[\boldsymbol{y}(n);\boldsymbol{\theta}\right] = \sum_{k=1}^{M} P\left[\boldsymbol{y}(n)|a_k \in \tilde{C};\boldsymbol{\theta}\right] P[a_k] \quad (30)$$

where $\tilde{C} = \{a_1, a_2, \ldots, a_M\}$ is the constellation and $\boldsymbol{\theta} = [S_1, S_2, \ldots, S_{N_a}, \sigma^2]^T$ is the vector of unknown parameters. Furthermore, it is easy to see that

$$P\left[\boldsymbol{y}(n)|a_k \in \tilde{C};\boldsymbol{\theta}\right] = \prod_{i=1}^{Na} \frac{1}{2\pi\sigma^2}\exp\left(-\frac{|y_i(n) - S_i a_k|^2}{2\sigma^2}\right). \quad (31)$$

A. Expectation Maximization (EM) Algorithm for Per-Antenna NDA ML SNR Estimation

Unfortunately, unlike the DA case, we were not able to derive a closed-form solution for the ML SNR estimator in the NDA case. This is basically due to the complexity of the likelihood function. However, it is well known from the open literature that the NDA ML estimates can be efficiently computed using the expectation-maximization (EM) algorithm [10]. Therefore, in the following, the NDA ML SNR estimator based on the EM algorithm is developed with a significantly lower computational load than other numerical NDA ML approaches. In fact, conditioned on an observation interval of N independent samples $\boldsymbol{y}(n)$ and an estimate $\widehat{\boldsymbol{\theta}}^{(p-1)}$ of the parameter vector $\boldsymbol{\theta}$ (computed in the step $p-1$ of the iterative procedure), the EM algorithm tries to find a more refined estimate $\widehat{\boldsymbol{\theta}}^{(p)}$ as detailed in the sequel.

Indeed, a_k can take one of M different values and, when they are assumed iid, we have $P[a_k \in \tilde{C}] = 1/M$. From (31), we can write

$$P\left[\boldsymbol{y}(n)|a_k \in \tilde{C};\boldsymbol{\theta}\right] = \prod_{i=1}^{Na} \frac{1}{2\pi\sigma^2} \times \exp\left(-\frac{|y_i(n)|^2 + S_i^2|a_k|^2 - 2S_i\Re\{y_i(n)^* a_k\}}{2\sigma^2}\right). \quad (32)$$

The expectation step (E-step) of the suggested EM algorithm is established as follows:

$$Q\left(\boldsymbol{\theta}|\widehat{\boldsymbol{\theta}}^{(p-1)}\right)=\sum_{n=1}^{N} E_a\left\{\ln\left(P\left[\boldsymbol{y}(n)|a_k \in \tilde{C};\boldsymbol{\theta}\right]\right)|\widehat{\boldsymbol{\theta}}^{(p-1)},\boldsymbol{y}(n)\right\} \quad (33)$$

where $E_a\{.\}$ denotes expectation with respect to $a \in \tilde{C}$. Moreover, from (32), the log-likelihood function is given by

$$\ln\left(P\left[\boldsymbol{y}(n)|a_k \in \tilde{C};\boldsymbol{\theta}\right]\right) = -N_a \ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^{N_a}\left(|y_i(n)|^2 + S_i^2|a_k|^2 - 2S_i\Re\{y_i(n)^* a_k\}\right). \quad (34)$$

Therefore, the E-step reduces simply to

$$Q\left(\boldsymbol{\theta}|\widehat{\boldsymbol{\theta}}^{(p-1)}\right) = -N_a N \ln(2\pi\sigma^2) - \frac{N}{2\sigma^2}\sum_{i=1}^{N_a}\left(M_2^i + S_i^2 A^{(p)} - 2S_i B_i^{(p)}\right) \quad (35)$$

where M_2^i is the second moment[3] of the received signal on the i^{th} antenna element, i.e., $M_2^i = E\{|y_i(n)|^2\}$, and $A^{(p-1)}$ and $B_i^{(p-1)}$ are defined as follows:

$$A^{(p)} = \frac{1}{N}\sum_{n=1}^{N}\sum_{k=1}^{M}|a_k|^2 P_{k,n}^{(p)} \quad (36)$$

$$B_i^{(p)} = \frac{1}{N}\sum_{n=1}^{N}\sum_{k=1}^{M}\Re\{y_i(n)^* a_k\} P_{k,n}^{(p)} \quad (37)$$

with

$$P_{k,n}^{(p)} = P\left[a_k|\boldsymbol{y}(n);\widehat{\boldsymbol{\theta}}^{(p-1)}\right] = \frac{P\left[\boldsymbol{y}(n)|a_k;\widehat{\boldsymbol{\theta}}^{(p-1)}\right]P[a_k]}{P\left[\boldsymbol{y}(n);\widehat{\boldsymbol{\theta}}^{(p-1)}\right]} \quad (38)$$

where, using (30) and (32), $P[\boldsymbol{y}(n);\boldsymbol{\theta}]$ is given by

$$P[\boldsymbol{y}(n);\boldsymbol{\theta}] = \frac{1}{M}\sum_{k=1}^{M}\prod_{i=1}^{Na}\frac{1}{2\pi\sigma^2} \times \exp\left(-\frac{|y_i(n)|^2 + S_i^2|a_k|^2 - 2S_i\Re\{y_i(n)^* a_k\}}{2\sigma^2}\right). \quad (39)$$

[3]Note that, in practice, the second moment M_2^i is estimated using simple sample mean, i.e., $M_2^i = 1/N\sum_{n=1}^{N}|y_i(n)|^2$.

In words, the maximization step (M-step) of the EM algorithm is used to update the estimates. Therefore, maximizing (35) with respect to $\{S_i\}_{i=1}^{N_a}$ and σ^2, the estimates of the elements of the parameter vector at the p^{th} iteration, $\widehat{\boldsymbol{\theta}}^{(p)}$, are given by

$$\widehat{S}_i^{(p)} = \frac{B_i^{(p)}}{A^{(p)}}, \quad i = 1,2,\ldots,N_a \quad (40)$$

$$\widehat{\sigma^2}^{(p)} = \frac{1}{2N_a}\sum_{i=1}^{N_a}\left(M_2^i - \frac{B_i^{(p)^2}}{A^{(p)}}\right). \quad (41)$$

Actually, starting with $P_{k,n}^{(0)} = 1/M$ and assuming a symmetric constellation, it can be easily verified that the value of $B_i^{(0)} \to 0$ makes the EM approach fail. To circumvent this problem, we calculate $\{\widehat{S}_i^{(0)}\}_{i=1,2,\ldots N_a}$ and $\widehat{\sigma^2}^{(0)}$ in the initialization step differently from the ones calculated in the iteration steps. That is we take the absolute value of $\Re\{y_i(n)^* a_k\}$ so that $B_i^{(0)}$ will not approach zero. Using these initializations, we have observed convergence of the $N_a + 1$ parameter estimates in all cases tested.

In the next subsection, to assess the performance of the proposed iterative algorithm against the the fundamental limit, we introduce an algorithm which numerically finds the SNR CRLBs in the NDA scenario.

B. Derivation of NDA SNR CRLBs Over SIMO Channels

In this section, we derive the CRLB for NDA SNR estimates in SIMO systems. Contrarily to the DA case, a closed-form solution seems to be prohibitive due to the complexity of the log-likelihood function. A simplified numerical evaluation is, therefore, derived and an efficient algorithm is presented for application to any linearly modulated signal in the presence of AWGN.

The CRLB on the i^{th} antenna element, similarly to (19), is given by

$$\text{CRLB}_{\text{NDA}}(\rho_i) = \left[\boldsymbol{I}_{\text{NDA}}^{-1}(\boldsymbol{\alpha})\right]_{i,i}, \quad i = 1,2,\ldots,N_a, \quad (42)$$

where $\boldsymbol{I}_{\text{NDA}}(\boldsymbol{\alpha})$ is the Fisher information matrix (FIM) that is obtained when the transmitted symbols are completely unknown to the receiver. Its entries are defined as:

$$[\boldsymbol{I}_{\text{NDA}}(\boldsymbol{\alpha})]_{ij} = \text{E}_{\boldsymbol{Y}}\left\{\frac{\partial\ln\left(P[\boldsymbol{Y};\boldsymbol{\alpha}]\right)}{\partial\alpha_i}\frac{\partial\ln\left(P[\boldsymbol{Y};\boldsymbol{\alpha}]\right)}{\partial\alpha_j}\right\}, \quad i,j = 1,2,\ldots,N_a+1. \quad (43)$$

Since the transmitted symbols are iid and the noise components are temporally white, the FIM is written as

$$[\boldsymbol{I}_{\text{NDA}}(\boldsymbol{\alpha})]_{ij} = N\,\text{E}_{\boldsymbol{y}}\left\{\frac{\partial\ln\left(P[\boldsymbol{y};\boldsymbol{\alpha}]\right)}{\partial\alpha_i}\frac{\partial\ln\left(P[\boldsymbol{y};\boldsymbol{\alpha}]\right)}{\partial\alpha_j}\right\}, \quad i,j = 1,2,\ldots,N_a+1 \quad (44)$$

where the time index n was intentionally dropped and $\boldsymbol{y}$ could be any of $\{\boldsymbol{y}(n)\}_{n=1}^{N}$, and $P[\boldsymbol{y};\boldsymbol{\alpha}]$ the probability distribution of $\boldsymbol{y}$ in the NDA case that is given by

$$P[\boldsymbol{y};\boldsymbol{\alpha}] = \frac{1}{M}\sum_{k=1}^{M}\prod_{i=1}^{N_a}\frac{1}{2\pi\sigma^2} \times \exp\left(-\frac{|y_i|^2}{2\sigma^2} - \rho_i|a_k|^2 + 2\sqrt{\frac{\rho_i}{2\sigma^2}}\Re\{y_i^* a_k\}\right). \quad (45)$$

Since $\mathrm{E}_{\boldsymbol{y}}\{.\}$ symbolizes the expectation with respect to the transmitted data and the additive noise, the expectation operation in (44) is given by

$$\mathrm{E}_{\boldsymbol{y}}\left\{\left(\frac{\partial \ln P[\boldsymbol{y};\boldsymbol{\alpha}]}{\partial \alpha_i}\right)\left(\frac{\partial \ln P[\boldsymbol{y};\boldsymbol{\alpha}]}{\partial \alpha_j}\right)\right\} = \mathrm{E}_{\tilde{C}}\left\{\int_{-\infty}^{+\infty} \frac{P_{\alpha_i}[\boldsymbol{y};\boldsymbol{\alpha}]P_{\alpha_j}[\boldsymbol{y};\boldsymbol{\alpha}]}{\left(P[\boldsymbol{y};\boldsymbol{\alpha}]\right)^2} P(\boldsymbol{\eta})d\boldsymbol{\eta}\right\} \quad (46)$$

where $\{P_{\alpha_i}[\boldsymbol{y};\boldsymbol{\alpha}]\}_{i=1}^{N_a}$ stands for the partial derivative of (45) with respect to α_i (the i^{th} element of $\boldsymbol{\alpha}$) and $\boldsymbol{\eta}$ is the $2N_a$-dimensional vector of the real and imaginary zero-mean noise samples defined as

$$\boldsymbol{\eta} = \left[\eta_1^I, \eta_1^Q, \ldots, \eta_{N_a}^I, \eta_{N_a}^Q\right] \quad (47)$$

where $\eta_i^I = \Re\{\eta_i\}$ and $\eta_i^Q = \Im\{\eta_i\}$. Then, plugging $|y_i|^2 = 2\sigma^2|\sqrt{\rho_i}a + \eta_i|^2$ and $\Re\{y_i^* a_k\} = \sqrt{2\sigma^2}\Re\{(\sqrt{\rho_i}a + \eta_i)a_k^*\}$ in the expression of these partial derivatives, we obtain the following results:

$$P_{\rho_i}[\boldsymbol{y};\boldsymbol{\alpha}] = \frac{1}{M}\sum_{k=1}^{M}\left(\frac{\Re\left\{(\sqrt{\rho_i}a+\eta_i)a_k^*\right\}}{\sqrt{\rho_i}} - |a_k|^2\right)\mathrm{C}_k, \quad i = 1, 2, \cdots, N_a \quad (48)$$

$$P_{\sigma^2}[\boldsymbol{y};\boldsymbol{\alpha}] = \frac{1}{M}\sum_{k=1}^{M}\sum_{i=1}^{N_a}\left(-\frac{1}{\sigma^2} + \frac{|\sqrt{\rho_i}a+\eta_i|^2}{\sigma^2} - \sqrt{\rho_i}\frac{\Re\left\{(\sqrt{\rho_i}a+\eta_i)a_k^*\right\}}{\sigma^2}\right) \times \mathrm{C}_k \quad (49)$$

where

$$\mathrm{C}_k = \prod_{i=1}^{N_a}\frac{1}{2\pi\sigma^2}\exp\left(-|\sqrt{\rho_i}a+\eta_i|^2 - \rho_i|a_k|^2 + 2\sqrt{\rho_i}\Re\left\{(\sqrt{\rho_i}a+\eta_i)a_k^*\right\}\right). \quad (50)$$

Observe here that $P[\boldsymbol{y};\boldsymbol{\alpha}] = (1/M)\sum_{k=1}^{M}\mathrm{C}_k$ and, therefore, the noise power $2\sigma^2$ cancels out in the inverse of $\boldsymbol{I}_{\mathrm{NDA}}(\boldsymbol{\alpha})$ and the CRLB emerges only as a function of $\{\rho_i\}_{i=1,2,\ldots,N_a}$ and the used modulation scheme. Moreover, the probability density function of the noise components, $\boldsymbol{\eta}$, over which (46) must be averaged is given by

$$P[\boldsymbol{\eta}] = \frac{1}{\pi^{N_a}}\prod_{i=1}^{N_a}\exp\left(-\left(\eta_i^I+\eta_i^Q\right)^2\right). \quad (51)$$

Then, denoting the integrand function in (46) by $\boldsymbol{\psi}(a, \{n_i^I, \eta_i^Q\}_{i=1,2,\ldots,N_a})$, one can write

$$\mathrm{E}_{\boldsymbol{y}}\{\boldsymbol{\psi}\} = \frac{1}{M\pi^{N_a}}\sum_{a\in\tilde{C}}\int\cdots\int_{-\infty}^{+\infty}\boldsymbol{\psi}\left(a, \left\{n_i^I, \eta_i^Q\right\}_{i=1,2,\ldots,N_a}\right) \times \prod_{i=1}^{N_a} e^{-\left(\eta_i^I+\eta_i^Q\right)^2} d\eta_1^I\, d\eta_1^Q \ldots d\eta_{N_a}^I\, d\eta_{N_a}^Q. \quad (52)$$

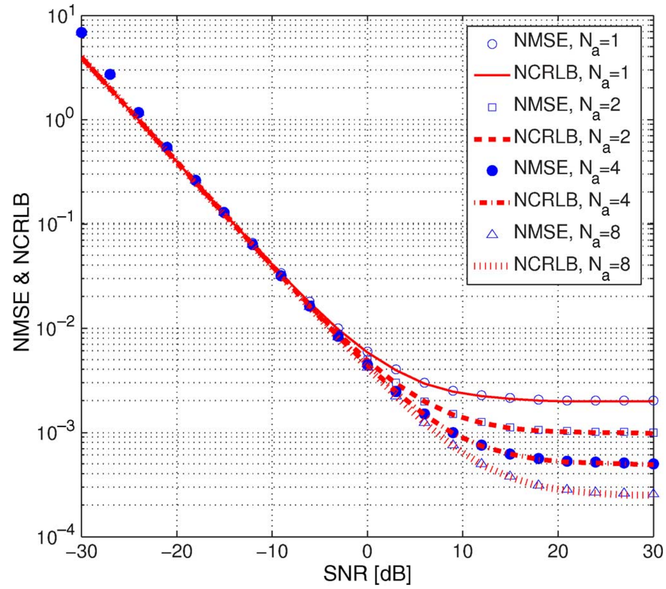

Fig. 1. NMSE and NCRLB of the DA SNR estimates for different numbers of antennas $N = 512$.

As aforementioned, it seems that no closed-form solution is available for this problem and a numerical approach may be envisaged. It is well known from the open literature that Gauss-Hermite quadrature rules [14] are designed to approximate these types of integral over the infinite interval $]-\infty, +\infty[$. To compute the NDA SNR CRLBs, these integrals were, therefore, numerically evaluated following the approach introduced in [14].

V. Simulation Results

In this section, we will assess the performance of our SIMO DA and NDA maximum-likelihood per-antenna SNR estimators. We will begin by the DA mode. First, we mention that, since the bias of the biased DA estimator is sufficiently negligible, then the biased and unbiased versions of the DA ML estimator exhibit the same performance in terms of their NMSEs.

In Fig. 1, the theoretical[4] NMSE of the unbiased DA estimator is compared to its normalized CRLB (NCRLB), i.e., $\mathrm{NCRLB}_{\mathrm{DA}}(\rho_i) = \mathrm{CRLB}_{\mathrm{DA}}(\rho_i)/\rho_i^2$. We see from this figure that the use of a receiving antenna array improves the achievable performance of the ML SNR estimator over the practical SNR range of 0 to 30 dB. At high SNR values, the NCRLB asymptotically saturates at $1/N_aN$ and hence decreases linearly with an increasing number of antennas. We also see that the performance of the estimator reaches its CRLB over a wide SNR range of -20 to 30 dB, and that it slightly deviates from it only for extremely low SNR values below -20 dB. Hence, the derived DA ML estimator is efficient over a wide SNR range.

We now consider the NDA mode. Unfortunately, unlike the DA ML estimator, we were not able to derive closed-form expressions for the bias and the variance of the EM-based NDA ML estimator. Therefore, Monte Carlo simulations were run

[4]We have been able to verify that the NMSE curves obtained both in closed form (i.e., theoretical) and by simulations coincide perfectly. Here in the DA case, we only plot the theoretical curves to avoid encumbering figures unnecessarily.

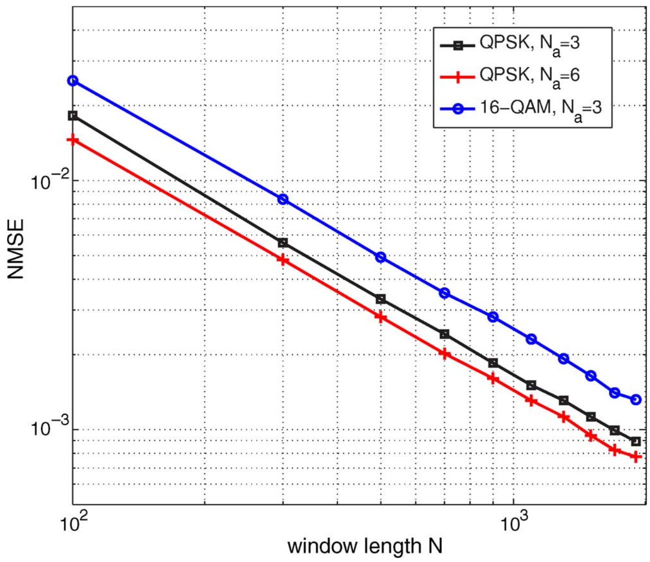

Fig. 2. NMSE of the NDA SNR estimates versus the observation window length N for different modulations and numbers of antennas $\mathrm{SNR} = 2\ \mathrm{dB}$.

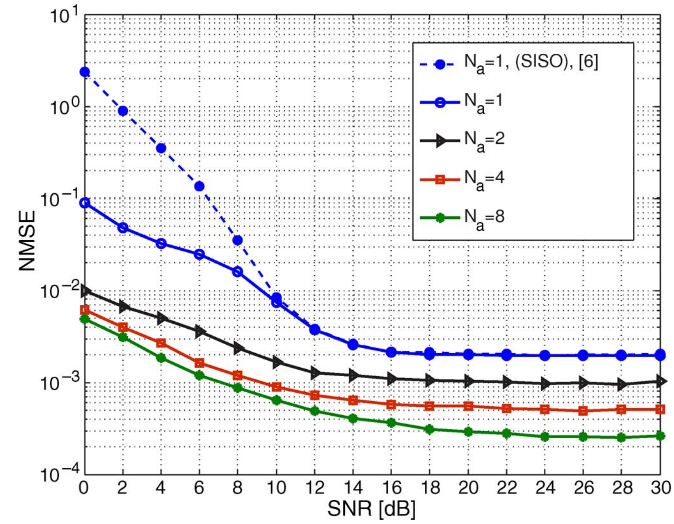

Fig. 3. NMSE of the NDA SNR estimates for different numbers of antennas, $N = 512$, 16-QAM.

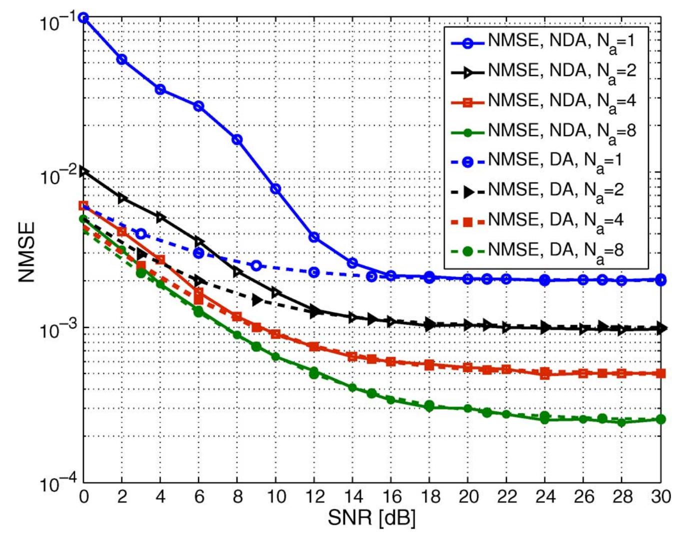

Fig. 4. NMSE of both the DA and NDA SNR estimators for different numbers of antennas, $N = 512$, 16-QAM.

over 5000 realizations to empirically compute its NMSE. In the following, the EM-based NDA ML estimator will be evaluated for two constellations, QPSK and 16-QAM, to serve as representative examples of constant- and nonconstant-modulus constellations, respectively. Note also that in Figs. 2to 6, we considered a scenario where the modulated received signal components are of equal instantaneous power at all antenna elements over the estimation interval. Therefore, the antenna branches are assumed to exhibit the same SNR.

In Fig. 2, we focus on the impact of varying the observation window length N on the performance of the proposed SIMO NDA maximum-likelihood SNR estimator. It can be seen that this estimator provides sufficiently accurate SNR estimates even with a few samples received at low SNR values (e.g., 2 dB). Despite the fact that increasing N translates into a linear increase in the estimation accuracy,[5] the relative performance improvement with respect to N for the different considered antenna-array sizes and modulation orders is the same. Consequently, from now on, any conclusion on the relative performance of the estimator will be assumed to hold regardless of the observation interval length (this was validated empirically from multiple simulations).

Fig. 3 depicts the NMSE of the proposed SIMO NDA ML estimator for 16-QAM constellation. For $N_a = 1$, note that our SIMO per-antenna estimator reduces simply to a SISO NDA ML estimator. It should be noted that the SISO NDA ML solution recently developed in [6] is based on an approximation which is valid only for sufficiently high SNR values. In contrast, our SIMO NDA ML estimator was derived without any restrictive assumption. Thus, as expected, even with a single receiving antenna (i.e., $N_a = 1$), it outperforms the one introduced in [6] in the low SNR region, as shown in Fig. 3. Using more receiving antennas further improves the achievable performance of the ML SNR estimator over the entire SNR range. This is due to the optimal exploitation of the statistical dependence provided by SIMO channels.

In Fig. 4, we compare the performance behavior of the proposed DA and NDA ML estimators for different antenna-array sizes (i.e., $N_a = 1$, 2, 4, 8). At relatively high SNR values, it can be seen that both estimators perform nearly the same. This means that in this SNR region, the NDA ML estimator exhibits accuracy levels equivalent to those that can be achieved if the transmitted data were perfectly known at the receiver. This advantage can be even preserved at low SNR values by increasing the receiving antenna-array size. Indeed, the two estimators tend to exhibit the same performance in the low SNR region as the number of receiving antenna elements increases. This is due to the optimal exploitation of the increasing mutual information across the antenna branches that renders the advantage of the DA version over its NDA counterpart relatively negligible.

[5]In the DA case, this property was proved theoretically in (27).

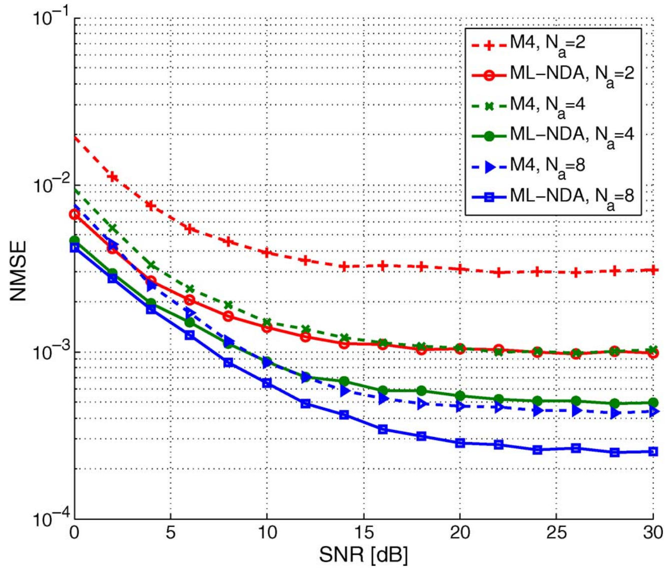

Fig. 5. NMSE of the NDA ML and the M_4 SNR estimates for different numbers of antennas, $N = 512$, QPSK.

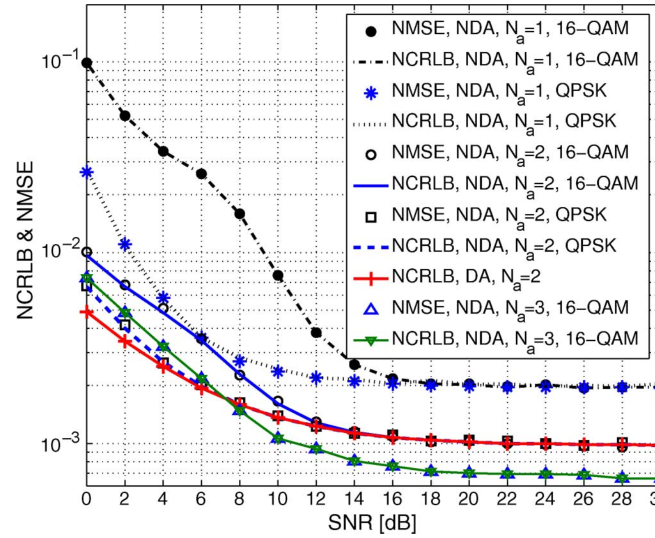

Fig. 6. NMSE and NCRLB of the NDA SNR estimates for different numbers of antennas, $N = 512$, QPSK, 16-QAM.

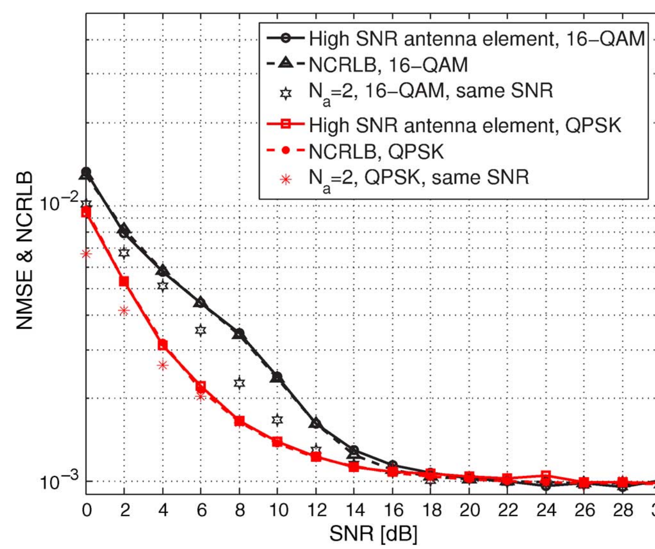

Fig. 7. NMSE of the NDA SNR estimates when the antenna elements experience different SNRs, $N_a = 2$, QPSK, 16-QAM.

Fig. 5 depicts the NMSE of both the NDA ML estimator and the recently derived moment-based M_4 NDA SIMO SNR estimator, for a QPSK constellation and different antenna-array sizes (i.e., $N_a = 2{,}4{,}8$). The M_4 estimator was shown in [11] to outperform current state-of-the-art techniques. For a given N_a, however, the EM-based ML estimator outperforms the M_4 estimator over the entire SNR range. Actually its performance is approximately equivalent to the one that can be achieved by the M_4 estimator but with $2N_a$ receiving antenna branches. Therefore, our I/Q-based ML estimator lends itself to integration into more compact antenna-array configurations. This is hardly surprising, since ML-based approaches yield in general better performance than moment-based approaches because the former do not discard any information about the received signal phase. Hence, relatively less mutual spatial information from smaller antenna arrays is required in order to achieve the same performance.

In Fig. 6, we compare the NMSE of the NDA ML estimator with the achievable fundamental limits, i.e., the NCRLBs in both the NDA and the DA cases over SIMO channels. First we notice that the NDA ML estimator also turns out to reach the corresponding CRLB over the entire practical SNR region. Moreover, we can easily see that the NDA CRLB approaches the DA CRLB at higher SNR, more so as the modulation order decreases. On the other hand, the DA CRLB is lower than the NDA CRLB in the low SNR region from the perfect *a priori* knowledge of the transmitted symbols. On the other hand, at high SNR values, following equivalent derivation lines as done in [9], it can be shown that the two CRLBs attain the same value asymptotically as the SNR goes to infinity.

We now consider the more general case in which the modulated signal components are of unequal average power over the different antenna elements and hence do not exhibit the same SNR. The SIMO channel is, however, still assumed time-invariant over the observation interval. Considering only two receiving antenna branches, our next scenario supposes that the second antenna element has 15 dB lower SNR than the SNR experienced on the first one. Fig. 7 illustrates the NCRLB and the estimated NMSE of the NDA ML estimator on the antenna element that experiences high SNR. It shows that the NDA estimator reaches its CRLB over the entire SNR range. Contrarily to the DA case, where the SNR accuracy on a given antenna branch holds irrespective of the SNR level on the other antenna elements [see (16), (17), and (27) where both the DA NMSE and DA NCRLB over a given antenna depend only on the SNR level at that branch], the performance of the NDA SNR estimator on a given antenna element depends on the SNR level experienced on the other antenna branches. This is due to the contribution of the estimate, in the E-step, of the transmitted symbols, associated with the low SNR antenna element which are inaccurate compared to those estimated from the high SNR antenna branch.

Fig. 8 depicts a 2-D plot of the NDA CRLBs in a SIMO system with two receiving antenna branches as function of the

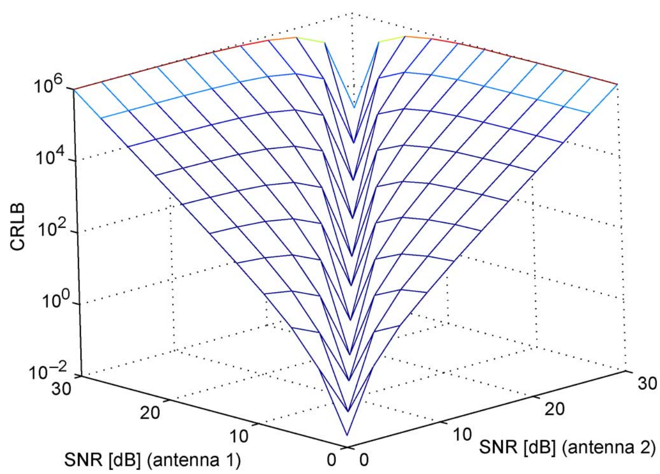

Fig. 8. CRLB in 2-D plot as function of ρ_1 and ρ_2 for a two-antenna SIMO system.

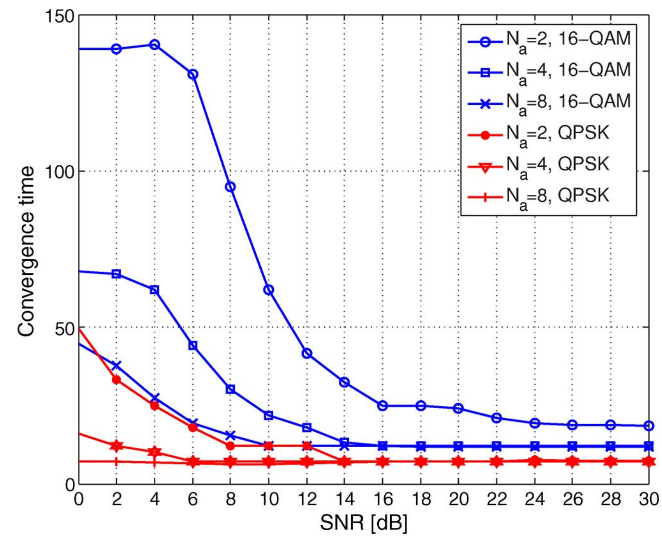

Fig. 9. Convergence time (in average iterations number) of the NDA SNR estimator for different numbers of antennas $N = 512$.

SNRs. It is seen that the CRLB sharply increases as soon as the two SNRs become different. This means that it is minimal when the experienced SNR is the same across the two receiving antenna elements. Actually, the fact that the unknown parameters to be estimated are different or the same affects the achievable performance only when these parameters are coupled as assumed in our case. In fact, the different SNRs are coupled, in the NDA case, since the off-diagonal elements of the NDA FIM are not equal to zero, contrarily to the DA case. However, when the parameters are decoupled, the FIM is diagonal and the estimation problems associated with each one of them become disjoint. The achievable performance holds, therefore, irrespectively of the unknown parameters number, i.e., the *a priori* knowledge of a given parameter does not bring any additional information about the others.

Note also that even if the SNR goes to zero over a given antenna element, this does not mean that this antenna element is useless. This is because noise-only measurments are experienced on this antenna branch and this brings information about the unknown noise variance assumed the same accross the two antenna elements. This affects in turn the achievable performance on SNR estimation over the active antenna element (higher-SNR antenna branch).

Finally, we focus on the complexity of our EM-based NDA ML estimator that depends in large part on its speed of convergence, i.e., the number of iterations required by the EM algorithm to converge. In fact, we decide that the EM algorithm has reached its steady state (or converged to the ML solution) when the absolute value of the difference between the SNR estimate in the p^{th} iteration and its estimate in the $(p-1)^{\text{th}}$ iteration is inferior to a sufficiently small value. In other words, the algorithms converges to the ML solution when $|\hat{\rho}_i^{(p)}\,(\text{dB}) - \hat{\rho}_i^{(p-1)}\,(\text{dB})| < \epsilon\;(\text{dB})$. In this context, Fig. 9 plots the average number of iterations required by the EM algorithm to reach convergence (i.e., convergence time) for different antenna-array sizes when ϵ is set[6] to 0.01 dB. We see clearly the advantage of SIMO configurations in reducing convergence time to obtain the ML solution by increasing the number of receiving antenna elements.

[6]It was verified by simulations that $\epsilon = 0.01$ dB provides best tradeoff between accuracy and complexity (i.e., for smaller ϵ, the required iterations number increases without any noticeable improvement in performance, and larger ϵ degrades seriously the performance of the NDA estimator).

VI. Conclusion

In this paper, we derived new DA ML and EM-based NDA ML SNR estimators over SIMO channels from any linear modulation. The ML DA solution was derived in closed form and the NDA ML estimate was iteratively computed. It was shown that these new SIMO SNR estimators achieve near-optimal accuracy over a wide SNR range. Both our EM iterative algorithm and ML DA solution assume that the noise components on all the antenna elements can be adequately modelled by complex Gaussian variables temporally and spatially white. The bias and the variance of the DA estimator were analytically derived in closed form, thereby showing that increasing the number of receiving antenna branches or window length can improve the performance of the DA estimator. The CRLBs for per-antenna SNR estimates from linearly modulated signals, in SIMO configurations, were also derived in closed form in the DA case and numerically computed in the NDA mode. The proposed DA and NDA SIMO SNR estimators were shown to reach their CRLBs and to exhibit remarkable performance improvements over the SISO ML SNR estimator recently derived in [6]. For relatively high SNR values, our new NDA ML SNR estimator exhibits performances equivalent to those achieved if the transmitted data were perfectly known. Furthermore, the use of I/Q-based ML estimators can lead to remarkable improvements over moment-based estimators and allows reduction of the required number of antenna elements by half. Moreover, the use of SIMO configurations can contribute to decreasing the required number of iterations in the EM algorithm and therefore to reducing its complexity. To the best of our knowledge, we are the first to derive ML per-antenna SNR estimators as well as their CRLBs both in the NDA and the DA cases over SIMO channels.

Appendix A
Derivation of the Exact Bias and Variance of the DA ML SNR Estimator

In order to analytically determine the statistical properties of $\widehat{\rho}_i$, its pdf must be found. Consider first the pdf of $\widehat{S}_i$ for which, using (2) and (9) and recalling that $(1/N)\sum_{n=1}^{N}|a(n)|^2 = 1$ since the constellation is of normalized energy, we find

$$\widehat{S}_i - S_i = \frac{\sqrt{2\sigma^2}}{N}\sum_{n=1}^{N}\Re\left\{\eta_i(n)^* a(n)\right\}. \tag{53}$$

Since $\widehat{S}_i - S_i$ is Gaussian with

$$E\{\widehat{S}_i - S_i\} = 0, \quad \mathrm{Var}\{\widehat{S}_i - S_i\} = \frac{\sigma^2}{N} \tag{54}$$

the pdf of $(N/\sigma^2)(\widehat{S}_i - S_i)^2$ is central chi-square [12] with one degree of freedom. Therefore $(N/\sigma^2)\widehat{S}_i^{\,2}$ has a noncentral chi-square distribution with one degree of freedom ($v_1 = 1$) and noncentrality parameter $\lambda = 2N\rho_i$. It will be denoted by $\chi^2_{v_1}$.

Now consider the distribution of $\widehat{2\sigma^2}$. We have

$$\begin{aligned}\widehat{2\sigma^2} &= \frac{1}{NN_a}\sum_{i=1}^{N_a}\left(\sum_{n=1}^{N}\left|y_i(n) - \widehat{S}_i a(n)\right|^2\right)\\ &= \frac{1}{NN_a}\sum_{i=1}^{N_a}\left(\sum_{n=1}^{N}\left(\Re\{y_i(n)\} - \widehat{S}_i\Re\{a(n)\}\right)^2\right.\\ &\qquad\left. + \sum_{n=1}^{N}\left(\Im\{y_i(n)\} - \widehat{S}_i\Im\{a(n)\}\right)^2\right)\end{aligned} \tag{55}$$

where $\Im\{.\}$ stands for the imaginary part of any complex number. It is easy to verify that $\mathrm{E}\{\Re\{y_i(n)\} - \widehat{S}_i\Re\{a(n)\}\} = 0$ and $\mathrm{Var}\{\Re\{y_i(n)\} - \widehat{S}_i\Re\{a(n)\}\} = \sigma^2$. Hence, the PDFs of both $\sum_{n=1}^{N}((\Re\{y_i(n)\} - \widehat{S}_i\Re\{a(n)\})^2/\sigma^2)$ and $\sum_{n=1}^{N}((\Im\{y_i(n)\} - \widehat{S}_i\Im\{a(n)\})^2/\sigma^2)$ are chi-square distributions with $(N-1)$ degrees of freedom. From the reproductive property of the chi-square distribution [13], we deduce that the PDF of $(NN_a/\sigma^2)\widehat{2\sigma^2}$ is a central chi-square with $v_2 = 2N_a(N-1)$ degrees of freedom denoted by $\chi^2_{v_2}$.

Consequently, $\widehat{\rho}_i$ is a scaled noncentral F-distributed variable [12] with

$$F = \frac{\frac{\chi^2_{v_1}}{v_1}}{\frac{\chi^2_{v_2}}{v_2}} = 2(N-1)\widehat{\rho}_i. \tag{56}$$

F is a noncentral F-distribution with v_1 and v_2 degrees of freedom and λ as a noncentrality parameter. Its mean E_F and variance Var_F are given by [12]

$$\mathrm{E}_F = \frac{v_2(v_1+\lambda)}{v_1(v_2-2)}, \quad v_2 > 2 \tag{57}$$

$$\mathrm{Var}_F = 2\left(\frac{v_2}{v_1}\right)^2\frac{(v_1+\lambda)^2 + (v_1+2\lambda)(v_2-2)}{(v_2-2)^2(v_2-4)}, \quad v_2 > 4. \tag{58}$$

Using (56)–(58) and substituting v_1 and v_2, it follows:

$$\begin{aligned}\mathrm{E}\{\widehat{\rho}_i\} &= \mathrm{E}\left\{\frac{F}{2(N-1)}\right\}\\ &= \frac{N_a N}{N_a(N-1)-1}\left(\rho_i + \frac{1}{2N}\right)\end{aligned} \tag{59}$$

$$\begin{aligned}\mathrm{Var}\{\widehat{\rho}_i\} &= \frac{N_a^2N^2}{\left(N_a(N-1)-1\right)^2\left(N_a(N-1)-2\right)}\\ &\quad\times\left(\rho_i^2 + \rho_i\left(\frac{2N_a(N-1)}{N} - \frac{1}{N}\right)\right.\\ &\quad\left. + \frac{N_a(N-1)}{2N^2} - \frac{1}{4N^2}\right).\end{aligned} \tag{60}$$

Appendix B
Proof of (27)

To derive a closed-form expression for the DA CRLB, we establish the inverse, $\boldsymbol{I}_{\mathrm{DA}}^{-1}(\boldsymbol{\alpha})$, of $\boldsymbol{I}_{\mathrm{DA}}(\boldsymbol{\alpha})$ as follows: [see (61) at the bottom of the page]. Hence, according to (19), the DA SNR CRLB on the i^{th} antenna element is derived as

$$\mathrm{CRLB}_{\mathrm{DA}}(\rho_i) = \frac{\frac{c}{a_i} - \sum\limits_{\substack{l=1\\ l\neq i}}^{N_a}\frac{b_l^2}{a_i a_l}}{c - \sum\limits_{l=1}^{N_a}\frac{b_l^2}{a_l}}. \tag{62}$$

$$\boldsymbol{I}_{\mathrm{DA}}^{-1}(\boldsymbol{\alpha}) = \frac{1}{c - \sum\limits_{l=1}^{N_a}\frac{b_l^2}{a_l}}\begin{pmatrix} \frac{c}{a_1} - \sum\limits_{\substack{l=1\\ l\neq 1}}^{N_a}\frac{b_l^2}{a_1a_l} & \frac{b_1b_2}{a_1a_2} & \cdots & \frac{b_1b_{N_a}}{a_1a_{N_a}} & -\frac{b_1}{a_1}\\ \frac{b_2b_1}{a_2a_1} & \frac{c}{a_2} - \sum\limits_{\substack{l=1\\ l\neq 2}}^{N_a}\frac{b_l^2}{a_2a_l} & \ddots & \vdots & -\frac{b_2}{a_2}\\ \vdots & \ddots & \ddots & \frac{b_{N_a-1}b_{N_a}}{a_{N_a-1}a_{N_a}} & \vdots\\ \frac{b_{N_a}b_1}{a_{N_a}a_1} & \cdots & \frac{b_{N_a}b_{N_a-1}}{a_{N_a}a_{N_a-1}} & \frac{c}{a_{N_a}} - \sum\limits_{\substack{l=1\\ l\neq N_a}}^{N_a}\frac{b_l^2}{a_{N_a}a_l} & -\frac{b_{N_a}}{a_{N_a}}\\ -\frac{b_1}{a_1} & -\frac{b_2}{a_2} & \cdots & -\frac{b_{N_a}}{a_{N_a}} & 1 \end{pmatrix}. \tag{61}$$

The required partial derivatives of (25) involved in the expressions of c, $\{a_i\}_{i=1}^{N_a}$, and $\{b_i\}_{i=1}^{N_a}$ are given by

$$\frac{\partial^2 \ln\left(P[\boldsymbol{Y};\boldsymbol{\alpha}]\right)}{\partial {\sigma^2}^2} = \frac{NN_a}{\sigma^4} - \sum_{i=1}^{N_a}\sum_{n=1}^{N}\frac{|y_i(n)|^2}{\sigma^6} + \sum_{i=1}^{N_a}\sum_{n=1}^{N}\sqrt{\frac{\rho_i}{2}}\frac{3}{2\sigma^5}C_i(n) \quad (63)$$

$$\frac{\partial^2 \ln\left(P[\boldsymbol{Y};\boldsymbol{\alpha}]\right)}{\partial {\rho_i}^2} = -\sum_{n=1}^{N}\frac{C_i(n)}{2\sqrt{2\sigma^2\rho_i^3}}, \quad i = 1,2\ldots,N_a \quad (64)$$

$$\frac{\partial^2 \ln\left(P[\boldsymbol{Y};\boldsymbol{\alpha}]\right)}{\partial \rho_i \partial \sigma^2} = -\sum_{n=1}^{N}\frac{C_i(n)}{2\sqrt{2\rho_i}\sigma^3}, \quad i = 1,2\ldots,N_a \quad (65)$$

where $\{C_i(n) = \Re\{y_i(n)^* a(n)\}\}_{i=1}^{N_a}$. Then, we show that the expected values of the partial derivatives are given by

$$E\left\{\frac{\partial^2 \ln\left(P[\boldsymbol{Y};\boldsymbol{\alpha}]\right)}{\partial {\sigma^2}^2}\right\} = -\sum_{i=1}^{N_a}\frac{N(2+\rho_i)}{2\sigma^4} \quad (66)$$

$$E\left\{\frac{\partial^2 \ln\left(P[\boldsymbol{Y};\boldsymbol{\alpha}]\right)}{\partial {\rho_i}^2}\right\} = -\frac{N}{2\rho_i}, \quad i=1,2,\ldots,N_a \quad (67)$$

$$E\left\{\frac{\partial^2 \ln\left(P[\boldsymbol{Y};\boldsymbol{\alpha}]\right)}{\partial \rho_i \partial \sigma^2}\right\} = -\frac{N}{2\sigma^2}, \quad i=1,2,\ldots,N_a. \quad (68)$$

Finally, injecting (66)–(68) in (62) yields the closed-form CRLB for the DA SNR estimates as follows:

$$\mathrm{CRLB}_{\mathrm{DA}}(\rho_i) = \frac{\rho_i^2 + 2N_a\rho_i}{N_a N}, \quad i = 1,2,\ldots N_a. \quad (69)$$

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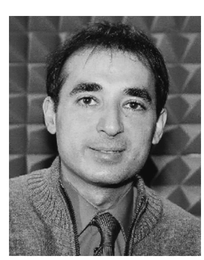

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